LECTURE ON CARTEL AND COLLUSION

November 13, 2012

Definition of cartel: A cartel is a group of firms that jointly decide on prices and/or quantities and then try to enforce this decision.

- A cartel typically works by raising prices above some competitive or Cournot level. Because of this, the individual member of the cartel have an incentive to raise their production levels above their quota. By doing this, they are guaranteed the relatively high cartel prices, but higher than cartel quantities. This imposes an externality on the other cartel members.

- The biggest problem for legal cartel is to prevent cheating. Each individual member has strong incentives to cheat. There are several potential mechanisms by which cartel can deter cheating. The best of available is to "eliminate" the cheaters. But this isn’t often available. Generally, then the cartel can force a member out of the cartel but this isn’t necessarily helpful, or try some other actions. But the punishment that is most typically used is to raise quantities. E.g., Saudi Arabia will raise quantities in response to cheating by other OPEC members.

Unfortunately this is a costly punishments. Saudi Arabia and all OPEC members are hurt by the lower resulting prices.

- The difference between cartel and collusion- the former has more of an active group who meets regularly. Cartel doesn’t happen often since cartels are illegal in most countries. Instead, what is more typical is that firms will collude with many members informally being part of the collusion at one time or another. This is roughly what happened in the LCE panel industry.

- Applying game theory to analyze sustaining cartel using repeated game: Consider the 2-firm Cournot model where we’ll only look at 2 strategies,

  (I) \( q^M \) = monopoly quantity and
  (II) \( q^C \) = Cournot quantity

Let’s consider the example earlier where demand is \( Q(P) = 10 - P \) and \( TC(q) = 4q \). We worked out earlier that monopoly quantity is \( Q = 3 \) which we’ll interpret as \( q_1 = q_2 = \frac{3}{2} \) and Cournot quantities are: \( q_1 = q_2 = 2 \). Let’s write down the profits for this \( 2 \times 2 \) game:

<table>
<thead>
<tr>
<th>Player 2</th>
<th>( q_c )</th>
<th>( q_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_c )</td>
<td>(4,4)</td>
<td>(5,\frac{15}{4})</td>
</tr>
<tr>
<td>( q_M )</td>
<td>(\frac{15}{4},5)</td>
<td>(\frac{9}{2},\frac{9}{2})</td>
</tr>
</tbody>
</table>

In the monopoly, \( P=7 \) so profit for each firm is

\[ \Pi_1 = (P - AC) \times q_1 \]

Note that the only nash equilibrium of this game is \((q_c, q_c)\). In fact this game is the prisoner’s dilemma game with slightly different payoff from earlier. So, one possibility is that we can sustain collusion by playing this game repeatedly, which is the prisoners dilemma again.
Last class, we went through the 2 period version of the production game. We found that there was no possibility for collusion in the second period.

The reason for this is that there is no future, staring in the second period. Thus, going back to the first period, no matter what I do now, the only Nash equilibrium starting in the second period is to not collude. Again then, there is no relevant future in the first period. Thus we should always play $q_c$. Implicitly what we are doing here is looking at NE starting at each period. This is called the **subgame perfect equilibrium**. You are not responsible for knowing the formal definition here.

We can take this same argument and show that in any game of finite length, there is no possibility of collusion. Specifically in the last period, collusion can’t happen as before. Now going to the second to the last period, we again find that collusion can’t happen, etc. We can extend this to the first period and find that collusion is not a **subgame perfect equilibrium**.

Let’s now consider infinite horizon case.

- **Discount factor**: The discount factor specifies the current value of earning $1 next period. E.g. suppose the discount factor is $\beta = 0.8$. Then what is the present value of $(q_c, q_c)$ forever? In the first period, player 1 earns a present value of 4.
  
  In the second period, player 1 earns a present value of $4 \times 0.8 = 3.2$
  
  In the third period, player 1 earns a present value of $4 \times 0.8 \times 0.8 = 2.56$

  Adding it all up, player 1 gets a present value payoff of: $4(1+\beta+\beta^2+\beta^3+\beta^4+\ldots) = \frac{4}{1-\beta} = \frac{4}{0.2}$.

  In finance, this is called a perpetuity.

- **Punishment strategies**

  When this game is infinitely repeated, then it may be possible to sustain collusion. To understood whether this is possible, we need to consider punishment strategies. There are two commonly discussed punishment strategies.

  (I) **Grim trigger**

  Play $q^m$ unless someone has play $q^c$ in this current period. In this case, pull the ”trigger” and play $q^c$ forever.

  (II) **Tic-for-tat**

  Play $q^m$ unless your rival plays $q^c$. At this point, play $q^c$ until your rival goes back to play $q^m$ after which point you switch back.

  (III) Now that we’ve proposed these strategies, let’s see when these strategies will actually be subgame perfect equilibria of the game.

  Let’s consider first grim trigger. There are two possible information sets:

  (a) No one has played $q^c$ in the past.

  (b) Some has played $q^c$ in the past.

  • Let’s verify the optimality of my strategy in both cases. Start with case (2): In these states, both firms are supposed to play $q^c$. If one firm unilaterally deviates, it will reduce its payoff from 4 to $3\frac{3}{4}$. This will not affect the future actions of its rival, since its rival is committed to play $q^c$ forever after, in this case no unilateral deviation is profitable.

  • Now let’s consider case 1. In this case, I get $4\frac{1}{2}$ now, $4\frac{1}{2}$ next period, etc if I don’t deviate. The present value of this is $\frac{4.5}{1-\beta}$. Suppose instead that I deviate and play $q^c$. In this case I earn 5 in the first period and 4 in the following periods

  $$5 + \frac{4}{1-\beta} \beta$$

  For the strategy profile $\{(q^m, q^m), (q^m, q^m), \ldots\}$ that we analyzed to be subgame perfect equilibrium, the value of not deviating from it must be at least as high as the value of deviating from it.

  $$\frac{4.5}{1-\beta} \geq 5 + \frac{4}{1-\beta} \beta$$

  Prepared by Chrystie Burr
Let’s solve for when this occurs.

\[
\Rightarrow \frac{4.5}{1-\beta} \geq \frac{5(1-\beta) + 4\beta}{1-\beta}
\]
\[
\Rightarrow \frac{4}{2} \geq 5(1-\beta) + 4\beta
\]
\[
\Rightarrow \frac{4}{2} \geq 5 - 5\beta + 4\beta
\]
\[
\Rightarrow \beta \geq 0.5
\]

What have we learned?

- Collusion can be sustained in the infinitely repeated game provided that the discount factor is sufficiently high. Alternately put, collusion can be sustained provided that the firms play the game pretty frequently.

- Let’s now consider tit-for-tac. I play \(q^c\) this period and then play \(q^m\) next period. Then play \(q^m\) after that.

In that case, I get \(5 + \beta \frac{3}{4} + \beta^2 \frac{4}{1-\beta}\)

For the strategy profile \(\{(q^m, q^m), (q^m, q^m)\ldots\}\) that we analyzed to be subgame perfect equilibrium, the value of not deviating from it must be at least as high as the value of deviating from it.

\[
\frac{4.5}{1-\beta} \geq 5 + \beta \frac{3}{4} + \beta^2 \frac{4}{1-\beta}
\]
\[
\Rightarrow \frac{4}{2} + \beta \frac{1}{2} \geq 5 + \beta \frac{3}{4}
\]
\[
\Rightarrow \frac{3}{4} \beta \geq \frac{1}{2}
\]
\[
\Rightarrow \beta \geq \frac{4}{6} = \frac{2}{3}
\]

- We can see that collusion is now sustainable in less cases with tit-for-tac then with grim trigger only when \(\beta \geq \frac{2}{3}\) not \(\beta \geq \frac{1}{2}\)

- Counterintuitively in game theoretic model the ability to punish severely will generates higher payoff.