Evaluating Punishment on Recidivism: A Probationary Selection Model with Judge Assignment Randomization

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Abstract

The effect of punishment on subsequent criminal behavior is a crucial topic in law and economics. In evaluating the influence of punishment on recidivism, substantial research was conducted on active sentencing cases, whereas 87.39% of crimes fell into categories for which probation is an option. When the policy changes the punishment scheme, it may induce the judges to change the sentencing practices, and their responses will vary. In order to examine the total impact of policy changes on judges’ decisions and their implications on recidivism, it is essential to take into account how probationary decisions are made. We estimate the effect of comprehensive punishment on recidivism using a probationary selection model with judge randomization. In our model, the judges make various decisions, including whether or not to impose probation, its duration, and the (potential) length of the sentence. We illustrate how this can be done using North Carolina data from 1995 to 2010.

Keywords: Recidivism, Probation, Sentencing, Judge Randomization

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1 Introduction

Understanding the effect of a judge’s decisions on punishment on the subsequent criminal behavior of defendants is central to empirical law and economics. Incarceration, as the major method of punishment, causes numerous challenges, such as rising financial burdens and prison overcrowding, and as a result, there are growing calls for change. One tendency of the judicial system’s reform has been to allow for more probation cases. In our dataset, 87.39% of the cases are eligible for probation, and 82.63% are on probation. Although there are guidelines for probation eligible categories, the regulations for the duration of probation are not as clear as those for sentencing.

The earliest literature on the economics of punishment dates back to Becker (1968). Following this seminal paper, there are several empirical papers on the effect of punishment on recidivism. The inference method focuses on policy change. For instance, Kessler and Levitt (1999) used sentence enhancements from California’s Proposition 8, Abrams (2012) used sentence enhancements from different states’ add-on gun laws to analyze the deterrence effect of sentencing. Kuziemko (2013) used Georgia Parole Reform to analyze the different effects of parole and sentencing on recidivism. Recently, as the volume of micro-level case data has grown, an increasing number of articles have employed judge randomization to examine the causal effect of punishment on recidivism. For example, Mueller-Smith (2015); Zapryanova (2020); Kuziemko (2013) examined incarceration’s effect on recidivism using judge randomization as the exogenous variation. However, these papers either focus on punishment or probation, without considering both of them and the selection of probation status.

We begin by modeling the judge’s decision matrix. In our model, the judge makes a number of decisions, such as whether or not to give probation, how long probation will last, and whether or not the length of the sentence will be suspended. From the copious case documents and encounters with the defendants, judges are able to identify characteristics
that cannot be observed by econometricians, such as the defendants’ social ties and risk preferences. Beyond the literature that uses judges’ one-dimensional leniency level, defined as leave-one-out sentence length decisions as instrument variables (Dobbie et al., 2018; Mueller-Smith, 2015; Aizer and Doyle Jr, 2015), we use judges’ multidimensional leniency as the source of identification, including probationary propensity, active sentence length, suspended sentence length, and probationary length. Our model nests the case when there is no judge to decide on probation or the case where probationary or non-probationary cases has constant effects on recidivism.

To recover the correlation between different decisions and recidivism, and to recover the effect of punishment, we employ a multi-step estimate procedure. Our estimation equation contains the conditional expectation, likelihood function in many scenarios, and then combined with the estimated information the final identification can reach the level of no selection. Our estimation method can find out each judge’s recidivism effects, which lets us judge how well judges can find defendants who are unlikely to commit crimes again. On this basis, we can conduct a counterfactual analysis to determine which defenders will be affected if the punishment scheme changes, as well as how this will affect public safety in the long run.

We estimate the model using data from North Carolina. We found that active sentencing has relatively bigger effects on recidivism comparing with probation decisions. One year of active sentencing can reduce recidivism by twelve percent. In cases involving probation, the effects of suspended sentencing and probation length are similar in magnitude but have opposite signs. Suspended sentencing as a threat can reduce recidivism, whereas the length of probation increases recidivism, and their combination as a component decision offsets each other.
2 Related Literature

The relationship between punishment and recidivism is fundamental in law and economics. Numerous economics and sociology literature has written extensively on the topic. We contribute to this literature by introducing a selection model into the judicial decision-making process to investigate the impact of probation and sentencing decisions on recidivism. The sociology and criminology literature examined a variety of punishments, including jail, probation, and parole, separately (Petersilia and Reitz, 2012; Mears et al., 2016; Nagin et al., 2009; Harding et al., 2017). In contrast, economics research focuses on the causal relationship between punishment and recidivism, and until relatively recently, the majority of publications were limited to the impacts of incarceration. Mueller-Smith (2015) is a leading paper that assessed the influence of jail on recidivism using the Texas judge rotation. He studied the comprehensive consequences of judges’ decisions and demonstrated that multi-dimensional and non-monotonic punishment cause bias in the evaluation of the treatment effect, but he did not consider how endogenous selection procedure involved in judges’ decisions. Some other papers examined the conditions of incarceration (Chen and Shapiro, 2007), the effects of juvenile incarceration (Aizer and Doyle Jr, 2015) on recidivism. The decision of parole and the influence of parole term on recidivism is another important aspect of incarceration. Kuziemko (2013), for instance, investigated whether these parolees re-offend using a change in parole guidelines and found that parole boards set prison time in an efficient manner. Zapryanova (2020) studied the impact of parole duration using judge randomization and found that time on parole has no significant effect on recidivism. None of them incorporates the multiple decisions of the judges in estimating the effects of punishment on recidivism.

Rose and Shem-Tov (2021) and Rose (2021) are two recent papers that shed light to probationary cases. Using discontinuities in judges’ sentence tables, Rose and Shem-Tov (2021) evaluated the varied effects of different lengths of active sentencing on recidivism.
When defining recidivism, they evaluated probation decisions and found that risks of probation violations and offenses are independent. Rose (2021) studied the racial disparity in the probation judgment specifically and discovered that 40% of rule breakers would commit more offenses if spared harsh punishment. However, both papers focus solely on either incarceration or probation cases, ignoring the reality that judges may make endogenous choices when separating probationary cases. Our paper investigates how judges’ probationary decisions as well as probation and active and suspended sentencing lengths to estimate how those decisions affect recidivism.

Beyond judge randomization, the issue of endogenous selection may also occur in a broader examiner randomization context. For example, Doyle Jr et al. (2015) used randomization of ambulance referrals to examine if high-cost hospitals can produce superior health outcomes. The referral process for ambulances might impact hospital and physician assignment. In various random examiner scenarios, the examiner may also make multidimensional decisions or impact the outcome by several selections and those selections are not randomized. Only examining the influence of one choice may ignore a number of crucial policy perspectives. In this sense, our study provides a method for analyzing how the random examiner’s multiple decisions would jointly impact estimates.

From a theoretical standpoint, this paper contributes to the development of many instruments in the non-linear setting. As a growing number of empirical investigations use examiner randomization as an exogenous shock, many theoretical works address the application of many instruments. For example, Hausman et al. (2012) looked at how to correctly estimate the variance and its inference for linear models with many instruments in the presence of heterogeneity. Ackerberg and Devereux (2009) described a better way of estimating the variance, while Mikusheva and Sun (2022) explored how to apply the AR test in the presence of many weak instruments. More broadly, Goldsmith-Pinkham et al. (2020) described the application of shift-share instruments, using the judge instrument as an example. In all of these papers, however, the primary outcome equations are linear
models, and the approaches presented concern how to appropriately infer in the two-stage least square framework. In this paper, the outcome variable recidivism is a dummy variable, and we use judge randomization as the source of exogeneity. Hence, the issue we are exploring is the use of many instruments in a nonlinear outcome equation. Our model estimation in this paper resolves the endogeneity problem in nonlinear models using many instruments.

3 Institutional Details and Data

In this section, we introduce the North Carolina sentencing guidelines, which are used to determine the appropriate level of punishment for felony offenses. Moreover, we discuss the sources from which we obtained our data, explain how we came up with the sample for our primary study, and present summary statistics.

3.1 Structured Sentencing in North Carolina

We focus on North Carolina criminal cases from 1995 to 2010, in which the court followed North Carolina’s mandatory sentencing standards. As part of a nationwide shift toward rule-based criminal sentencing, these guidelines were created with the intention of decreasing sentencing discrepancies among judges. North Carolina maintained to enact structured sentencing rules for all felony offenses on October 1, 1994, despite the fact that many states merely consider it as advisory after *Blakely v. Washington (2004)*.

The rotation of judges is one of the distinguishing characteristics of the North Carolina Superior Courts. In each district, judges are chosen every 8 years, and they can be reelected indefinitely. Despite the fact that judges are elected by voters in their own districts, the state constitution requires judges to rotate every six months between the districts within their divisions. The rotation schedule is decided by the Administrative Office of the Courts. In our dataset, there are 160 judges in all. Due to this rotation system, there is a large
discrepancy in the decisions of North Carolina’s judges, despite the existence of a set framework.

3.2 Judges’ Decisions

The general timeline for a normal criminal proceeding in superior court is as follows: investigation; charge; initial hearing/arrangement; discovery; plea bargaining; pre-trial motions; trial; and sentencing. Throughout the procedure, the case will be handled by a judge in the district, and the judges will randomly rotate among districts every 6 months. Consequently, even in the vast majority of cases (98% of our dataset), the final judgment is dependent on plea bargaining, the judges’ intentions are represented throughout the entire procedure.

In order to maintain accurate and consistent sentencing regulations, judges follow the structured sentencing guidelines for all felony cases, and for each case, they must take the following steps: Judges determine the offense class for each felony conviction, and offenses are assigned to one of ten offense classes by statute; judges determine the history points for the offense; judges consider aggravating and mitigating factors such as being armed with a deadly weapon or having a support system in the community; judges select a minimum and maximum sentence \(^1\) from the appropriate sentence range; and determine the sentence disposition.

Sentence disposition is where endogenous selection may occur. The judge may select among active punishment, intermediate punishment, and community punishment based on the class of the offense and the defendant’s criminal history. The minimum and maximum sentence will be activated and the prisoner will be sent to prison if the judge decides to impose active punishment. If the judge decides to impose an intermediate punishment or community punishment, the minimum and maximum sentences will be suspended and the length of probation will be determined by the judge. Intermediate punishment is

\(^1\) Maximum sentences are always 120% of the minimum and minimum sentence is reported in our dataset.
accompanied by supervised probation, while community punishment is accompanied by unsupervised probation.

The specific punishment chart can be found in the appendix A. When the judge decides the case class and the historical points of the defendant, the respective grid in the judgment chart can be located. For first time offender, if the case class is E, F, G, or H, the judge may select between intermediate punishment and active punishment, i.e., he or she may decide whether the case should be placed on probation.

Probation is a very significant form of punishment within the legal system. By the end of 2020, the Bureau of Justice Statistics estimates that 1 in 66 adult U.S. residents were under community supervision. Among our data, 81.78% of cases were fatal Thus, among the numerous judgments made by judges, even if they are not as well-defined as sentencing, probation decisions are quite essential.

3.3 Data Source

We use two primary datasets. The first comprises of Administrative Office of the Courts of North Carolina (AOC) records from 1995 to 2010. The AOC dataset contains information on defendants, offenses, initial charges, and convictions for all cases heard in the Superior Court of North Carolina. The data also specifies the judge hearing each case, which is a key variable in our identification strategy. The second dataset we use is from the North Carolina Department of Public Safety (DPS), which has full information on every person who earned supervised probation or jail terms between 1995 and 2010. This information allows us to examine prison records. By individual identification (name and birthdate) and charge date, we merge these two databases and create a comprehensive dataset containing both jail records and charge information. We then define reoffending based on recurrent records in this comprehensive datasets.
3.4 Measuring Reoffending and New Offender Definition

Our primary measure of reoffending is an indicator of whether a person is charged within a predetermined time frame (six months, one year, two years, three years, or five years) after being released from jail. If the individual was not incarcerated at the time of the most recent offense, the date of the most recent charge is used as the starting point for reoffending assessment.

Literature provides several definitions of recidivism. For instance, Rose and Shem-Tov (2021) defines reoffending using the recidivate rate from the date of the offense. They employed distinct approaches that took into account the discontinuity in the sentencing guideline. Alexeev and Weatherburn (2022) considers recidivism using judge randomization, but concentrates on financial punishment and drug addiction in an Australian scenario. The recidivism in Alexeev and Weatherburn (2022) is defined as the commission of any crime within two years. The paper by Mueller-Smith (2015) has the most pertinent setting with us. As primary outcomes, he utilized the one-year and three-year recidivism rates for the felony caseload. Kuziemko (2013) defined reoffending as an inmate returning to prison within three years of his release. We employ a similar definition to Kuziemko (2013) and investigate the heterogeneity of the judge’s decision-making.

Using the date of release from jail to define reoffending allows us to focus on a highly policy-relevant component of conviction: the possible deterrent and rehabilitative impacts of incarceration or probation. Using our definition, in the whole sample the reoffending rates after 1, 3, and 5 years are 9.3%, 18.7%, and 25.6%, respectively. To compare our findings to those in the literature, we also used the date beginning with the date of the charge to define reoffending and the results are consistent between these two definitions.

First-time offenders and repeat offenders are two very distinct categories for sentencing purposes. In order to focusing on the judge’s decisions and excluding the influence of the defendant, we restrict our analysis to first-time offenders. From an empirical standpoint, it
is also interesting to understand the potential reoffending propensity of first-time offenders. After narrowing the sample to the first offender, the crime rates at six months, one year, two years, three years, and five years were 6.92%, 9.64%, 13.71%, 17.11%, and 22.78%, respectively.

3.5 Sample Construction

Our identification strategy relies on judge rotation on the superior court, and because only a portion of misdemeanor cases are processed in the superior court, our analytic sample concentrated on individuals convicted of felony offenses between 1995 and 2010. As a baseline for our analysis, we use the charge-level data from the AOC. We can observe offenders’ characteristics, such as their date of birth, gender, race, ethnicity, and criminal history points. For each charge, we observe the offender’s name and the date of the charge, which we combine as the charge identifier. The AOC record includes the judge’s initials, allowing us to construct the judge rotation data.

Due to possible probation, charge modifications, and other factors, the actual prison term may differ from the sentenced term. Thus, we combine DPS data to determine an individual’s actual incarceration length in order to define reoffending. Offenders can be sentenced to concurrent incarceration terms for offenses committed on different dates if they face multiple charges at once. To address this issue, we combine the two datasets by individual identifier and disposition year to consider them as a single record, and we cluster the standard error by individual. Since our primary outcome is reoffending within three and five years, these definitions are consistent to alternative measures.

The past experience of the offender strongly influences the judges’ punishment decisions. In order to concentrate on homogeneous effects, we restrict our research to first-time offenders.

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2 According to the severity of the offense and the number of criminal history points accrued, felonies are classified into ten different classes. The system assigns one point for misdemeanor offenses and two to ten points for felony offenses. If the violations occur while on probation, the offender will receive additional points.
offenders. Using the information from history points, we limit the sample to convicts with 0 history points.

### 3.6 Summary Statistics

Table 3.1 outlines several fundamental aspects of the database. The average age of the convicts is 25.57 years, 21.0% of them are female, and 49.0% of them are black, which is consistent with the data from previous work. Among all the judges’ decisions, 86 of the judge’s decisions were probationary. The average sentence length, including both active and suspended sentences, was 7.4 months. On average, probation lasted 25 months.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>44,821</td>
<td>25.57311</td>
<td>10.26721</td>
<td>13.07324</td>
<td>88.48734</td>
</tr>
<tr>
<td>Female</td>
<td>44,825</td>
<td>.2110206</td>
<td>.4080375</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Black</td>
<td>44,825</td>
<td>.4916676</td>
<td>.4999361</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1(Probation)</td>
<td>44,825</td>
<td>.8605912</td>
<td>.3463765</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Sentence Length</td>
<td>43,923</td>
<td>7.402495</td>
<td>8.565643</td>
<td>.0327869</td>
<td>668.1687</td>
</tr>
<tr>
<td>Probation Length</td>
<td>38,576</td>
<td>25.38189</td>
<td>16.76096</td>
<td>.0327869</td>
<td>432</td>
</tr>
</tbody>
</table>

Table 3.1: Summary Statistics

Figure 3.1 plots the recidivism rate within 5 years for active sentence cases and probationary cases. There is a discrepancy between the recidivism rates of these two groups. This discrepancy may be attributable to the judge’s observation of recidivism-related characteristics, such as risk preference and social network, when reaching the probation decision, or it may be due to the different effects of the two punishments. Consequently, it is essential to differentiate the role of punishment in the judge’s judgment and to assess the impact of punishment on recidivism.
Figure 3.1: Recidivism trend for probationary cases and active sentencing cases

4 Model

We assume judges needs to first decide whether to assign probation to a given case. $p_{ji}$ is the indicator of whether a case $i$ handled by judge $j$. If the case is probationary, we can observe probation length $t_{ji}$ and suspended sentence length $s_{ji}$. If the case is non-probationary, we can observe active sentence length $a_{ji}$.

\[ p_{ji} = 1 \left( \gamma_{pj} + x_i^t \beta_{pj} + u_{pji} > 0 \right) \]

\[ t_{ji} = \gamma_{tj} + x_i^t \beta_{tj} + u_{tji} \text{ if } p_{ji} = 1 \]

\[ s_{ji} = \gamma_{sj} + x_i^t \beta_{sj} + u_{sji} \text{ if } p_{ji} = 1 \]

\[ a_{ji} = \gamma_{aj} + x_i^t \beta_{aj} + u_{aji} \text{ if } p_{ji} = 0 \]
where \( x_i \) are case characteristics including case class dummies, gender, race and age. Error term \((u_{pji}, u_{tji}, u_{sji}, u_{a0i})\) are unobserved judges’ preference in their decision process.

Given the punishment, we observe whether the crime is committed again or not. If the case is probationary, we observe \( y_{1ji} \) and if the case is non-probationary, we observe \( y_{0ji} \).

In the recidivism outcome equation, the subscript \( j \) refers to the judge who handled the individual \( i \)’s case. We add subscript \( j \) to indicate the effect of the punishment decision of the judgment on the outcome. \( u_{1ji} \) and \( u_{0ji} \) are individual unobserved characteristics that affects recidivism behavior. \( u_{1ji} \) and \( u_{0ji} \) are distinct as defendant under probationary would have different social interactions with the defendants in the prison. Probation and punishment choices are endogenous because judges can detect some unobserved characteristics that econometrician cannot including risk preference, social networks and many others. Thus, our goal is to identify the impact of judges’ decisions on recidivism, \( \alpha_t \), \( \alpha_s \) and \( \alpha_a \), when these unobservables are all twisted. Here, \( \alpha_t \) is the effect of probation length on recidivism, \( \alpha_s \) is the effect of suspended sentence length on recidivism and \( \alpha_a \) is the effects of active sentence length.

\[
y_{1ji} = 1 \{ \alpha_t l_{tji} + \alpha_s s_{sji} + x'_i \beta_1 + u_{1ji} > 0 \} \text{ if } p_{ji} = 1
\]
\[
y_{0ji} = 1 \{ \alpha_a a_{aji} + x'_i \beta_0 + u_{0ji} > 0 \} \text{ if } p_{ji} = 0
\]

All the error terms are assumed to be jointly normal:
In our model, identification is complicated by the possibility that all error terms are correlated, and the correlation between these unobservable components is the source of endogeneity in the decision to judge. If there is no selection for probationary decisions, this model is reduced to an endogenous Probit model with suspended sentence length, probation term, and active sentence as endogenous variables. A prominent way to estimate is using judge dummies to calculate judges’ leniency level and use judges’ leniency as instrument variables. In the conventional Probit model, the coefficients are identified up to the scale of the error terms $\sigma_{0j}$ and $\sigma_{1j}$, respectively.

5 Identification and Estimation

In this section, we will show how to recover the correlation between several unobserved error terms using conditional expectation, and then recover the parameters, as well as the underlying assumptions necessary for identification. We will focus on introducing the identification for active sentencing case and the detailed steps of probational case identification can be found in appendix C.

For the active sentencing cases, the equations involved are:

$$p_{ji} = 1 (\gamma_{pj} + x'_i \beta_{pj} + u_{pji} > 0)$$

$$a_{ji} = \gamma_{aj} + x'_i \beta_{aj} + u_{aji} \text{ if } p_{ji} = 0$$

$$y_{0ji} = 1 \{\alpha_a a_{ji} + x'_i \beta_0 + u_{0ji} > 0\} \text{ if } p_{ji} = 0$$
and we assume:

\[
\begin{pmatrix}
  u_{pji} \\
  u_{aji} \\
  u_{0i}
\end{pmatrix} \sim \mathcal{N}
\begin{pmatrix}
  0 \\
  0 \\
  0
\end{pmatrix},
\begin{pmatrix}
  \sigma_{pj}^2 & \rho_{pa} \sigma_{aj} \sigma_{pj} & \rho_{0pj} \sigma_{pj} \sigma_{0} \\
  \rho_{pa} \sigma_{aj} \sigma_{pj} & \sigma_{aj}^2 & \rho_{0aj} \sigma_{aj} \sigma_{0} \\
  \rho_{0pj} \sigma_{pj} \sigma_{0} & \rho_{0aj} \sigma_{aj} \sigma_{0} & \sigma_{0}^2
\end{pmatrix}
\]

In this set up, our goal is to identify how active sentencing length affect recidivism outcome. The difficulty with this problem is that while the judges are making an active sentence length decision, they are also deciding not to award probation in this case. In order to determine the effects of active sentence when probation is an option, we need to consider the correlation between individual unobserved error and judges’ preference error. We first introduce the parametric identification steps and discuss the semiparametric steps that slightly relax the joint normal distribution.

5.1 Parametric Identification

First, we examine the probation decisions. This decision is a 0-1 variable and it is observable from the data, hence a Probit Model can be estimated:

\[
p_{ji} = 1 (\gamma_{pj} + x_i^j \beta_{pj} + u_{pji} > 0)
\]

where \(x_i\) are case characteristics and \(u_{pji}\) is error term that contains judges’ preference. When the case is determined non-probationary, we can observe the length of the active
sentence and $E (a_{ji}|x_i, j, p_i = 0)$ contains a Heckman-correction term:

$$E (a_{ji}|x_i, j, p_i = 0) = E (\gamma_{aj} + x_i'\beta_{aj} + u_{aji}|x_i, j, p_{ji} = 0)$$

$$= \gamma_{aj} + x_i'\beta_{aj} + E (u_{aji}|x_i, j, 1 (u_{pji} \leq -\gamma_{pj} - x_i'\beta_{pj}) = 1)$$

$$= \gamma_{aj} + x_i'\beta_{aj} + \frac{\text{cov}(u_{aji}, u_{pji})}{\sigma_{pji}} - \Phi \left( \frac{\gamma_{pj} + x_i'\beta_{pj}}{\sigma_{pji}} \right)$$

where the Mills’ ratio part $-\frac{\phi\left(\frac{\gamma_{pj} + x_i'\beta_{pj}}{\sigma_{pji}}\right)}{1 - \Phi\left(\frac{\gamma_{pj} + x_i'\beta_{pj}}{\sigma_{pji}}\right)}$ can be identified by the Probit equation (1). The Heckman two step procedure can identify $\frac{\text{cov}(u_{aji}, u_{pji})}{\sigma_{pji}}$ and $\sigma_{aj}$.

To construct the correlation between decisions and the outcome, we can write out $E (y_{0ji}|x_i, j, p_{ji} = 0)$. The endogeneity here comes from two sources: correlation between judges’ preference of probation decision selection error term $u_{pji}$ and individual unobserved error term $u_{0ji}$, active sentence length decision term $u_{aji}$. It can be seen by writing out the conditional expectations:

$$E (y_{0ji}|x_i, j, p_{ji} = 0)$$

$$= E \{1 \{\alpha_{a_{aji}} + x_i'\beta_{0} + u_{0ji} > 0\} |x_i, j, p_{ji} = 0\}$$

$$= \text{Pr} \left( \{\alpha_{a_{aji}} + u_{0ji} > -\alpha_{a}\gamma_{aj} - x_i' (\beta_{aj}\alpha_{a} + \beta_{0})\} \cap \left\{\frac{u_{pji}}{\sigma_{pji}} \leq -\left(\frac{\gamma_{pj} + x_i'\beta_{pj}}{\sigma_{pji}}\right)\right\} |x_i, j\right)$$

$$= \Phi_{2} \left( \frac{\alpha_{a}\gamma_{aj} + x_i' (\beta_{aj}\alpha_{a} + \beta_{0})}{\sqrt{\text{Var}(\alpha_{a_{aji}} + u_{0ji})}} - \left(\frac{\gamma_{pj} + x_i'\beta_{pj}}{\sigma_{pji}}\right), \rho_{a_{aji}} |x_i, j\right)$$

By maximizing the likelihood functions of $\{y_{0ji}, x_i, j, p_{ji}\}$:

$$\prod_{i=1}^{n} \left\{E (y_{0ji}|x_i, j, p_{ji} = 0)\right\}^{1(y_{0i}=1)} \left\{1 - E (y_{0ji}|x_i, j, p_{ji} = 0)\right\}^{1(y_{0i}=0)}$$
we can identify $\frac{\alpha_a}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}$ and $\rho_{asj} = \text{corr}\left(\frac{\alpha_a u_{aji} + u_{0ji}}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}, \frac{u_{pji}}{\sigma_{pj}}\right)$.

The coefficient we identified $\frac{\alpha_a}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}$ is up to scale of $\text{Var}(\alpha_a u_{aji} + u_{0ji})$, which contains the judges' preference, individual unobserved characteristics and their correlation.

The correlation of $u_{aji}$ and $u_{0ji}$ is one source of the endogeneity. To disentangle the pure effect of $\alpha_a$, we need to trace out the correlation of $u_{0ji}$ and $u_{aji}$. An available conditional mean function is:

$$E(a_{ji}|x_i, j, y_{0i} = 0, p_{ji} = 0)$$
$$= E \{\gamma_{aj} + x_i' \beta_{aj} + u_{aji}|x_i, j, y_{0i} = 0, p_{ji} = 0\}$$
$$= E \{\gamma_{aj} + x_i' \beta_{aj} + u_{aji}|x_i, j, u_{0ji} \leq - (\alpha_a (\gamma_{aj} + x_i' \beta_{aj} + u_{aji}) - x_i' \beta_0), u_{pji} \leq - (\gamma_{pj} + x_i' \beta_{pj})\}$$
$$= \gamma_{aj} + x_i' \beta_{aj} + E \left\{u_{aji} | \tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} \leq \tilde{c}_{3j}, x_i, j\right\}$$
$$= \gamma_{aj} + x_i' \beta_{aj} + \frac{\sigma_{1j} (\rho_{12j} - \rho_{23j} \rho_{13j})}{1 - \rho_{23j}^2} E \left(\tilde{U}_{2j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} \leq \tilde{c}_{3j}\right)$$
$$+ \frac{\sigma_{1j} (\rho_{13j} - \rho_{23j} \rho_{12j})}{1 - \rho_{23j}^2} E \left(\tilde{U}_{3j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} \leq \tilde{c}_{3j}\right)$$

(4)

where

$$\tilde{U}_{2j} = \frac{u_{0ji} + \alpha_a u_{aji}}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}, \tilde{c}_{2j} = \frac{-\alpha_a \gamma_{aj} - \alpha_a x_i' \beta_{aj} - x_i' \beta_0}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}$$

$$\tilde{U}_{3j} = \frac{u_{pji}}{\sigma_{pj}}, \tilde{c}_{3j} = \frac{-\gamma_{pj} + x_i' \beta_{pj}}{\sigma_{pj}}$$

$$\rho_{23j} = \text{Cov}\left(\tilde{U}_{2j}, \tilde{U}_{3j}\right) = \text{Cov}\left(\frac{u_{0ji} + \alpha_a u_{aji}}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}}, \frac{u_{pji}}{\sigma_{pj}}\right)$$
\[
\rho_{12j} = \frac{1}{\sigma_{aj}} \text{Cov}
\left( u_{aji}, \frac{u_{0ji} + \alpha_a u_{aji}}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}} \right)
\]
\[
= \frac{1}{\sigma_{aj}} \frac{1}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}} \text{Cov}(u_{aji}, u_{0ji}) + \frac{\alpha_a}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}} \sigma_{aj}
\]
\[
\rho_{13j} = \frac{1}{\sigma_{aj}} \text{Cov}
\left( u_{aji}, \frac{u_{pji}}{\sigma_{pj}} \right)
\]

Regressing \( E(a_{ji}|x_i, j, y_{0i} = 0, p_{ji} = 0) \) on \( E(\tilde{U}_{2j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} \leq \tilde{c}_{3j}) \) and \( E(\tilde{U}_{3j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} \leq \tilde{c}_{3j}) \) can help us identify \( \sigma_{1j}(\rho_{12j} - \rho_{23j}\rho_{13j}) \) and \( \sigma_{1j}(\rho_{13j} - \rho_{23j}\rho_{12j}) \). Note that \( \rho_{23j} \) contains the correlation between the judges’ preference \( u_{aji} \) and the combination of individual unobservable \( u_{0ji} \) and it has been identified from the likelihood functions of \( \{y_{0ji}, x_i, j, p_{ji}\} \). \( \rho_{13j} \) has been identified from the Mills’ ratio term in \( E(a_{ji}|x_i, j, p_{ji} = 0) \). Thus, we are able to identify \( \rho_{12j} \) that contains correlation between \( u_{aji} \) and \( u_{0ji} \).

To conclude, we can solve for two values from equation (3) and (4):

\[
\frac{1}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}} \text{Cov}(u_{aji}, u_{0ji}) = \frac{\rho_{0aj}\sigma_{0j}\sigma_{aj}}{\sqrt{\alpha_a^2 \sigma_{aj}^2 + \sigma_{0j}^2 + 2\alpha_a \rho_{0aj}\sigma_{0j}\sigma_{aj}}} \tag{5}
\]
\[
\frac{\alpha_a}{\sqrt{\text{Var}(\alpha_a u_{aji} + u_{0ji})}} = \frac{\alpha_a}{\sqrt{\alpha_a^2 \sigma_{aj}^2 + \sigma_{0j}^2 + 2\alpha_a \rho_{0aj}\sigma_{0j}\sigma_{aj}}} \tag{6}
\]

and we can identify \( \alpha_a/\sigma_{0j} \) from the equation system.

**Proposition 1.** \( \alpha_a/\sigma_{0j} \) and \( \rho_{0aj} \) are uniquely identified in equation (5) and (6)

**Proof.** See Appendix B.1

The identification for probationary cases can be solved analogously. We can identify \( \frac{\alpha_a}{\sigma_{sy}} \) and \( \frac{\alpha_s}{\sigma_{sj}} \) by calculating \( E(s_{ji}|x_i, j, p_{ji} = 1) \), \( E(t_{ji}|x_i, j, p_{ji} = 1) \), \( E(y_{1ji}|x_i, j, p_{ji} = 1) \), \( E(s_{ji}|x_i, j, y_{i1} = 0, p_{ji} = 1) \). The detailed steps can be found in appendix C.
5.2 Semiparametric Identification

This section discusses semiparametric identification results and the extent to which the joint normal assumption can be relaxed. In our model, the probation decision of each judge is a Probit model, so 

\[ \frac{-\phi(\gamma pj + x'_i \beta pj)}{1-\Phi(\gamma pj + x'_i \beta pj)} \]

is nonparametrically identified.

\[ Pr (y_0 i = 1|x_i, p_{ji} = 0) = F_{-(u_0 i + \alpha_a u_{aji})} (\alpha_a [\gamma aj + x'_i \beta 1i] + x'_i \beta 0 | x_i, p_{ji} = 0) \]

\[ \alpha_a \text{ and } \beta_0 \text{ are identified up to } \sqrt{Var (u_0 i + \alpha_a u_{aji})}. \]

We need to analyze two layers of endogeneity here, one is the decision of punishment itself, which is endogenous since judges can perceive some personal features of the defendant that the econometrician cannot see. The other is within the realm of punishment, where the decision of probation and the subsequent decision of sentence duration, probation length, are correlated. The first layer of endogeneity can be addressed by randomizing judges, while the second level of endogeneity requires that judges have variation in their decisions at multi-dimensions including the propensity of probation decisions, length of probation and suspended sentencing, length of active sentencing.

To identify \( \alpha_a \), we need variation of active sentence length \( a_{ji} \) or more specifically, \( \gamma aj + x'_i \beta 1i \). The distribution of the judge’s degree of leniency for class H active sentence cases is depicted in Figure 5.1. In this scenario, the judge needs to make a decision on the probation as well as a decision on the length of the active sentence. The leniency degree of judge can be shown to vary along two dimensions. Similarly, in order to identify the role of the judge’s decision on the probationary case, the judge needs to make decisions in three dimensions: probation, probation length, and suspended sentence length. Figure 5.2 illustrates these decisions in three dimensions. In summary, the variation in multiple dimensions of the judge can help us to distinguish the variation in multiple decisions and thus obtain the pure impact of the judge decision on recidivism.
Figure 5.1: Judges’ leniency distribution for active sentencing cases in Class H
Figure 5.2: Judges’ leniency distribution for probationary cases in Class H
5.3 Estimation

In this section, we present estimation steps to recover the effects of judges’ decisions on recidivism. We present the estimating steps for active sentencing cases; probationary cases can be estimated in the similar manner.

We first estimate the probationary decision model:

\[ p_{ji} = 1 (\gamma_{pj} + x_i' \beta_{pj} + u_{pji} > 0) \]

and get the estimated coefficients up to variance of \( u_{pji} \), and then derive the Mills’ ratio value

\[ -\frac{\phi(z_{pji} + x_i' \tilde{\beta}_{pj})}{1 - \phi(z_{pji} + x_i' \tilde{\beta}_{pj})}. \]

Then, we regress \( a_{ji} \) on case characteristics \( x_i \), Mills’ ratio to estimate \( \frac{\sigma_{a}}{\sigma_{pj}} \) and \( \sigma_{a_{ji}} \). Then we use maximum likelihood of equation (3) to estimate \( \frac{{\sigma_{a}}}{\sqrt{\text{Var}(\alpha_{a}u_{a_{ji}} + u_{0_{ji}})}} \) and \( r_{a_{xj}} \). From the above steps we can derive estimated value of \( \hat{\sigma}_{2} = -\frac{\hat{r}_{a_{xj}} + x_{i}^{'} \hat{\beta}_{pj} - x_{i}^{'} \hat{\beta}_{a}}{\sqrt{\text{Var}(\alpha_{a}u_{a_{ji}} + u_{0_{ji}})}} \), \( \hat{\sigma}_{3} = -\frac{\gamma_{pj} + x_{i}^{'} \hat{\beta}_{pj}}{\sigma_{pj}} \) the estimated correlation between \( \hat{U}_{3j} \) and \( \hat{U}_{2j} \). We can then estimated the conditional mean of \( E(\hat{U}_{3j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) \) and \( E(\hat{U}_{3j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) \), where

\[
E_{n}(\hat{U}_{3j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) = -\Phi\left(\frac{\hat{c}_{3} - \rho_{a_{xj}} \hat{c}_{2}}{\sqrt{1 - \rho_{a_{xj}}^{2}}}\right) \phi(\hat{c}_{2}) - \rho_{a_{xj}} \Phi\left(\frac{\hat{c}_{2} - \rho_{a_{xj}} \hat{c}_{3}}{\sqrt{1 - \rho_{a_{xj}}^{2}}}\right) \phi(\hat{c}_{3})
\]

\[
E_{n}(\hat{U}_{3j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) = -\Phi\left(\frac{\hat{c}_{3} - \rho_{a_{xj}} \hat{c}_{2}}{\sqrt{1 - \rho_{a_{xj}}^{2}}}\right) \phi(\hat{c}_{2}) - \rho_{a_{xj}} \Phi\left(\frac{\hat{c}_{2} - \rho_{a_{xj}} \hat{c}_{3}}{\sqrt{1 - \rho_{a_{xj}}^{2}}}\right) \phi(\hat{c}_{3})
\]

Regressing \( a_{ji} \) on \( \hat{E}(\hat{U}_{2j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) \) and \( \hat{E}(\hat{U}_{3j}|\hat{U}_{2j} \leq \hat{c}_{2}, \hat{U}_{3j} \leq \hat{c}_{3}) \) conditional on \( p_{ji} = 0 \) and \( y_{ji} = 0 \) can yield the values of equation (5) and (6). Then we solve for \( \hat{\alpha}_{a}/\sigma_{0j}. \)

---

\(^3\) A result help calculating conditional expectations can be found in appendix Lemma 2.
6 Results

6.1 Simulation Results

We show the simulation results for a given judge \(j\) and he handles 10,000 cases. The error term \((u_{pji}, u_{tji}, u_{sji}, u_{1ij}, u_{0ij}) \sim \mathcal{N}(0, \Omega)\), where \(\Omega_{ij} = 0.5\) for \(i \neq j\) and \(\Omega_{ii} = 1\). We generate the control variables \(x_{1i}, x_{2i}, x_{3i}, x_{4i} \sim \mathcal{N}(0, 1)\), \((\beta_{pj}, \beta_{tj}, \beta_{sij}, \beta_{sij}) = (0.5, 0.2, 1, 1)\), \((\beta_1, \beta_0) = (0.6, 0.6)\) \(\alpha_t = 1\), \(\alpha_s = -1\) and \(\alpha_a = -2\).

Figure 6.1 shows the results based on 10,000 simulations. We show the simulation results for active sentencing case here and probationary cases results can be seen from Appendix A1. In each sub-figure, the red vertical line is the true value. Figure 1(a) plots the correlation of \(\frac{\alpha_a u_{a_{ji}} + u_{0_{ji}}}{\sqrt{Var(\alpha_a u_{a_{ji}} + u_{0_{ji}})}}\) and \(u_{pji}/\sigma_{pji}\) that we estimated from the maximum log likelihood estimator of equation 3. Figure 1(c) represents the parameter \(\alpha_a\) up to scale of \(Var(\alpha_a u_{a_{ji}} + u_{0_{ji}})\). Figure 1(b) is the coefficients we derive from conditional regression of equation 4. By constructing the equation values, we are able to recover the true parameter up to scale of \(\sigma_{0i}\) in figure 1(d).

6.2 Data

Table 6.1 presents the estimate for three key punishment effects coefficient: \(\alpha_a\), the effect of active sentencing, \(\alpha_t\), the effect of probation length for probationary cases and \(\alpha_s\), the effects of suspended sentencing for probation cases. We estimate the whole model of the recidivism rate within one, two, three, four and five years. The estimated variances in the bracket are derived by 100 bootstrapped samples.

Comparing to probation choices such as probationary length and suspended sentence length, active sentencing has relatively bigger effects on recidivism. One month of active sentence can reduce recidivism by one percent; therefore, one year of active sentencing can reduce recidivism by twelve percent. In cases involving probation, the effects of suspended
Figure 6.1: Simulation Distribution for Active Sentencing Cases
sentencing and probation length are similar in magnitude but have opposite signs. To be more explicit, suspended sentencing as a threat can reduce recidivism, whereas the length of probation increases recidivism, and their combination as a component decision offsets each other.

<table>
<thead>
<tr>
<th></th>
<th>Year1</th>
<th>Year2</th>
<th>Year3</th>
<th>Year4</th>
<th>Year5</th>
</tr>
</thead>
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<tr>
<td>(\alpha_a)</td>
<td>-0.01176</td>
<td>-0.00385</td>
<td>-0.00501</td>
<td>-0.00748</td>
<td>-0.01036</td>
</tr>
<tr>
<td></td>
<td>(0.01084)</td>
<td>(0.00616)</td>
<td>(0.00647)</td>
<td>(0.00762)</td>
<td>(0.00879)</td>
</tr>
<tr>
<td>(\alpha_t)</td>
<td>0.00112</td>
<td>0.00117</td>
<td>0.00119</td>
<td>0.00119</td>
<td>0.00118</td>
</tr>
<tr>
<td></td>
<td>(0.01843)</td>
<td>(0.02083)</td>
<td>(0.02101)</td>
<td>(0.02103)</td>
<td>(0.01970)</td>
</tr>
<tr>
<td>(\alpha_s)</td>
<td>-0.00113</td>
<td>-0.00118</td>
<td>-0.00119</td>
<td>-0.00119</td>
<td>-0.00119</td>
</tr>
<tr>
<td></td>
<td>(0.03868)</td>
<td>(0.04068)</td>
<td>(0.04325)</td>
<td>(0.04252)</td>
<td>(0.04094)</td>
</tr>
</tbody>
</table>

Table 6.1: Estimation Results for Recidivism Rate within 5 years

7 Conclusion

How punishment influences recidivism is a crucial question in law and economics and policy making, and the literature offers a variety of perspectives. In this paper, we attempt to answer this question from the standpoint of judge decisions by constructing a selection model.

We model the decision matrix of the judge. In our model, the judge takes a variety of decisions, including whether or not to grant probation, the duration of probation, and whether or not the sentence will be suspended. Judges can identify features that cannot be observed by econometricians, such as the defendants’ social links and risk preferences, based on the extensive case papers and interactions with the defendants. Our approach nests circumstances in which there is no judge to decide on probation or in which probationary or non-probationary cases have consistent impacts on recidivism. To recover the association between various decisions and recidivism, as well as the effect of punishment, we use a multi-step estimation technique. Using North Carolina data to estimate the model, we find that
one year of active sentencing can cut recidivism by 12 percent. For probationary cases, probation and suspended sentences length offset each other.

References


# A Punishment Chart

## FELONY PUNISHMENT CHART

**PRIOR RECORD LEVEL**

<table>
<thead>
<tr>
<th>OFFENSE CLASS</th>
<th>I 0-1 Pt</th>
<th>II 2-5 Pts</th>
<th>III 6-9 Pts</th>
<th>IV 10-13 Pts</th>
<th>V 14-17 Pts</th>
<th>VI 18+ Pts</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>B1</td>
<td>240 - 300</td>
<td>275 - 345</td>
<td>317 - 397</td>
<td>365 - 456</td>
<td>336 - 420</td>
<td>386 - 483</td>
</tr>
<tr>
<td>B2</td>
<td>192 - 240</td>
<td>221 - 276</td>
<td>254 - 317</td>
<td>292 - 365</td>
<td>314 - 393</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>125 - 157</td>
<td>144 - 180</td>
<td>165 - 207</td>
<td>190 - 238</td>
<td>219 - 273</td>
<td>251 - 314</td>
</tr>
<tr>
<td>E</td>
<td>94 - 125</td>
<td>108 - 144</td>
<td>124 - 165</td>
<td>143 - 190</td>
<td>164 - 219</td>
<td>189 - 251</td>
</tr>
<tr>
<td>F</td>
<td>73 - 92</td>
<td>83 - 104</td>
<td>96 - 120</td>
<td>110 - 138</td>
<td>127 - 159</td>
<td>146 - 182</td>
</tr>
<tr>
<td>G</td>
<td>58 - 73</td>
<td>67 - 83</td>
<td>77 - 96</td>
<td>88 - 110</td>
<td>101 - 127</td>
<td>117 - 146</td>
</tr>
<tr>
<td>H</td>
<td>44 - 58</td>
<td>50 - 67</td>
<td>58 - 77</td>
<td>66 - 88</td>
<td>76 - 101</td>
<td>87 - 117</td>
</tr>
</tbody>
</table>

**DISPOSITION**

- Aggravated Range
- Mitigated Range

**DEATH OR LIFE WITHOUT PAROLE**

Defendant Under 18 at Time of Offense: Life With or Without Parole

**PRESumptive RANGE**

- A
- B
- C
- D
- E
- F
- G
- H
- I

### Notes

- A – Active Punishment
- I – Intermediate Punishment
- C – Community Punishment

Numbers shown are in months and represent the range of minimum sentences.

Revised: 09-09-13

---

# Table

## Table of Rates

<table>
<thead>
<tr>
<th>Class</th>
<th>Rate 1</th>
<th>Rate 2</th>
<th>Rate 3</th>
</tr>
</thead>
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<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>B</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>C</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>D</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>E</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>F</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>G</td>
<td>1.9</td>
<td>2.0</td>
<td>2.1</td>
</tr>
<tr>
<td>H</td>
<td>2.2</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td>I</td>
<td>2.5</td>
<td>2.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>
B  Some useful results for the identification

Lemma 1. Suppose

\[
\begin{pmatrix}
X_1 \\
X_2 \\
X_3
\end{pmatrix} \sim N
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \sigma_{13} \\
\sigma_{12} & \sigma_2^2 & \sigma_{23} \\
\sigma_{13} & \sigma_{23} & \sigma_3^2
\end{pmatrix}
\]

\[ Z_1, Z_2, Z_3 \text{ are standardized variables, with } X_1 = \sigma_1 Z_1, X_2 = \sigma_2 Z_2, X_3 = \sigma_3 Z_3, \text{ For } i \neq j, \text{ Cov}(Z_i, Z_j) = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \rho_{ij}. \text{ Then,} \]

\[ E(X_1|X_2 \leq c_2, X_3 \leq c_3) = E(\sigma_1 Z_1|\sigma_2 Z_2 \leq c_2, \sigma_3 Z_3 \leq c_3) \]

\[ = \sigma_1 \left[ \gamma_2 E \left( Z_2|Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3} \right) + \gamma_3 E \left( Z_3|Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3} \right) \right] \]

where

\[ \gamma_1 = \frac{\rho_{12} - \rho_{23} \rho_{13}}{1 - \rho_{23}^2} \]

and

\[ \gamma_2 = \frac{\rho_{13} - \rho_{23} \rho_{12}}{1 - \rho_{23}^2}. \]

Proof. If \( \rho_{23} \neq 0, \)

\[ \begin{pmatrix}
\gamma_1 \\
\gamma_2
\end{pmatrix} = E \left( Z_2 Z_3 \right) \begin{pmatrix}
Z_2 \\
Z_3
\end{pmatrix}^{-1} E \left( Z_1 Z_2 \right) \begin{pmatrix}
Z_1 \\
Z_3
\end{pmatrix} \]

\[ = \begin{pmatrix}
1 & \rho_{23} \\
\rho_{23} & 1
\end{pmatrix}^{-1} \begin{pmatrix}
\rho_{12} \\
\rho_{13}
\end{pmatrix} = \frac{1}{1 - \rho_{23}^2} \begin{pmatrix}
\rho_{12} - \rho_{23} \rho_{13} \\
\rho_{13} - \rho_{23} \rho_{12}
\end{pmatrix} \]
Thus,

\[
E(X_1|X_2 \leq c_2, X_3 \leq c_3) = \frac{\sigma_1}{1 - \rho_{23}^2} \left[ (\rho_{12} - \rho_{23}\rho_{13}) E\left(Z_2|Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3}\right) + (\rho_{13} - \rho_{23}\rho_{12}) E\left(Z_3|Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3}\right) \right]
\]

If \(\rho_{23} \to 0\), then

\[
E\left(Z_2|Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3}\right) = \frac{\int_{-\infty}^{c_2} \int_{-\infty}^{c_3} z_2 \phi_{2,3}(z_2, z_3, \rho_{23}) d\phi d\phi_{2,3}(z_2, z_3, \rho_{23}) d\phi d\phi_{2,3}(z_2, z_3, \rho_{23}) d\phi d\phi_{2,3}(z_2, z_3, \rho_{23}) d\phi d\phi_{2,3}(z_2, z_3, \rho_{23}) d\phi}{\int_{-\infty}^{c_2} \int_{-\infty}^{c_3} \phi_{2,3}(z_2, z_3, \rho_{23}) d\phi d\phi_{2,3}(z_2, z_3, \rho_{23}) d\phi}
\]

and the result remains.

Lemma 2.

\[
E(Z_2|Z_2 \leq \tilde{c}_2, Z_3 \leq \tilde{c}_3) = -\Phi\left(\frac{\tilde{c}_3 - \rho_{23}\tilde{c}_2}{1 - \rho_{23}^2}\right) \phi(\tilde{c}_2) - \rho_{23} \Phi\left(\frac{\tilde{c}_2 - \rho_{23}\tilde{c}_3}{1 - \rho_{23}^2}\right) \phi(\tilde{c}_3)
\]

where \(Z_2\) and \(Z_3\) follow standard normal distribution, \(\rho_{23} = \text{Cov}(Z_2, Z_3)\), \(\Phi_2\) is the standard bivariate normal conditional density function.

Proof.

\[
E(Z_2|Z_2 \leq \tilde{c}_2, Z_3 \leq \tilde{c}_3) = \frac{E(Z_2 I(Z_3 \leq \tilde{c}_3)|Z_2 \leq \tilde{c}_2)}{E(1(Z_3 \leq \tilde{c}_3)|Z_2 \leq \tilde{c}_2)}.
\]
Note that $Z_3|Z_2 = \mathcal{N}(\rho_{23}Z_2, (1 - \rho_{23}^2))$ so that 

$$P(Z_3 \leq \tilde{c}_3|Z_2) = P\left(\frac{Z_3 - \rho_{23}Z_2}{(1 - \rho_{23}^2)^{1/2}} \leq \frac{\tilde{c}_3 - \rho_{23}Z_2}{(1 - \rho_{23}^2)^{1/2}} | Z_2\right) = \Phi\left(\frac{\tilde{c}_3 - \rho_{23}Z_2}{(1 - \rho_{23}^2)^{1/2}}\right).$$

Therefore,

$$E(Z_2|Z_3 \leq \tilde{c}_3|Z_2 \leq \tilde{c}_2) = \frac{1}{\Phi(\tilde{c}_2)} \int_{-\infty}^{\tilde{c}_2} z_2 \Phi\left(\frac{\tilde{c}_3 - \rho_{23}z_2}{(1 - \rho_{23}^2)^{1/2}}\right) \phi(z_2) dz_2$$

$$= -\frac{1}{\Phi(\tilde{c}_2)} \int_{-\infty}^{\tilde{c}_2} \phi(\tilde{c}_2) - \frac{1}{\Phi(\tilde{c}_2)} \int_{-\infty}^{\tilde{c}_2} \Phi\left(\frac{\tilde{c}_3 - \rho_{23}z_2}{(1 - \rho_{23}^2)^{1/2}}\right) \phi(z_2) dz_2$$

$$= -\Phi\left(\frac{\tilde{c}_3 - \rho_{23}\tilde{c}_2}{(1 - \rho_{23}^2)^{1/2}}\right) \frac{\phi(\tilde{c}_2)}{\Phi(\tilde{c}_2)} - \frac{1}{\Phi(\tilde{c}_2)} \int_{-\infty}^{\tilde{c}_2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{z_2 - \tilde{c}_3\rho_{23}}{(1 - \rho_{23}^2)^{1/2}}\right)^2\right) dz_2$$

Thus,

$$E(Z_2|Z_2 \leq \tilde{c}_2, Z_3 \leq \tilde{c}_3) = \frac{E(Z_2|Z_3 \leq \tilde{c}_3|Z_2 \leq \tilde{c}_2)}{E(1|Z_3 \leq \tilde{c}_3|Z_2 \leq \tilde{c}_2)}$$

$$= -\Phi\left(\frac{\tilde{c}_3 - \rho_{23}\tilde{c}_2}{(1 - \rho_{23}^2)^{1/2}}\right) \frac{\phi(\tilde{c}_2)}{\Phi(\tilde{c}_2)} - \rho_{23} \Phi(\tilde{c}_2) \left(\frac{\tilde{c}_2 - \rho_{23}\tilde{c}_3}{(1 - \rho_{23}^2)^{1/2}}\right)$$

$$= \frac{P(Z_3 \leq \tilde{c}_3, Z_2 \leq \tilde{c}_2) / P(Z_2 \leq \tilde{c}_2)}{\Phi_2(\tilde{c}_3, \tilde{c}_2, \rho_{23})}$$
In conclusion,

\[
E(Z_1|Z_2 \leq c_2, Z_3 \leq c_3)
\]

\[
= \frac{\sigma_1}{1 - \rho_{23}^2} \left[ (\rho_{12} - \rho_{23}\rho_{13}) E \left( Z_2 | Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3} \right) + (\rho_{13} - \rho_{23}\rho_{12}) E \left( Z_3 | Z_2 \leq \frac{c_2}{\sigma_2}, Z_3 \leq \frac{c_3}{\sigma_3} \right) \right]
\]

\[
= \frac{\sigma_1}{1 - \rho_{23}^2} \left[ (\rho_{12} - \rho_{23}\rho_{13}) \left( -\Phi \left( \frac{\tilde{c}_2 - \rho_{23}\tilde{c}_3}{(1 - \rho_{23}^2)^{1/2}} \right) \Phi(\tilde{c}_2) - \rho_{23} \Phi(\tilde{c}_3) \Phi \left( \frac{\tilde{c}_3 - \rho_{23}\tilde{c}_2}{(1 - \rho_{23}^2)^{1/2}} \right) \right) \right.
\]

\[
+ \left. (\rho_{13} - \rho_{23}\rho_{12}) \frac{-\Phi \left( \frac{\tilde{c}_2 - \rho_{23}\tilde{c}_3}{(1 - \rho_{23}^2)^{1/2}} \right) \Phi(\tilde{c}_3) - \rho_{23} \Phi(\tilde{c}_3) \Phi \left( \frac{\tilde{c}_3 - \rho_{23}\tilde{c}_2}{(1 - \rho_{23}^2)^{1/2}} \right) }{\Phi_2(\tilde{c}_3, \tilde{c}_2, \rho_{23})} \right]
\]

B.1 Proof of Proposition 1

Let \( \tilde{\alpha}_a = \frac{\alpha_a}{\sigma_0} \), then we have

\[
\rho_{0a_j}\sigma_{0j}\sigma_{aj} \sqrt{\alpha_a^2 \sigma_{aj}^2 + \sigma_{0j}^2 + 2\alpha_a \rho_{0a_j}\sigma_{0j}\sigma_{aj}} = \rho_{0a_j}\sigma_{aj} \sqrt{(\tilde{\alpha}_a)^2 \sigma_{aj}^2 + 1 + 2\tilde{\alpha}_a \rho_{0a_j}\sigma_{aj}}
\]

\[
\frac{\alpha_a}{\sqrt{\alpha_a^2 \sigma_{aj}^2 + \sigma_{0j}^2 + 2\alpha_a \rho_{0a_j}\sigma_{0j}\sigma_{aj}}} = \frac{\tilde{\alpha}_a}{\sqrt{(\tilde{\alpha}_a)^2 \sigma_{aj}^2 + 1 + 2\tilde{\alpha}_a \rho_{0a_j}\sigma_{aj}}}
\]

and we want to show that:

\[
\left\{ \begin{aligned}
\rho_{0a_j}\sigma_{aj} \\
\tilde{\alpha}_a
\end{aligned} \right\} \sqrt{\alpha_a^2 \sigma_{aj}^2 + 1 + 2\alpha_a \rho_{0a_j}\sigma_{0j}\sigma_{aj}} = \left\{ \begin{aligned}
\rho_{0a_j}\sigma_{aj} \\
\tilde{\alpha}_a
\end{aligned} \right\} \sqrt{(\tilde{\alpha}_a)^2 \sigma_{aj}^2 + 1 + 2\tilde{\alpha}_a \rho_{0a_j}\sigma_{aj}} \Rightarrow \left\{ \begin{aligned}
\tilde{\alpha}_a = \tilde{\alpha}_a^* \\
\rho_{0a_j} = \rho_{0a_j}^*
\end{aligned} \right\}
\]
From the equalities we can get \( \rho_{0aj} = \frac{\tilde{\alpha}_a}{\tilde{\alpha}_a^*} \rho_{0aj}^* \). Then,

\[
\frac{\rho_{0aj}\sigma_{aj}}{\sqrt{(\tilde{\alpha}_a)^2 \sigma_{aj}^2 + 1 + 2\tilde{\alpha}_a \rho_{0aj}\sigma_{aj}}} = \frac{\tilde{\alpha}_a^* \rho_{0aj}^* \sigma_{aj}}{\sqrt{(\tilde{\alpha}_a^*)^2 \sigma_{aj}^2 + 1 + 2\tilde{\alpha}_a^* \rho_{0aj}^* \sigma_{aj}}}
\]

And we derive \( \tilde{\alpha}_a = \tilde{\alpha}_a^* \). The uniqueness of \( \rho_{0aj} \) can be derived analogously.

## C Identification for probationary cases

For probationary cases, the equations involved are:

\[
p_{ji} = 1 (\gamma_{pj} + x_i' \beta_{pj} + u_{pji} > 0)
\]

\[
t_{ji} = \gamma_{tj} + x_i' \beta_{tj} + u_{tji} \text{ if } p_{ji} = 1
\]

\[
s_{ji} = \gamma_{sj} + x_i' \beta_{sj} + u_{sji} \text{ if } p_{ji} = 1
\]

\[
y_{1ji} = 1 \{\alpha_t t_{ji} + \alpha_s s_{ji} + x_i' \beta_1 + u_{1ji} > 0\} \text{ if } p_{ji} = 1
\]

where \( p_{ji} \) is judge \( j \)'s decision on the probation status of case \( i \). \( t_{ji} \) is probationary length, \( s_{ji} \) is suspended sentence length and \( y_{1ji} \) is recidivism outcome of individual \( i \) given probation status \( p_{ji} = 1 \). Similar with the active sentence case, the subscript \( j \) for outcome equation is to identify case \( i \) is charged with judge \( j \), and we assume that all the error term are
Our goal is to identify how judges' decisions of suspended sentence $s_{ji}$ and probationary length $t_{ji}$ affect recidivism outcome. We claim that we can identify $\alpha_{s_{1j}}$ and $\alpha_{t_{1j}}$.

From the probationary decisions, we can identify a Probit model:

$$p_{ji} = 1 (\gamma_{pj} + x'_i \beta_{pj} + u_{pji} > 0) \quad \text{(A1)}$$

where $x_i$ are case characteristics and $u_{pji}$ is error term that contains judges' preference. When the case is determined non-probationary, we can observe the length of the suspended sentence and $E (s_{ji}|x_i, j, p_i = 1)$ contains a Heckman-correction term:

$$E (s_{ji}|x_i, j, p_i = 1) = E (\gamma_{sj} + x'_i \beta_{sj} + u_{sji}|x_i, j, p_{ji} = 1) = \gamma_{aj} + x'_i \beta_{aj} + \frac{cov(u_{sji}, u_{pji})}{\sigma_{pj}^2} \frac{\phi\left(\frac{x'_i \beta_{pj}}{\sigma_{pj}}\right)}{\Phi\left(\frac{\gamma_{pj} + x'_i \beta_{pj}}{\sigma_{pj}}\right)} \quad \text{(A2)}$$

Similar results hold for probation length decisions:

$$E (t_{ji}|x_i, j, p_{ji} = 1) = E (\gamma_{tj} + x'_i \beta_{tj} + u_{tji}|x_i, j, p_{ji} = 1) = \gamma_{tj} + x'_i \beta_{tj} + \frac{cov(u_{tji}, u_{pji})}{\sigma_{pj}^2} \frac{\phi\left(\frac{x'_i \beta_{pj}}{\sigma_{pj}}\right)}{\Phi\left(\frac{\gamma_{pj} + x'_i \beta_{pj}}{\sigma_{pj}}\right)} \quad \text{(A3)}$$

where the Mills' ratio part $\frac{\phi\left(\frac{x'_i \beta_{pj}}{\sigma_{pj}}\right)}{1-\Phi\left(\frac{\gamma_{pj} + x'_i \beta_{pj}}{\sigma_{pj}}\right)}$ can be identified by the Probit equation. The
Heckman two step procedure can identify \( \frac{\text{cov}(u_{sji}, u_{pji})}{\sigma_{pj}} \), \( \sigma_{sj} \), \( \frac{\text{cov}(u_{sji}, u_{pji})}{\sigma_{pj}} \) and \( \sigma_{tj} \). To construct the correlation between decisions and the outcome, we can write out \( E(y_{tji} | x_i, j, p_{ji} = 1) \).

The endogeneity here comes from two sources: correlation between judges’ preference of probation decision selection error term \( u_{pji} \) and individual unobserved error term \( u_{tji} \), suspended sentence length decision term \( u_{sji} \) and probationary length term \( u_{tji} \). It can be seen by writing out the conditional expectations:

\[
E(y_{tji} | x_i, j, p_{ji} = 1) = E\{1 \{ \alpha_t j + \alpha_s s_{ji} + x_i' \beta_1 + u_{tji} > 0 \} | x_i, j, p_{ji} = 1 \} = \Phi_2 \left( \frac{-\alpha_t (\gamma_{tji} + x_i' \beta_1) - \alpha_s (\gamma_{sji} + x_i' \beta_3) - x_i' \beta_1}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{tji})}}, \frac{\gamma_{pji} + x_i' \beta_{pj}}{\sigma_{pj}}, \rho_{pj} | x_i, j \right) \tag{A4}
\]

By maximizing the likelihood functions of \( \{ y_{tji}, x_i, j, p_{ji} \} \):

\[
\prod_{i=1}^n \{ E(y_{tji} | x_i, j, p_{ji} = 1) \}^{1(y_{tji} = 1)} \{ 1 - E(y_{tji} | x_i, j, p_{ji} = 1) \}^{1(y_{tji} = 0)}
\]

we can identify \( \frac{\alpha_t}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{tji})}}, \frac{\alpha_s}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{tji})}} \) and \( \rho_{pj} = \text{corr} \left( \frac{\alpha_t u_{tji} + \alpha_s u_{sji} + u_{tji}}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{tji})}} \right) \). To disentangle the pure effect of \( \alpha_t \) and \( \alpha_s \), we need to trace out the correlation of \( u_{tji} \), \( u_{tji} \) and \( u_{sji} \). We have two available conditional mean functions:

\[
E(s_{ji} | x_i, j, y_{1i} = 0, p_{ji} = 1)
\]

\[
= E \{ \gamma_{sji} + x_i' \beta_{sji} + u_{sji} | x_i, j, y_{1i} = 0, p_{ji} = 1 \}
\]

\[
= \gamma_{sji} + x_i' \beta_{sji} + \frac{\sigma_{sji}}{1 - \rho_{23j}^2} \left[ (\rho_{12j} - \rho_{23j} \rho_{13j}) E(\tilde{U}_{2j} | \tilde{Y}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}) \right]
\]

\[
+ (\rho_{13j} - \rho_{23j} \rho_{12j}) E(\tilde{U}_{3j} | \tilde{Y}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}) \right]
\]

(A5)
\[ E(t_{ji}|x_i, j, y_{i1} = 0, p_{ji} = 1) \]
\[ = E\{\gamma_{tj} + x'_t \beta_{tj} + u_{tji}|x_i, j, y_{i1} = 0, p_i = 1\} \]
\[ = \gamma_{tj} + x'_t \beta_{tj} + \frac{\sigma_{tj}}{1 - \rho_{23j}^2} \left[ (\rho_{42j} - \rho_{23j}\rho_{43j}) E\left(\tilde{U}_{2j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}\right) \right. \]
\[ + \left. (\rho_{43j} - \rho_{23j}\rho_{42j}) E\left(\tilde{U}_{3j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}\right) \right] \]
\[ \text{where} \]
\[ \tilde{U}_{1j} = u_{sji}, \quad \tilde{U}_{4j} = u_{tji} \]
\[ \tilde{\tilde{U}}_{2j} = \frac{\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji}}{\sqrt{\text{Var}(\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji})}}, \quad \tilde{\tilde{c}}_{2j} = -\frac{-\alpha_{i}\left(\gamma_{tj} + x'_t \beta_{tj}\right) - \alpha_{s}\left(\gamma_{s} + x'_s \beta_{s}\right) - x'_t \beta_1}{\sqrt{\text{Var}(\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji})}} \]
\[ \tilde{\tilde{U}}_{3j} = \frac{u_{tji}}{\rho_{pj}}, \quad \tilde{\tilde{c}}_{3j} = -\frac{\gamma_{pj} + x'_t \beta_{pj}}{\rho_{pj}} \]
\[ \rho_{12j} = \frac{1}{\sigma_{sji}} \text{Cov}\left(u_{sji}, \frac{\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji}}{\sqrt{\text{Var}(\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji})}}\right) \]
\[ \rho_{42j} = \frac{1}{\sigma_{tj}} \text{Cov}\left(u_{tji}, \frac{\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji}}{\sqrt{\text{Var}(\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji})}}\right) \]
\[ \rho_{23j} = \text{Cov}\left(\tilde{U}_{2j}, \tilde{U}_{3j}\right) = \text{Cov}\left(\frac{\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji}}{\sqrt{\text{Var}(\alpha_{i}u_{tji} + \alpha_{s}u_{sji} + u_{1ji})}}, \frac{u_{tji}}{\rho_{pj}}\right) \]

Regression \( E(s_{ji}|x_i, j, y_{i1} = 0, p_{ji} = 1) \) and \( E(t_{ji}|x_i, j, y_{i1} = 0, p_{ji} = 1) \) on \( E\left(\tilde{U}_{2j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}\right) \) and \( E\left(\tilde{U}_{3j}|\tilde{U}_{2j} \leq \tilde{c}_{2j}, \tilde{U}_{3j} > \tilde{c}_{3j}\right) \) can help us identify \( \frac{\sigma_{tj}}{1 - \rho_{23j}^2} (\rho_{42j} - \rho_{23j}\rho_{43j}) \) and \( \frac{\sigma_{tj}}{1 - \rho_{23j}^2} (\rho_{42j} - \rho_{23j}\rho_{43j}) \). Note that \( \rho_{23j} \) contains the correlation between the judges’ preference \( u_{sji} \) and the combination of individual unobservable \( u_{1ji} \) and it has been identified from the likelihood functions of \( \{y_{1ji}, x_i, j, p_{ji}\} \). \( \rho_{13j} \) and \( \rho_{43j} \) has been identified from the Mills’ ratio term in \( E(s_{ji}|x_i, j, p_i = 1) \) and \( E(t_{ji}|x_i, j, p_i = 1) \). Thus, we are able to identify \( \rho_{42j} \) that contains correlation between \( u_{sji}, u_{tji} \) and \( u_{1ji} \).
To conclude, we can solve for four values from equation (A5) and (A6):

\[
\frac{\rho_{1s_j} \sigma_{1j}}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{1ji})}}
\]

\[
\frac{\rho_{1t_j} \sigma_{1j}}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{1ji})}}
\]

\[
\frac{\alpha_t}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{1ji})}}
\]

\[
\frac{\alpha_s}{\sqrt{\text{Var}(\alpha_t u_{tji} + \alpha_s u_{sji} + u_{1ji})}}
\]

and we can identify $\alpha_s/\sigma_{1j}$ and $\alpha_t/\sigma_{1j}$ from the equation system.

D Simulation Results
Figure A1: Simulation Distribution for Active Sentencing Cases