ACOUSTIC LOGIC GATES IMPLEMENTED USING A PHASE-CONTROLLING PHONONIC CRYSTAL

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ABSTRACT

In this thesis a phononic crystal (PC) consisting of a square array of cylindrical Polyvinylchloride (PVC) inclusions in air is used to construct a variety of acoustic logic gates. The use of Plane Wave Expansion (PWE) method is employed to obtain the band structure and equifrequency contours. In a certain range of operating frequencies, the PC band structure shows square-like equifrequency contours centered off the Gamma point. This attribute allows for the realization of non-collinear wave and group velocity vectors in the PC wave vector space. This feature can be utilized to control with great precision, the relative phase between propagating acoustic waves in the PC. By altering the incidence angle of the impinging acoustic beams or varying the PC thickness, interferences occur between acoustic wave pairs. It is recognized that information can be encoded with this mechanism (e.g. wave amplitudes/interference patterns) and accordingly to construct a series of logic gates emulating Boolean functions. The NAND, XOR, and NOT gates are demonstrated with Finite-Difference Time-Domain (FDTD) simulations.
CHAPTER 1

INTRODUCTION

1. A brief history of phononic crystals

Phononic crystals are composite materials which are periodically composed in 1, 2, or 3 dimensions of two or more materials which differ in physical and elastic properties. This periodicity and difference in elastic impedances of the constituent materials can lead to Bragg scattering and/or local resonance, giving rise to frequencies where wave propagation is forbidden. The field of phononic crystals (PC) and acoustic metamaterials came to fruition out of the field of photonic crystals (e.g. composites which differ in dielectric properties) and electromagnetic metamaterials. It should be mentioned that the initial idea of periodic composite structures differing in dielectric properties was first introduced by E. Yablonovitch in 1987 [1]. E. Yablonovitch postulated that for a three dimensional composite material there might be frequencies where no wave modes can propagate. The notion of “gaps” in the dispersion curves of such structures ignited the field of photonic crystals and soon to follow phononic crystals. The backbone of phononic has its foundation in the acoustics and elastodynamics, the discipline of physics concerned with the propagation of sound and elastic waves in materials. Naming conventions are usually designated by the supported wave modes, for example, sonic crystals correspond to structures which support only low frequency longitudinal modes whereas the use of phononic crystals is used for structures which
support both transverse and longitudinal over a range of frequencies. In this Thesis they are used interchangeably.

Much of the progress in the field of phononic crystals has been carried out within the last two decades, but wave propagation in composite structures has been investigated for several decades [1-4]. It was theoretically demonstrated by Achenbach et al. [3] that for an infinite three-dimensional structure of stacked parallel planes consisting of rectangular arrays of spherical voids in each layer; at a low-frequency there would exist a region on the dispersion curve where no longitudinal modes could exist. This spark was a launching point and propelled the theoretically and experimentally investigation of the possibilities of band gaps in periodic elastic composite systems. These gaps result from inhibiting the transmission of phonons. In the early 1990’s the existence of frequency gaps in the band structure for acoustic and elastic waves was demonstrated [4-7]. This began with M.M. Sigalas and E.N Economou [4-5], who theoretically demonstrated that for an infinite two dimensional array of high-density parallel cylinders embedded in a low-density host material a complete band gap should exist. Shortly after, both E.N. Economou et. al [6]and M. Kushwaha et. al [7] reported the first full band structure calculation for vibrations of transverse polarization in a two dimensional PC. In 2001 J.O. Vasseur et. al [8] theoretically and experimentally reported the existence of absolute band gaps in the band structure for a two dimensional PC consisting of steel cylinders arranged in a cubic structure inside an epoxy matrix. Complete band gaps mean that both longitudinal and transverse are forbidden for all directions of propagation. It was quickly the objective of many researchers to exploit these band gaps for a variety of applications,
such as, filtering, trapping, and guiding waves [8-16]. Researchers continued to investigate the effects of symmetry in a PC structure, impedance mismatch between dissimilar materials, and resonances associated with the inclusions in a PC and their effects on the band structure. It soon became evident that the tailoring of elastic wave dispersion was very much possible [13-16].

Other functions which were exploited due to the spectral (ω-space) band gaps present made use of intentional defects in phononic crystals [17-22]. This is usually done by removal of a few or more periodic inclusions. The result, lead to the ability to trap specific modes which could exist within a PC, resulting in resonant cavities and waveguides [23-27]. For point like defects inside a PC, an acoustic source could be coupled to the defect, to yield a resonant cavity capable of producing a highly directional acoustic source [28-29]. In addition, the removal of a line of periodic inclusions allowed for the ability to use these structures as waveguides within the PC. Many researchers have theoretically and experimentally shown that with small line defects there is little loss in transmission of guided waves [24-27]. However, increases in the width of the line defects induced loss in transmission due to interference effects. The bending of waves inside these waveguides has also been displayed [26]. The ability to guide waves has been used to produce multiplexing and demultiplexing devices [27]. This kind of device was based on the fact that inserting two types of defects which alternate along a row of inclusions, allow for two waves of differing frequencies to travel together (i.e. multiplexing). Then by adding a Y-shape split to the end of the two defect channel by
which only one frequency was supported, the waves would travel different paths (i.e. demultiplexing.)

Additionally, properties relative to the wave vector ($k$-space) in PCs became of interest due to unique characteristics refractive behavior [30]. The first application of this was shown with photonic crystals by Pendry et al. [31]. They demonstrated that focusing electromagnetic waves was possible by exploiting negative refraction in a flat lens. It followed that phononic crystals displayed certain passing bands in the dispersion curves which exhibited negative indices of refraction [32-36]. In phononic crystals, negative refraction is achieved when the wave group velocity is antiparallel to the wave vector. Bragg scattering in PCs results in band-folding whereby several bands with negative slope (negative group velocity and positive phase velocity) are produced, a prerequisite for negative refraction. In addition it was demonstrated that some phononic crystals would exhibit positive, negative, and zero angle refraction [37]. Researchers began to show both experimentally and theoretically that phononic crystals exhibited such characteristics and could be used as flat lenses, allowing for sub wavelength resolution [15]. This was demonstrated in a two-dimensional PC constituted of a triangular lattice of steel rods immersed in a liquid. High fidelity imaging is obtained when all-angle negative refraction conditions are satisfied, that is, the equifrequency contour of the PC is circular and matches that of the medium in it is embedded. This PC was able to provide sub wavelength resolution by manipulating the transmittance of evanescent modes via the excitation of vibrational modes bound to the PC slab. This is in contrast to a conventional lens which only transmits the propagating components of a source.
Recently, progress has been made in the extension of the properties of PCs beyond the $\omega$-$k$ space and into the space of acoustic wave phase ($\varphi$-space) [38-39]. The relative phase between pairs of acoustic waves propagating in a PC medium can be fully understood with analysis of the PC band structure and equifrequency contours (EFCs). It was demonstrated that PCs showing EFCs with non-collinear wave and group velocity vectors are ideal systems for controlling the relative phase between propagating acoustic waves on account of the fact that excited Bloch modes within the PC travel at different phase velocities [39]. In this situation, the relative phase between waves can be precisely modulated by either changing the incident angles of the incoming acoustic beams (i.e. changing the phase velocity at which the beams travel in the PC) or varying the thickness of the PC. In addition to this feature is another phase-property of PCs stemming from having the EFC of the incident homogeneous medium being larger than the first Brillouin zone of the PC. In this instance, the same Bloch modes can be excited within the wave vector space ($k$-space) of the PC by incident waves with different incident angles. Impinging beams with this characteristic are referred to as complementary angle inputs or complementary waves. The complementary waves travel the same path within the PC (i.e. the path consistent with the angle of refraction) and, upon exiting the PC, contribute to beam splitting [38,39]. Altering the relative phase between the incident beams changes the manner at which these modes superpose within the PC. Specifically, if two complementary waves enter the PC out-of-phase, then along the path of the refracted beam, destructive interferences will cancel out the amplitudes of the propagating waves. These PCs which have distinctive phase-properties can greatly enrich the field of
phononics. PCs with full spectral, wave-vector, and phase-space properties ($\omega, k, \text{and } \varphi$) have the potential of impacting a broad range of technologies. Past efforts in developing acoustic devices utilizing PCs with just spectral or refractive properties have led to inherently passive designs. Adding the dimension of phase-control enhances the functionalities of PCs and can lead to more active acoustic based technologies.

It has been the objective of the research presented in this thesis to exploit the above mentioned phase-phenomena in a two dimensional PC to develop a new paradigm in acoustic wave based Boolean logic functions. The model presented here offers precise control of phase between acoustic waves within a finite volume of the PC. Past work in acoustic wave based logic devices utilized the addition or subtraction of out-of-phase sources; however, this work did not include the use of a phononic crystal [40]. Other work in photonic crystals, utilized non-linear effects in the temporal transmission spectra [41, 42]. The work presented here constitutes a significant effort to broaden the functions of two dimensional PC’s. This thesis is organized as follows: In Chapter 1, the mathematical foundation from atomic crystal theory is used to derive an understanding of waves in periodic media. In Chapter 2, a derivation of an analytical model for controlling the phase between acoustic waves propagating through a PC is presented. Also, a brief discussion on the elastic wave equation and Plane Wave Expansion (PWE) method is given. In Chapter 3, the Finite-Difference Time-Domain (FDTD) method is detailed and its application for simulating acoustic based Boolean logic gates, specifically, the NAND, XOR and NOT functions. Thereafter, the results of the FDTD simulations are reported. The conclusions are drawn at the end of the chapter.
2. Formalism for understanding phononic crystals

It is the purpose of this section to introduce the reader to the formalism used to describe wave propagation in periodic media. The fundamental underlying mathematical description is taken from atomic crystal theory and is applied to develop a lattice of inclusions in a matrix. In Fig. 1.1 the arrangement of inclusions in a matrix are shown to give the reader some insight to how these phononic crystals might physically appear.

Beginning with the concept of the lattice, which is defined to be an infinite collection of points (basis) that have spatial position which appears the same at any viewing point [43]. A lattice may be described mathematically by a vector \( \mathbf{r} \) and takes the following form:

\[
\mathbf{r} = x \mathbf{b}_1 + y \mathbf{b}_2 + z \mathbf{b}_3
\]

Eq (1.1)

In Eq. (1.1) \( \mathbf{b}_1, \mathbf{b}_2, \) and \( \mathbf{b}_3 \) are referred to as the basis vectors. The variables \( x, y, \) and \( z \) are integer values describing the location of lattice sites. It is standard practice to select basis vectors which are primitive, constructing what is known as a Bravais lattice. In Fig. 2.1 a three dimensional cubic system is shown with examples of different selection of basis vectors. Another, and possibly more important aspect to embrace when dealing with waves in periodic structures, is the concept of identifying the Wigner-Sitz cell (also known as Voronoi cell). This cell is the smallest and contains the symmetry of the Bravais lattice. The Wigner-Seitz cell is constructed by selecting a lattice point and drawing line segments to its closest nearby lattice points. Then at each line segments midpoint another line is drawn normal to the first line. The use of the Wigner-Sitz cell is more common when in reciprocal space and is given the name, Brillouin zones. It is now
relevant to discuss the transformation of the lattice from physical/real space to what is known as reciprocal space. This transformation is convenient because it happens to have dimensions in relation to wave propagation (i.e. $m^{-1}$). Mathematically any periodic function (i.e. points of a lattice) may be represented by a Fourier series.

Figure 1.1: 1-D and 2-D arrangements for two material Phononic crystal (a) 1-D arrangement with two materials which differ in density and elastic constants, having $\rho_1, c_1$ and $\rho_2, c_2$. The structure is periodic in the $x$ direction and considered infinite in $y$ and $z$ direction (b) 2-D arrangement for differing materials. The structure is periodic in the $x$ and $y$ directions and considered infinite in the $z$ direction.
Fig. 1.2: A schematic of a two dimensional square lattice displaying the variety of choices for lattice vectors $\mathbf{b}_1$ and $\mathbf{b}_2$. The selection of $\mathbf{b}_1^*$ and $\mathbf{b}_2^*$ is the primitive lattice basis.
Thus, imposing the conclusion of periodicity on the lattice points leads to:

$$\sum_{\vec{G}} f(\vec{G}) e^{i\vec{G} \cdot (f+\vec{R})} = \sum_{\vec{G}} f(\vec{G}) e^{i\vec{G} \cdot \vec{r}}$$  \hspace{1cm} \text{Eq. (1.2)}

Where \( f(G) \) is the Fourier coefficient, \( R \) is some lattice translation, and \( G \) is the span of reciprocal lattice vectors, thus Eq.1.2 can only be true for any lattice vector if:

$$e^{i\vec{G} \cdot \vec{R}} = 1$$  \hspace{1cm} \text{Eq. (1.3)}

Thus, we now can define the reciprocal lattice and as a consequence of Eq. 1.3:

$$\vec{G} \cdot \vec{R} = 2\pi m$$  \hspace{1cm} \text{Eq. (1.4)}

Where \( m \) takes on integer values. Using Eq.1.1 for our real lattice and the following expression for \( G \):

$$\vec{G} = \frac{2\pi}{n} \vec{B}_1 + \frac{2\pi}{n} \vec{B}_2 + \frac{2\pi}{n} \vec{B}_3$$  \hspace{1cm} \text{Eq. (1.5)}

Evaluating Eq. 1.4, one arrives at:

$$\vec{B}_1 = 2\pi \frac{\vec{b}_2 \times \vec{b}_3}{\vec{b}_1 \cdot (\vec{b}_2 \times \vec{b}_3)}$$  \hspace{1cm} \text{Eq. 1.6 a-c)}

$$\vec{B}_2 = 2\pi \frac{\vec{b}_3 \times \vec{b}_1}{\vec{b}_1 \cdot (\vec{b}_2 \times \vec{b}_3)}$$

$$\vec{B}_3 = 2\pi \frac{\vec{b}_1 \times \vec{b}_2}{\vec{b}_1 \cdot (\vec{b}_2 \times \vec{b}_3)}$$

Where \( \vec{b}_1, \vec{b}_2, \vec{b}_3 \) are the primitive basis lattice vectors of the real space crystal and \( \vec{B}_1, \vec{B}_2, \) and \( \vec{B}_3 \) are the primitive basis lattice vectors for reciprocal space.
**Fig. 1.3:** (a) primitive cubic real space lattice, $|\mathbf{b}_1|=|\mathbf{b}_2|=|\mathbf{b}_3|$. (b) Resulting reciprocal lattice due to equations 1.6a-c, $|\mathbf{B}_1|=|\mathbf{B}_2|=|\mathbf{B}_3|$.

**Fig. 1.4:** The representation of Brillouin zones for primitive cubic lattice. Filled in black region represents irreducible Brillouin zone with high symmetry points $\Gamma$, $M$, and $X$. Solid lines represent 1st Brillouin zone and dashed lines 2nd Brillouin zone.
The resulting transformations due to equations 1.6a-c for a cubic system can be seen in Fig. 1.3. It turns out that for a cubic system the real space lattice yields a cubic reciprocal lattice. As mentioned earlier one may construct the Wigner-Sitz cell in reciprocal space to obtain a unit cell, which is named the Brillouin zone. Depending on the selection of reciprocal basis lattice vectors, the constructed cell is given a numerical number associated with the Brillouin zone. The irreducible Brillouin zone is the smallest cell that contains all symmetry of the crystal. It therefore can give all information needed to for understanding waves traveling in a crystal. In Fig. 1.4, the Brillouin zones for a two dimensional simple cubic lattice are shown. Are shown, the numbers correspond to the zones; usually the irreducible zone is all that is needed, to fully describe the reciprocal space but in the case where more information is required, other zones may be investigated. Zones outside the irreducible Brillouin zone are usually referred to as extended zones. As was mentioned above the use for the reciprocal space and the Brillouin zones are very convenient to describe wave propagation in periodic structures. We will see later that the use of high symmetry points in the irreducible Brillouin zone is used to display the dispersive information of a phononic crystal. The 1st and 2nd Brillouin zones are also useful in mapping out the refractive properties of waves with use of the equifrequency contours in the Brillouin zones.
CHAPTER 2

PHASE-CONTROLLING WITH SOLID/AIR PHONONIC CRYSTALS

1. The Elastic wave-equation

It is the purpose of this chapter to introduce the formulation of elastic/acoustics waves travelling within a PC. Beginning with the presentation of the elastic wave equation and an overview of PWE. Following, is the resulting band structure and equifrequency contours for the system of interest, PVC-Air PC. Then an analytical model is developed for understanding wave phase (φ-space) in a phononic crystal.

The elastic wave equation for a composite which is isotropic but considered inhomogeneous, that is that the elastic parameters and density for the system are functions of position, is:

\[
\frac{\partial^2 u_i}{\partial t^2} = \frac{1}{\rho(\vec{r})} \frac{\partial T_{ij}}{\partial x_j} \quad \text{Eq. (2.1)}
\]

Where \( u_i \) is the \( i^{th} \) component of the displacement and \( T_{ij} \) is the stress tensor. The stress tensor for an isotropic material can be defined by using Hooke’s law and symmetry properties of the elastic constant tensor to define a stress–strain relationship as:

\[
T_{ij} = \lambda(\vec{r}) \varepsilon_{kk} \delta_{ij} + 2\mu(\vec{r}) \varepsilon_{ij} \quad \text{Eq. (2.2)}
\]

Where:

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad \text{Eq. (2.3)}
\]
In equation (2.2) \( \lambda \) and \( \mu \) are known as the lame’ coefficients and are related to the transverse, \( c_t \), and longitudinal, \( c_l \), speeds of sound and density, \( \rho \), of the medium:

\[
\lambda = \rho c_t^2, \mu = \rho c_l^2 - 2\rho c_t^2 \quad \text{Eq. (2.4)}
\]

The elastic wave equation allows for the numerical study of elastic and acoustic waves in Phononic crystals. The research presented in this work is grounded on the basis of numerical methods for solving the elastic wave equation (i.e. PWE and FDTD).

2. Plane-wave expansion method

For the two dimensional solid-fluid PC reported in this thesis it is necessary for one to assume that the solid cylinders (i.e. PVC inclusions) are infinitely rigid [44]. This assumption means that for the inclusions the modulus of compressibility and density are infinite (i.e. very large for compared to air). This is well justified for the PVC-Air system because it implies the sound does not penetrate the inclusions and hence the wave propagation is confined to the air. Therefore, the following derivation is valid for longitudinal modes because the transverse speed of sound for fluids is zero. In this method the wave equation is formulated as an eigenvalue problem by taking the Fourier expansion of the elastic displacement field and the physical parameters of the constituent materials along reciprocal lattice vectors. The substitution of Eq. 2.1 – 2.4 and setting \( c_t = 0 \); the wave equation for longitudinal modes may be written in Cartesian form as:

\[
\rho(\vec{r}) \frac{\partial^2 \vec{u}}{\partial t^2} = \vec{\nabla} \left( \rho(\vec{r})c_t(\vec{r})^2 \vec{\nabla} \cdot \vec{u} \right) \quad \text{Eq. (2.5)}
\]
Where $\rho_{c1}$ is defined to be the elastic constant $C_{11}$. One can now expand $C_{11}$, and the density by a Fourier series, this is due to the periodic nature of the two dimensional PC.

$$
\rho(\vec{r}) = \sum_{\vec{G}} \sigma(\vec{G}) e^{i\vec{G} \cdot \vec{r}} \quad \text{Eq. (2.6 a-b)}
$$

$$
C_{11}(\vec{r}) = \sum_{\vec{G}} \beta(\vec{G}) e^{i\vec{G} \cdot \vec{r}}
$$

Where $\vec{G}$ and $\vec{r}$ are the reciprocal and direct lattice vectors, respectively. Assuming a plane wave solution and the periodicity of the medium; one can invoke Bloch theorem to solve for Eq. 2.5, for which the displacement field is:

$$
\bar{u}(\vec{r}, t) = e^{i(\vec{K} \cdot \vec{r} - \omega t)} \sum_{\vec{G}} \bar{u}_k(\vec{G}) e^{i\vec{G} \cdot \vec{r}} \quad \text{Eq. (2.7)}
$$

Here $\vec{K}$ is the two dimensional Bloch vector. In short, the combination of Eq. 2.6a-b with Eq. 2.7, one can formulate an infinite set of equations for which the eigenvalues, $\omega(K)$, and eigenvectors, $\bar{u}_k(G)$, can be found:

$$
\sum_{\vec{G}} \left[ \sigma(\vec{G} - \vec{G}')(\vec{K} + \vec{G}') \cdot (\vec{K} + \vec{G}') - \beta(\vec{G} - \vec{G}') \omega^2 \right] \bar{u}_k(\vec{G}') = 0 \quad \text{Eq. (2.8)}
$$

One now can investigate the two dimensional reciprocal lattice for a PC with filling fraction $f$. The density and elastic constants take the form:
\[ q(\vec{G}) = \begin{cases} \quad xf + y(1 - f) \equiv z', \text{ for } \vec{G} = 0 \\ \quad (x - y)F(\vec{G}) \equiv \Delta zF(\vec{G}), \text{ for } \vec{G} \neq 0 \end{cases} \]

Eq. (2.9)

Where \( q \) is stands for by \( \sigma \) or \( \beta \) and \( x, y \) stands for the density or elastic constants for the inclusions and matrix. The structure factor \( F(\vec{G}) \) is given by:

\[ F(\vec{G}) = \frac{1}{A} \int_{\Omega} d^2 r e^{-i\vec{G} \cdot \vec{r}} \]

Eq. (2.10)

Where \( A \) is the area of the inclusion unit cell and the integration is performed over the inclusion. One can now recast Eq. 2.8 as:

\[ \sum_{\vec{G} \neq \vec{G}'} F\left(\vec{G} - \vec{G}'\right)\left[\Delta \alpha (\vec{K} + \vec{G}) \cdot (\vec{K} + \vec{G}') - \Delta \beta \omega^2\right] \vec{u}_k(\vec{G}') + [\rho^{-1}|\vec{K} + \vec{G}|^2 - \beta' \omega^2] \vec{u}_k(\vec{G}) = 0 \]

Eq. (2.11)

The previous formulation for waves in a periodic material demonstrates that it can be solved numerically to determine the eigen frequencies and eigenvectors for a given set of wave vectors. It is common practice to find the eigen frequencies for wave vectors which are within the irreducible Brillouin zone (i.e. directions of high symmetry). In order to obtain good convergence with this method, the number of plane waves is chosen to be 441 [44]. In appendix A.2, I provide a MATLAB code which utilizes PWE was used to calculate the band structures and equifrequency contours in this work.
3. Band structure and equifrequency calculations for PVC-Air crystal

The previous section discussed the PWE method which is used to determine the band structure and EFC of the PVC-Air phononic crystal. A band structure or dispersion diagram relates the frequency of a wave with its wave-number. These diagrams are extremely useful in determining the available frequencies for propagation and the phase and group velocity of waves in materials (i.e. velocity of the waves and velocity of energy). These diagrams are usually formatted in such a way that they take advantage of symmetry in crystals so that only directions of high symmetry need only be displayed. On the other hand if one is interested in the complete propagation of waves in a PC, then the use of EFC need be investigated. EFC can be obtained by PWE method, the only variation is that all eigenvalues for k at a specific frequency be found. These curves are particularly useful in analyzing the refraction of a wave impinging on a PC and the Bloch modes which may be excited within the PC.

As discussed earlier the system of interest in this research is a two dimensional PC consisting of a square array of PVC cylinder inclusions which are embedded in an air matrix. The lattice parameter is, \( a = 27.0 \) mm, and the radius of the inclusions is \( r = 12.9 \) mm, see Fig. 2.1(a). The physical parameters of the constitutive materials of this PC are: \( \rho_{pvc} = 1264 \text{ kg/m}^3, C_{t,pvc}=1000 \text{ m/s}, C_{l,pvc} = 2300 \text{ m/s}, \rho_{air}=1.3 \text{ kg/m}^3, C_{t,air}=0 \text{ m/s}, C_{l,air} = 340 \text{ m/s} \), where \( \rho \) is the mass density, \( C_t \) is the transverse speed of sound, and \( C_l \) is the longitudinal speed of sound. With the use of the PWE method the resulting band structure for this system can be seen in Fig. 2.1(b). It is pointed out that there exist a complete band gap between 4 and 10 kHz. It also is noted that there exist a partial band
gap between 13.5 to 17.0 kHz (fourth band) in the ΓX direction. The corresponding EFC for frequencies in the range of 13.5 and 17.0 kHz are shown in Fig. 2.2(a). It allows for the investigation of the group velocities which are the gradient of the \( \omega (k) \) on the dispersion surfaces. For application purposes, in the rest of this thesis, the frequency is chosen to be 13.5 kHz, Fig. 2.2(a) shows the EFC for this frequency in blue. Investigation of Fig. 2.2(a) shows that the contours are not centered on the Γ point, which is consistent with the partial band gap seen along the ΓX direction in this frequency range (see Fig. 2.1(a)). An extended zone scheme of the first Brillouin zone is shown in Fig. 2.2(b). One can see the square-like shape of these contours with this representation. For clarity, other pass bands in this system which exhibit partial band gaps and EFCs which are nearly square may be used to construct the logic gates presented later in this work.

The use of EFCs is particularly useful in understanding wave propagation in PCs. The extended zone scheme of the EFC in Fig. 2.2(b) is used in this thesis to analyze waves propagating in a finite slab of PVC-Air PC implanted in air enclosure. This representation highlights the various Bloch modes of the PC that will be excitable by incident acoustic waves with wave vectors exceeding the size of the first Brillouin zone (see Fig. 2.3). These waves are a result of the excitation of Bloch modes in the PC. For this system, it so happens that the EFC of air is twice as large as the PVC-Air crystal. In this thesis incident waves which excite the same Bloch modes in the PC are referred to as complementary waves, whereas, waves which do not excite the same Bloch modes are referred to as non-complementary waves. Another result of the EFC of Air being twice that of the EFC of the PVC-Air PC, results in beam splitting with angle dependent
amplitude. In spite of beam splitting, the research in this work is focused on the exiting
Wave vectors $\mathbf{k}_1$ (see Fig. 2.3).

Fig 2.1: (a) Structure of PVC-Air phononic crystal (PC) with lattice parameter, $a=27\text{mm}$, and radius, $r=12.9\text{mm}$. (b) Band structure determined by Plane wave expansion method (PWE) for PVC-Air phononic crystal (PC) with lattice parameter $a = 27\text{mm}$ and radius of inclusions $r = 12.9\text{mm}$. Shown are the principal directions of propagation in the 1st Brillouin zone for PVC-air system, dashed line at 13.5 KHz.
Fig. 2.2: (a) Equifrequency contours at frequencies 13.5-17.0 kHz (in increments of 0.7 kHz) for PVC-Air phononic crystal in 1st Brillouin zone with high points of symmetry and irreducible Brillouin zone displayed. Determined by PWE method. (b) Equifrequency contours at frequencies 13.0-17.0 kHz for PVC-Air phononic crystal in extended zone scheme
Fig 2.3: Equifrequency contour (EFC) of source host and exit medium (Air) and embedded PVC-Air PC at frequency 13.5 kHz. The EFC for air is twice that of the PC, thus Bloch waves are excited within the PC corresponding excited wave vectors are shown. \(V_g\) is the group velocity and \(\alpha\) is the angle it makes with the horizontal axis.
4. Phase-Controlling formulations

In this subsection, we cover the phase controlling properties of the PVC-Air PC that have been demonstrated by Swinteck et. al [39]. The phase of acoustic waves propagating through a finite thickness crystal (thickness d) arises from the fact that the group velocity and wave vector are not collinear. The group velocity vector is oriented along the gradient of the EFC, which is the direction of energy propagation. The wave vector inside the PC originates at the Gamma point and ends on the EFC where its parallel component is that of the incident wave. The direction of the wave vector is specified by the phase velocity (i.e. velocity of the wave). Let us consider a Bloch wave with wave vector \( \mathbf{k} \), the path traveled inside the PC by the wave is given by:

\[
\vec{R} = d \hat{i} + d \tan(\alpha) \hat{j}
\]

Where \( \hat{i} \) and \( \hat{j} \) are unit vectors in the horizontal (x) and vertical (y) directions, respectively, and \( \alpha \) is the angle the group velocity makes with the horizontal axis. We now take into account two non-complementary waves with wave vectors, \( \mathbf{k} \) and \( \mathbf{k}' \) (i.e. different group velocities, therefore different \( \alpha \)), entering the PC at the same location. These two waves will travel paths of different length and direction. This path difference is due to the non collinear group velocity and wave vector, which causes the two waves to exit the PC at different locations and exhibit relative phase shift. To calculate the phase shift in the PC between these two waves, we first write their spatial form when they exit as:

\[
e^{i\vec{k} \cdot \vec{R}} = e^{i \frac{2\pi}{a} d[k_xi + \tan(\alpha)kyj]}
\]

Eq. (2.13)
\[ e^{i(k \cdot R')} = e^{i \frac{2\pi}{a} [k_x \hat{i} + \tan(\alpha') k_y \hat{j}]} \quad \text{Eq. (2.14)} \]

Where \( R \) and \( R' \) are the path traveled by the two waves in the PC given by Eq. 2.12. The difference in the argument of the real parts for the two waves (i.e. phase shift) is therefore given by:

\[
\phi_{k,k'} = (\vec{k} \cdot \vec{R}) - (\vec{k}' \cdot \vec{R}')
\]

\[
= \frac{2\pi}{a} d \left\{ k_x \hat{i} + \tan(\alpha) k_y \hat{j} - k'_x \hat{i} - \tan(\alpha') k'_y \hat{j} \right\} \quad \text{Eq. (2.15)}
\]

From Eq. 2.15 the thickness, \( d \), of the crystal plays an integral part in the phase shift between waves in the PC. This can be seen in Fig. 2.4 which demonstrates the phase shift, for a finite crystal of 23 periods, as a function of incident angles. The validity of equation 4 was confirmed via FDTD method (see Chapter 3.1 for details).

**Fig. 2.4:** Phase shift as a function of plane wave incident angle. Eq. 2.15 is plotted as circles and is compared to FDTD (see Chapter 3.1) phase shift measurements.
Fig 2.5: (a) EFC diagram of PVC-Air PC embedded in air. Two different wave vectors, $k$ and $k'$, are incident on the left surface of the crystal. These two incident waves do not excite the same Bloch modes inside the PC and therefore have group velocity vectors with different angles, $\alpha$ and $\alpha'$. (b) In direct space there is a phase shift between $k$ and $k'$ due to the different paths, $R$ and $R'$ traveled in the PC, as well as, the paths traveled outside the PC. Beam splitting occurs but is not shown in direct space for simplicity.
It is illustrated in Fig. 2.5(a) the EFC’s of air and the PC with two acoustic waves in air incident upon the crystal and exiting to air. The wave vector that is parallel to the surface of the PC is conserved when entering and exiting the crystal. In this figure, the direct space diagram displays the phase shift which occurs between two non-complementary waves incident from air on to the same point of the PC. (See Fig 2.6(b)) Utilizing Eq. (2.15) it can be shown that a phase shift will occur in the PC between the two waves. This phase shift is preserved and upon exiting the crystal the two waves will cross paths where the vectors $\mathbf{R}_{\text{out}}$ and $\mathbf{R}_{\text{out}}'$ intersect (see Fig. 2.5(a)). These different paths, $\mathbf{R}_{\text{out}}$ and $\mathbf{R}_{\text{out}}'$, lead to an additional phase shift which is given by the first difference in Eq. (2.15).

$$\Phi_{\text{total}} = (\mathbf{k}_{\text{out}} \cdot \mathbf{R}_{\text{out}}) - (\mathbf{k}'_{\text{out}} \cdot \mathbf{R}'_{\text{out}}) + (\mathbf{k} \cdot \mathbf{R}) - (\mathbf{k}' \cdot \mathbf{R}')$$  Eq. (2.16)

Thus non complementary waves which travel from the input side, through the PC, and exit on the output side, will cross paths at some distance to constructively or destructively interfere.

Now consider two complementary waves with wave vectors, $\mathbf{k}$ and $\mathbf{k}'$, incident on the same location on the surface of the PC slab. These two complementary waves excite the same Bloch modes inside the PC. This situation is illustrated in Fig. 2.6(a); since the two waves excite the same Bloch modes their group velocity vectors are identical (i.e. same refraction angle $\alpha$) and therefore will occupy the exact volume. As a result of this, the two waves will follow an identical path as they propagate through the crystal (see Fig. 2.6(b)). Thus, according to Eq. (2.15) there is no phase shift between complementary waves inside the crystal. It is also necessary to note that the waves inside the crystal...
retain the phase difference that may have been imposed on the incident acoustic waves prior to entering the PC. Subsequently by controlling the phase difference between these two complementary waves, one may obtain constructive or destructive interferences inside the PC. Destructive interference will lead to a condition for which there is no propagation through the crystal. On the other hand, constructive interference will allow for acoustic waves to travel through the PC. Similarly to the case of non-complementary inputs, we now arrive at the capability to encode information within the amplitude of incident complementary acoustic waves and operate on this information via interference.
Fig. 2.6: (a) EFC diagram for two different wave vectors, $k$ and $k'$, incident on the left surface of the PC which excite the same Bloch modes (i.e. matching group velocity angle $\alpha$). (b) In direct space both waves experience identical paths traveled and will therefore superimpose in direct space and exit along the same point of the PC (Beam splitting is shown in direct space).
CHAPTER 3

REALIZATION OF ACOUSTIC LOGIC GATES

In this section it is demonstrated that with a phase-controlling PC it is possible to construct active devices. Boolean logic gates are a fundamental concept in Electronics, the idea that true and false statements are evaluated based on the state of the system. The formulation here is that the pressures at a point in space can be represented by on and off states.

1. Finite-difference Time-domain method

The Finite-difference Time-domain method is extremely useful in understanding and simulating the propagation of elastic/ acoustic waves inside a phononic crystal [46]. The basis for this method is that the elastic wave equation (see Chapter 2.1, Eq. 2.1) is discretized in both time and space on a finite size mesh. Beginning with Eq. 2.1, 2.2, and 2.3 in two dimensions for x and y is:

\[
\frac{\partial^2 u_x}{\partial t^2} = \frac{1}{\rho(\vec{r})} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) \quad \text{Eq. (3.1 a-b)}
\]

\[
\frac{\partial^2 u_y}{\partial t^2} = \frac{1}{\rho(\vec{r})} \left( \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right)
\]

For which the stresses are:

\[
\tau_{xx} = (\lambda(\vec{r}) + 2\mu(\vec{r})) \frac{\partial u_x}{\partial x} + \lambda(\vec{r}) \frac{\partial u_y}{\partial y} \quad \text{Eq. (3.2 a-c )}
\]

\[
\tau_{yy} = (\lambda(\vec{r}) + 2\mu(\vec{r})) \frac{\partial u_y}{\partial y} + \lambda(\vec{r}) \frac{\partial u_x}{\partial x}
\]

\[
\tau_{xx} = \mu(\vec{r}) \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)
\]
The set of equations above are all that is needed to describe wave propagation in a two dimensional phononic crystal. To do so computationally, they must be discretized over a mesh of $i_{\text{max}}$ and $j_{\text{max}}$ grid points, where by the grid points are spaced by $\Delta x$ and $\Delta y$.

Equally, one must carry out discretization in time with a time step of $\Delta t$. In Eq. 3.2 a-c the displacement derivatives are approximated in space by central differences which are given by the following:

\[
\frac{\partial u_x^k(i,j)}{\partial x} \approx \frac{u_x^k(i+1/2,j) - u_x^k(i-1/2,j)}{\Delta x} \quad \text{(Eq. 3.3 a-b)}
\]

\[
\frac{\partial u_y^k(i,j)}{\partial y} \approx \frac{u_y^k(i,j+1/2) - u_y^k(i,j-1/2)}{\Delta y}
\]

Where $i$ and $j$ are the index of grid points and $k$ is the time step. It should be mentioned that the above way for discretization of the equations insures second order accurate central difference for the space derivatives. As a result of this the displacement components $u_x$ and $u_y$ must be centered in different space points, in other words, the mesh grids are staggered. As a result of this half indexes are used and are computed by geometrically averaging the displacement component. Eq. 3.1a-b at time $k\Delta t$ are discretized through Eq. 3.3a-b. The time derivatives are approximated by a combination of forward and backward differences

\[
\frac{\partial u_x^{k+1}(i,j)}{\partial t} \approx \frac{u_x^{k+1}(i,j) - u_x^k(i,j)}{\Delta t} \quad \text{Eq. (3.4a-b)}
\]

\[
\frac{\partial u_y^{k+1}(i,j)}{\partial t} \approx \frac{u_y^k(i,j) - u_y^{k-1}(i,j)}{\Delta t}
\]
The stability and convergence of this method is acquired by ensuring that the so called courant conditions are meet, that is the time step and mesh cell spacing are small enough.

2. Boolean Logic construct

Simulations of the Boolean logic functions that can be generated using a finite size phase-controlling PC is carried out by the FDTD method previous section. The FDTD FORTRAN source code for simulation of the Acoustic Boolean logic gates has been provided in Appendix A.2. The PC is placed in an environment of air and the entire computation space is removed of any reflections and artifacts by the use of first order Mur absorbing boundary conditions on all sides of the simulation space [47]. In all the calculations reported in the next section, the PC consisted of a minimum of 23 periods in its thickness (x-direction) and 46 periods in its length (y-direction). The thickness and length are sufficient to ensure that the properties of the finite crystal approach those of the infinite crystal determined by the PWE method (see Chapter 2.1). For sources and inputs in the simulations, a diagonal line segment of nodes is given a forcing function. The forcing function prescribes displacement oscillations in the x and y direction by applying sine and cosine functions with appropriate phase. They take the following form in the FDTD code:

\[ u_x(i,j) = u_{xo}(i,j) + A \sin \theta \sin \omega t \Delta t \]  \hspace{1cm} \text{Eq. (3.5a-b)}

\[ u_y(i,j) = u_{yo}(i,j) - A \cos \theta \sin \omega t \Delta t \]

Where \( \theta, \omega, t, \Delta t \), and \( A \) are the source angle, angular frequency, current time step, time step size, and the source amplitude, respectively. The length of the sources and inputs is
sufficiently large to minimize the spread of angles, mimicking plane wave behavior. For the simulation of the logic gates, the sources and inputs may acquire any incident angle to the face of the PC that correspond with the nearly flat portion of the EFC, for the PVC-Air PC these angles correspond to 10 degrees to 50 degrees. Other angles will be reflected due to the partial band gap or will undergo too large of angels of refraction inside the PC to be useful for construction of the devices. The sources and inputs are also given initial wave phase relations. On the output side of the simulated system a detector, D, is located.

3. Results for logic gates

In this section we present the design of logic gates using the PVC-air PC and display the FDTD simulations for the NAND, XOR, and NOT gates. In these simulations the displacement fields were obtained and used to validate the possibility of achieving Boolean logic functions using propagation and interference between waves impinging on a PC.

A. NAND gate

The NAND gate is identified as a universal logic gate. Networks of universal gates such as the NAND gate can produce any of the other Boolean logic functions (i.e. all other gates)[46] . The NAND gate function consists of two inputs which are operated on by the gate to produce one output. To construct the NAND gate, a PC with two non-complementary plane wave sources, \( S_1 \) and \( S_2 \), are impinging onto the crystal at fixed angles. These angles are \( 10^\circ \) and \( 38^\circ \) with their centers incident upon the same point of the crystal. The two sources are oscillating in-phase on the input side of the PC. The
calculated total phase shift on the output side is $2\pi$ radians, by use of Eq. 2.7. This total phase shift results in constructive interference on the output side between the centers of the exiting beams. In the simulation space, the point of intersection between the centers of the exiting beams occurs at $410\ mm$ in y-direction and $1550\ mm$ in x-direction. The detector, $D$, is centered about this point and angled at $24^\circ$ ($10^\circ+38^\circ/2$) with a spatial width of $80\ mm$. Inputs, $I_1$ and $I_2$, for this system are the corresponding complementary waves to the sources, $S_1$ and $S_2$, respectively. Each input is set $\pi$ radians out of phase with the corresponding source to ensure destructive interference within the PC. These inputs are $-19^\circ$ and $-50^\circ$ and their centers are incident upon the same point on the surface of the crystal as the sources. The inputs are turned on or off to obtain the input identifiers 0 and 1. On the output side, the value of the average pressure where the exiting beams intersect on the detector $D$, is relative to a threshold and given the output identifiers 0 or 1. The average pressure is defined as the time average of the absolute value of the pressure over one period. A schematic of the operation for the NAND gate is displayed in Fig. 3.1(a).

In Figs. 3.1(b-e) the resulting displacement field and corresponding average pressure at the detector are shown for the various cases. The point where the exiting beams intersect on the detector is highlighted by the dotted line. In Fig. 3.1(b) the inputs are turned off ($I_1=0, I_2=0$), which allows for the waves produced by the sources to travel through the PC and exit on the output side. The exiting beams destructively interfere where they intersect and a maximum average pressure is given the identifier 1. In order to establish the system threshold we observe the case for which the average pressure is at a minimum. In Fig. 3.1(e) the inputs are turned on ($I_1=1, I_2=1$), these inputs destructively
interfere with the $S_1$ and $S_2$ in the PC. The resulting output is the lowest for all other input cases and for this reason; it is used as the system threshold by assigning the identifier 0. In Fig. 3.1(c-d), one may see that the output of the average pressure at the center of $D$ is above that of the threshold; in both instances the identifier 1 is assigned. The displacement field and average pressure results for the NAND gate demonstrate the interference between complementary and non-complementary waves.
Fig. 3.1: (a) Schematic for NAND gate with corresponding truth table. (b-e) Finite-difference time-domain (FDTD) results for NAND gate construction (b) both inputs are turned off ($I_1, I_2=0$). On the output side the time average of the absolute value of the pressure over one period (average pressure), is acquired at $D$ and the intersection between exiting beams (dashed line, 410 mm y-direction), is given the identifier 1. (c) $I_1$ is on and $I_2$ is off. Complementary waves $S_1$ and $I_1$, destructively interfere within PC leaving non-complementary wave to exit, output result is 1. (c) $I_1$ is off and $I_2$ is on, only non-complementary waves exit, output result is 1. (d) Both inputs are turned on ($I_1, I_2=1$) and destructively interfere with sources, output result is 0 (light gray line).
B. XOR gate

In this subsection, we discuss the construction and results for the XOR gate. Although the XOR is not a universal logic gate, it remains important in most logic gating applications. The XOR gate operates similarly to the NAND gate with two inputs into the system and one output. For the XOR gate system the sources, \( S_1 \) and \( S_2 \), consist of two non-complementary plane waves incident upon the same point of the crystal. The sources remain in phase with one another and their incident angles are fixed at 10° and 28.1°. The calculated total phase shift for exiting beams on the output side is \( \pi \) radians, given by Eq. 2.7. This occurs in the simulation cell at the location of 383\( mm \) in the \( y \)-direction and 1888\( mm \) in the \( x \)-direction. The detector \( D \) is centered about this point and given an angle of 19° (10°+28.1°/2) with a spatial width of 80 \( mm \). The inputs for the system consist of \( I_1 \) and \( I_2 \) which are the complementary waves of \( S_1 \) and \( S_2 \), respectively, and are out of phase by \( \pi \) radians. The incident angles for \( I_1 \) and \( I_2 \) are fixed at angles -50° and -28.1°, with their centers incident upon the same point as the sources. As in the NAND gate, the inputs are controlled and the output is compared to a threshold. A schematic of the XOR is shown in Fig. 3.2(a).

In Fig. 3.2(b-e) the displacement field and average pressure at \( D \) is displayed for all input cases. In Fig. 3.2(b) one may see that both inputs are turned off (\( I_1=0, I_2=0 \)), permitting both sources exit the PC. The average pressure at \( D \) is shown and there is a phase shift between exiting beams giving rise to destructive interference at the point where they intersect. As was done in the NAND gate, the threshold for the average pressure is identified. Since case one and four will have outputs of 0 (see Fig. 3.2(a)), the
one with the largest minimum average pressure is used to establish the system threshold. In case four both inputs are turned on \( (I_1=1, I_2=1) \) and the resulting average pressure output is larger than that of case one, for this reason it is used as the system threshold. In Fig. 3.2(c-d), case two \( (I_1=0, I_2=1) \) and case three \( (I_1=1, I_2=0) \), shows that the complementary inputs destructively interference inside the PC with the corresponding sources. The comparison between the NAND and XOR logic gates demonstrate the usage of a phase-controlling between non-complementary waves by the PC, this is because the first case for each gate is inherently different and thus the PC must operate on the inputs accordingly. At this point we interject that there is substantial energy loss for incident acoustic beams at the interface between the air and the PC. This loss is incident angle dependent and due to reflections. As a consequence of this, complementary waves will not undergo the same loss of energy at the interface, with one exemption. This does not affect the interference within the PC for angle 28.1° (i.e. zero angle refraction), because the complementary wave is -28.1° and therefore the loss of energy is equivalent.
Fig. 3.2: (a) Schematic for XOR gate with corresponding truth table. (b-e) FDTD results for XOR gate construction (a) Both inputs are turned off ($I_1$, $I_2=0$). On output side, average pressure at the detector $D$ is recorded. The intersection of exiting waves (dotted line, $383 \text{ mm}$ y-direction) is given the identifier 0. (b) $I_1$ is on and $I_2$ is off. Complementary waves destructively interfere within PC leaving non-complementary waves to exit, output result is 1. (c) $I_1$ is off and $I_2$ is on, only non-complementary waves exit, output result is 1. (d) Both sources are turned on ($I_1$, $I_2=1$) and complementary waves destructively interfere, output result is 0.
C. NOT gate

Similar to the XOR gate, the NOT gate remains important in logic gating applications. The NOT gate operates on one input and produces one output. For this reason, the construction of the NOT gate disregards the phase shift between non-complementary waves, but instead implements the interference of complementary waves. The NOT gate system is composed of a source, $S_1$, and Input, $I_1$. The source is a plane wave incident upon the PC and is set in a fixed angle of 28.1° and phase. The input for this system is the complementary plane wave of the source and is set at -28.1° and π radians out of phase with $S_1$. The input is turned off and on accordingly (see Fig. 3.3(a)). Since this gate does not operate on non-complementary beams, the detector is angled at 28.1° on the output side and centered about the exiting beam (538 mm in the y-direction and 1753 mm in the x-direction). The threshold for the system is determined by the output for case two, since its average pressure reading will be the lowest. In Fig. 3.3(b) the resulting displacement field and average pressure at $D$ are displayed. In this case the input is turned off ($I_1=0$) and the resulting average pressure at the detector results in an identifier 1. In the second case the input is turned on ($I_1=1$) and interferes destructively with the source inside the PC, this is shown in Fig. 3.3(c). From the latter, a minimum average pressure is detected and the output is given the identifier 0. As stated in the previous section, for angles 28.1° and -28.1° there is an equal loss of energy at the interface of the air and PC system. For this reason one may see in Fig. 3.3(c) that the displacement field on the output side is roughly nonexistent.
Fig. 3.3: (a) Schematic for NOT gate with corresponding truth table. (b-e) FDTD results for NOT gate construction (a) The input is turned off. On output side the average pressure at the detector $D$ is recorded. The center of the exiting wave (dotted line, 538 mm y-direction) is given the identifier 0. (b) $I_1$ is turned on and destructively interferes with $S_1$ inside the PC, output result is 1.
CONCLUSION

The employment of a phononic crystal that enables control of the phase for acoustic waves to implement Boolean logic functions through interferences. This PC is composed of Polyvinylchloride cylinders in an air matrix with a lattice parameter $a=27\ mm$ and a radius $r=12.9\ mm$. The phase-controlling is achieved through the features of the PC’s band structure and namely its nearly square equifrequency contour at a frequency of $13.5\ kHz$. For this case the wave vectors of Bloch modes are not collinear with the group velocity vectors. The result of this is a phase shift inside the PC for incident acoustic waves. It is important to note that these conditions are not limiting since, for a PVC-Air PC, any combination of lattice parameter and radius which results in a filling fraction of $f=0.7171$, will give rise to nearly square equifrequency contours at some operating frequency. Furthermore, waves with different incident angles which do not excite the same Bloch modes (non-complementary waves) in the PC, will exit on the output side and converge at some point to constructively or destructively interfere. In addition, the fact that the equifrequency contour of acoustic waves in the medium in which the PC is embedded is larger than the 1st Brillouin zone of the crystal enables the excitation of multiple Bloch modes. One can therefore take advantage of the spatial overlap of these modes (complementary waves) to achieve constructive and destructive interferences.

It is demonstrated with the Finite-Difference Time-Domain method, that the unique temporal, spectral, and phase properties of this PVC-Air PC can be utilized to
showcase linear Boolean logic operations. With this scheme, the NAND, XOR, and NOT logic gates are demonstrated. In addition, this method may be further adapted in a network of PC’s to construct all other gates and perhaps an all-acoustic logic circuit. The main advantage of these gates is that they rely on linear behavior of the acoustic materials. This linearity however requires that the acoustic logic gate device possesses at least a permanent source that can interfere with a corresponding input. In addition, this source has to be out of phase with respect to the input. The linearity of the Boolean function requires also system identified thresholds. These thresholds however may not be universal, that is they may depend on the circuit in which a particular gate of interest is inserted. A possible remedy to this issue is to pass any input through a beam splitting PC (e.g. PVC-air). One of these beams serves as an input for a given gate and the other beam subsequently serves as a reference. Instead of using absolute values of pressure as thresholds, then one would define thresholds relative to the pressure of the reference beam. In spite of these potential draw backs the present work is still a significant step forward in the design of acoustic devices with higher functions (logic functions) based on phononic crystals.

The Boolean logic functions presented in this thesis rely on the phenomenon of Bragg scattering of acoustic waves by a periodic array of PVC inclusions. Imperfections in the periodicity of the array as well as defects in the PC may lead to degradation of the operating conditions of the logic gates. In the work done by J. Bucay et al [38], experimental measurements of the transmission of acoustic waves through a PVC-air PC were reported. In that study, great care was taken to fabricate a periodic structure,
however, in spite of very small deviations (less than 1%) in position and alignment of the PVC cylinders, experimental and FDTD calculated transmissions showed very good agreement. It is therefore anticipated that the Boolean logic gates studied require as periodic a PC as possible but small imperfections may not impact its operation significantly.

Future work will consist of experimental evidence of the construction of acoustic based logic gates. Other acoustic logic gates such as the XNOR and XOR will also be demonstrated through FDTD. Furthermore, it has been proposed that more complex gates which are based on formulations from quantum logic gates may be realized through the use of beam splitting of waves from the PC to exiting medium. The two exciting beams experience different phase shifts with respect to one another and there amplitudes are also different. Thus information may be encoded within these two mechanisms and used to represent two occupied states. Reintroducing these exiting beams into another PC will lead to reconstruction of the initial wave incident on the PC. This satisfy the condition of reversibility.
APPENDIX A: Code

A.1 FDTD code for simulation of gate

!! elastic wave in a two-dimensional material
!! using FDTD applied to the canonical form of the elastic
!! equation: \( \frac{d^2u}{dt^2} = \frac{1}{\rho} \frac{dT_{ij}}{dx_i} \)
!!
!! \( u \) is the displacement field (computed at times \( dt, 2*dt \ldots \)),
!! \( \frac{dT_{ij}}{dx} \) is the divergence of the stress tensor.
!!
!! Spatial discretization scheme on the two staggered meshes:
!!  ux(i,j) and vx(i,j) correspond to the node (i,j) on the mesh
!!  uy(i,j) and vy(i,j) correspond to the centre (i+0.5,j+0.5)
!!
!! 1st-order Mur conditions are applied on both ends of the x and y
!! Units: length (m), time (ms), mass (1000 Kg), velocity (km/s), pressure (GP)

program FDTD_gate

    implicit none
    character (len=72) :: slab_comment
    integer, parameter :: nxmax=5650, nymax=5650, nmmax=10,
        nt=300000, fmax = 8092
    integer            :: nx, ny, nm
    integer            :: i, j, n, m, it, im, ip, jm, jp,
        mip, mim, mjp, mjm, mipjp, istep, jf, nf, nin, nou
    integer            :: k, l, js, ii2, jj0, ii,
        jj,iii,iii2,jjj,jss,iss

    integer            :: nsst, is
    integer            :: material(nxmax,nymax)
real               :: input, trans, data(2*nt),
spectrum(fmax), vari
real (kind(0D0))  :: dx, dy, yin, you, dxsdy, length, dt
real (kind(0D0))  :: c1l, c1n, a1
real (kind(0D0))  :: a0, ymin, ymax, y, u0, v0, freq
real (kind(0D0))  :: c11p, c11m, c12p, c12m, c44p, c44m
real (kind(0D0))  :: sxxp, sxxm, sxyp, sxym, syyp, syym,
syxp, syxm, fx, fy
real (kind(0D0))  :: fac, uy0(nymax), uy0p1(nymax),
y0(nymax)
real (kind(0D0)), dimension(nmmmax) :: cl, ct, rho, sc11,
sc12, sc44, dtsr, dtsr4
real (kind(0D0)), dimension(nxmax,nymax) :: ux, uy, uxpl,
uypl, vx, vy
real (kind(0d0))  :: llx, sl, sd, omega, xperiod,
xpress(nxmax,nymax), apress(nxmax,nymax)
real (kind(0d0))  :: cpress(nxmax,nymax),
cpressx(nxmax,nymax), cpressy(nxmax,nymax)
real (kind(0d0))  :: engyflxX(nxmax,nymax),
engyflxyY(nxmax,nymax)
real (kind(0D0))  :: theta, sig, gauswt, theta2
real (kind(0D0))  :: pi
!! Geometry of the slab
!! yin and you are the thickness of the media where the
!! incoming
!! outgoing wave are propagating.
!!
!! Files to be opened to store data
open(unit= 27, file= 'AP1028L46.csv',status='old')
!open(unit= 30, file= 'a2press19_33.csv',status='old')
!open(unit= 31, file= '20degree.csv',status='old')
open(unit= 32, file= 'XP1028L46.csv',status='old')
!! Mesh setup
dx = 1.0d-03    !(1.0d-03)/3.0d0
dy = dx
yin = 800.0d-03 !!300.0d-03
you = 1200.0d-03
nin = int(yin/dy)
ou = int(you/dy)
pi = 4.0d0*atan(1.0d0)
nsst = 299066 !!parameters for the source/lens
!!! creates crystal
call
slab_geometry(dx,dy,yin,you,nx,ny,nxmax,nymax,material, &
     nm,nmmax,cl,ct,rho,0,slab_comment)

dxsdy = dx/dy

!! Define the square root of the elastic constants

do n=1,nm
    sc11(n) = cl(n)*sqrt(rho(n))
    sc44(n) = ct(n)*sqrt(rho(n))
    sc12(n) = sqrt(sc11(n)**2 - 2.0d0*sc44(n)**2)
end do

!! Sound velocities on j=1 and j=ny layers (used for Mur conditions)

cl1 = cl(material(1,1))
cln = cl(material(1,ny))

!! Determination of stable Time step (courant)

istep = max(1,nt/1024)
length = 0.25d0/sqrt(1.0d0/dx**2 + 1.0d0/dy**2)
dt = length/cl1
    do n=1,nm
        dt = min(dt,length/cl(n))
        print *, "Timestep:", dt
    end do

    do n=1,nm
        dtsr(n) = dt/rho(n)/dx**2
    end do

    do n=1,nm
        dtsr4(n) = dtsr(n)**0.25d0  ! geometric average on four nodes
    end do
!! Initial conditions on u, v, apress, and express
!! u0 is the wave packet computed at time t=0
!! v0 is the time derivative of the wave packet at time t=-dt/2
!! a0 is the amplitude of the wave packet

ux(1:nx,1:ny) = 0.0d0
vx(1:nx,1:ny) = 0.0d0
uy(1:nx,1:ny) = 0.0d0
vy(1:nx,1:ny) = 0.0d0

do j=1,nx
    do i=1,ny
        apress(i,j) = 0.0d0
        xpress(i,j) = 0.0d0
        engyflxX(i,j) = 0.0d0
        engyflxY(i,j) = 0.0d0
    end do
end do

!! Wave Amplitude, Frequency
a0 = 3.0d-04
a1 = 8.0d-04
ymin = dy
ymax = (j-2)*dy
freq = 13.5d0  !!14.1d0
omega = 2.0d0*pi*freq
xperiod = 1/freq

do it=1,nt  !!!START TIME EVOLUTION

!!!! Sources, change theta,ii,ii2,jj according to needs
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!! Source 1
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
theta = (9.96d0*pi/180.0d0) !Angle to rotate source by
ii = 798 !beg of source x
ii2 = 994 !end of source x
jj = 94 !beg of source y

do is=ii,ii2
js = jj + nint((is-ii)*tan(theta))
ux(is,js) = ux(is,js) + \((-1\times a0\times d\sin(\theta)\times d\sin(\omega\times it\times dt)\)
vx(is,js) = vx(is,js) + \((-1\times a0\times \omega \times d\sin(\theta)\times d\cos(\omega\times (it-0.5)d\times dt)\)
uy(is,js) = uy(is,js) + a0\times d\cos(\theta)\times d\sin(\omega\times it\times dt)
vvy(is,js) = vy(is,js) + a0\times \omega \times d\cos(\theta)\times d\cos(\omega\times (it-0.5)d\times dt)
end do

!!==================================
!! Source 2
!!==================================
theta = (28.029d0*\pi/180.0d0) !Angle to rotate source by
ii = 1015 !beg of source x
ii2 = 1193 !end of source x
jj = 135 !beg of source y
do is=ii,ii2
    js = jj + nint((is-ii)*tan(theta))
ux(is,js) = ux(is,js) + a0\times d\sin(\theta)\times d\sin(\omega\times it\times dt)
vx(is,js) = vx(is,js) + a0\times \omega \times d\sin(\theta)\times d\cos(\omega\times (it-0.5)d\times dt)
uy(is,js) = uy(is,js) + \((-1\times a0\times d\cos(\theta)\times d\sin(\omega\times it\times dt)\)
vvy(is,js) = vy(is,js) + \((-1\times a0\times \omega \times d\cos(\theta)\times d\cos(\omega\times (it-0.5)d\times dt)\)
end do

!!==================================
!! Source 3
!!==================================
theta = (18.919d0*\pi/180.0d0) !Angle to rotate source by
ii = 482 !beg of source x
ii2 = 624 !end of source x
jj = 241 !beg of source y
do is=ii,ii2
    js = jj - nint((is-ii)*tan(theta))
ux(is, js) = ux(is, js) + (-1)*a0*dsin(theta)*dsin(omega*it*dt)
vx(is, js) = vx(is, js) + (-1)*a0*omega*dsin(theta)*dcos(omega*(it-0.5d0)*dt)
uy(is, js) = uy(is, js) + (-1)*a0*dcos(theta)*dsin(omega*it*dt)
vyy(is, js) = vy(is, js) + (-1)*a0*omega*dcos(theta)*dcos(omega*(it-0.5d0)*dt)
end do

!!==================================
!! Source 4 a1 > a0...mismatch loss
!!==================================
theta = (50.145d0*pi/180.0d0)  !Angle to rotate source by
  ii = 372  !beg of source x
  ii2 = 468  !end of source x
  jj = 365  !beg of source y

do is=ii, ii2
  js = jj - nint((is-ii)*tan(theta))
  ux(is, js) = ux(is, js) + (-1)*a1*dsin(theta)*dsin(omega*it*dt)
  vx(is, js) = vx(is, js) + (-1)*a1*omega*dsin(theta)*dcos(omega*(it-0.5d0)*dt)
  uy(is, js) = uy(is, js) + (-1)*a1*dcos(theta)*dsin(omega*it*dt)
  vy(is, js) = vy(is, js) + (-1)*a1*omega*dcos(theta)*dcos(omega*(it-0.5d0)*dt)
end do

!!! BEGIN SPATIAL CALC

do j=2, ny-1
  jp = j+1
  jm = j-1
  ! Calculate the stress tensor components that can be shifted from step i-1 to step i (here, i=1)
  im = 1
  m = material(1,j)
  mim = material(im,j)
  mjp = material(1,jp)
  c11m = sc11(mim)*sc11(m)
\[ c_{12m} = sc_{12}(mim) \cdot sc_{12}(m) \]
\[ c_{44p} = sc_{44}(mj)p \cdot sc_{44}(m) \]
\[ sxxp = c_{11m}(ux(2,j) - ux(im,j)) + c_{12m}(uy(im,j) - uy(im,jm)) \cdot dxsdy \]
\[ syxp = c_{44p}(uy(2,j) - uy(im,j) + (ux(2,jp) - ux(2,j)) \cdot dxsdy) \]

\[
\text{do } i=2,nx-1
\]
\[
\quad \quad ip = i+1
\]
\[
\quad \quad im = i-1
\]
\[
\quad \quad m = \text{material}(i,j)
\]
\[
\quad \quad mip = \text{material}(ip,j)
\]
\[
\quad \quad mjp = \text{material}(i,jp)
\]
\[
\quad \quad mjm = \text{material}(i,jm)
\]
\[
\quad \quad mipjp = \text{material}(ip,jp)
\]
\[
\quad \quad c_{11p} = sc_{11}(mip) \cdot sc_{11}(m)
\]
\[
\quad \quad c_{12p} = sc_{12}(mip) \cdot sc_{12}(m)
\]
\[
\quad \quad c_{44m} = sc_{44}(mjm) \cdot sc_{44}(m)
\]
\[
\quad \quad sxxm = sxxp
\]
\[
\quad \quad sxyp = syxp
\]
\[
\quad \quad sxym = c_{44m}(uy(i,jm) - uy(im,jm) + (ux(i,j) - ux(i,jm)) \cdot dxsdy)
\]
\[
\quad \quad c_{12p}(uy(i,j) - uy(i,jm)) \cdot dxsdy
\]
\[
\quad \quad sxyp = syxp
\]
\[
\quad \quad syym = c_{44m}(uy(i,j) - uy(i,jm) + (ux(i,j) - ux(i,jm)) \cdot dxsdy)
\]
\[
\quad \quad c_{11m}(ux(ip,jp) - ux(i,jp)) + c_{11p}(uy(i,jp) - uy(i,j)) \cdot dxsdy
\]
\[
\quad \quad syym = c_{12m}(ux(ip,j) - ux(i,j)) + c_{12p}(uy(ip,j) - uy(i,j)) \cdot dxsdy
\]
\[
\quad \quad sxym = sxyp
\]
\[
\quad \quad syxp = c_{44p}(uy(ip,j) - uy(i,j) + (ux(ip,jp) - ux(ip,j)) \cdot dxsdy)
\]
\[
\quad \quad xpress(i,j) = -0.5d0 \times ((sxxp+sxxm)/2.0d0 + (syyp+syym)/2.0d0)
\]
\[
\quad \quad fy = syxp-sxym + (syyp-syym) \cdot dxsdy
\]
\[
\quad \quad vx(i,j) = vx(i,j) + fx \cdot dtsr(m)
\]
vy(i,j) = vy(i,j) + fy*(dtsr4(m)*dtsr4(mip)*dtsr4(mjp)*dtsr4(mipjp))
uxp1(i,j) = ux(i,j) + dt*vx(i,j)
uyp1(i,j) = uy(i,j) + dt*vy(i,j)

end do

end do

!!! END SPATIAL CALC
!!

!! Apply 1st-order Mur conditions on both ends of the y and x

do i=1,nx
    uxp1(i,1) = ux(i,2) + (cl1*dt-dy)/(cl1*dt+dy)*(uxp1(i,2)-ux(i,1))
    uyp1(i,1) = uy(i,2) + (cl1*dt-dy)/(cl1*dt+dy)*(uyp1(i,2)-uy(i,1))
end do

do i=1,nx
    uxp1(i,ny) = ux(i,ny-1) + (cln*dt-dy)/(cln*dt+dy)*(uxp1(i,ny-1)-ux(i,ny))
    uyp1(i,ny) = uy(i,ny-1) + (cln*dt-dy)/(cln*dt+dy)*(uyp1(i,ny-1)-uy(i,ny))
end do

do j=1,ny
    uxp1(1,j) = ux(2,j) + (cl1*dt-dy)/(cl1*dt+dy)*(uxp1(2,j)-ux(1,j))
    uyp1(1,j) = uy(2,j) + (cl1*dt-dy)/(cl1*dt+dy)*(uyp1(2,j)-uy(1,j))
end do

do j=1,ny
    uxp1(nx,j) = ux(nx-1,j) + (cln*dt-dy)/(cln*dt+dy)*(uxp1(nx-1,j)-ux(nx,j))
    uyp1(nx,j) = uy(nx-1,j) + (cln*dt-dy)/(cln*dt+dy)*(uyp1(nx-1,j)-uy(nx,j))
end do

!!! Time shift

ux(1:nx,1:ny) = uxp1(1:nx,1:ny)
uy(1:nx,1:ny) = uyp1(1:nx,1:ny)
!! Calculate absolute pressure and Energy fluxes
if((it.gt.nsst).and.(it.lt.nsst+int(xperiod/dt))) then
    do i=1,nx
        do j=1,ny
            apress(i,j) = apress(i,j) + dabs(xpress(i,j))
            engyflxX(i,j) = engyflxX(i,j)+ xpress(i,j)*vx(i,j)
            engyflxY(i,j) = engyflxY(i,j)+ xpress(i,j)*vy(i,j)
        end do
    end do
end if

!! Time signature of displacement
if(it.GT.130000)then
    write(31,'(1x,i6,4f14.6)') it,ux(187,1301),uy(187,1301),ux(376,1274),uy(376,1274)
end if

end do  !!!!ENDS TIME EVOLUTION LOOP
close(unit=31)

!!! Average Pressure at Location 1

  iii = 142
  iii2 = 342
  jjj = 3007
  theta2 = ((19.0d0*pi)/180.0d0)

  do iss=iii,iii2
      jss = jjj + nint((iss-iii)*dtan(theta2))
      write(27,*) iss,"","jss","", apress(iss,jss)/(xperiod/dt)
  end do
  close(unit=27)

!!!Average Pressure at location 2

  iii = 361
  iii2 = 561
jjj = 1522
theta2 = ((28.0*pi)/180.0d0)

do iss=iii,iii2
  jss = jjj + nint((iss-iii)*dtan(theta2))
  write(30,*): iss,,jss,,
apress(iss,jss)/(xperiod/dt)
end do

!!! Instantaneous Pressure
do i=1,nx
  do j=1,ny
    write(32,*): i,,j,, xpress(i,j)
  end do
end do

close(unit=32)

!! Fourier transform
!
call four1(data,nt,1)
do jf=1,min(fmax,nt)
  spectrum(jf) = sqrt(data(2*jf-1)**2 + data(2*jf)**2)
end do

!! Propagation of the longitudinal pulse in the homogeneous
medium used to normalize the output

open(unit=13,file='pulse.out')
write(13,'(a)') 'n 2'
fac = (cl1/dy)**2*dt

do it=1,nt
  do j=2,ny-1
    jp = j+1
    jm = j-1
    fy = uy0(jp)-uy0(j) - (uy0(j)-uy0(jm))
    vy0(j) = vy0(j) + fac*fy
    uy0p1(j) = uy0(j) + dt*vy0(j)
  end do

uy0p1(1) = uy0(2)+(cl1*dt-dy)/(cl1*dt+dy)*(uy0p1(2)-uy0(1))
uy0p1(ny) = uy0(ny-1)+(cl1*dt-dy)/(cl1*dt+dy)*(uy0p1(ny-1)-uy0(ny))
uy0(1:ny) = uy0p1(1:ny)
data(2*it-1) = uy0(ny-1)
data(2*it) = 0.0
if(mod(it,istep) == 0) write(13,'(3e14.4)')
it*dt,uy0(ny-1),uy0(2)
end do
close(unit=13)

call fourl(data,nt,1)

!! Transmission coefficient
!!
trans = 1.0
open(unit=14,file='transcoefficient.dat')
write(14,*) ' '

nf = max(fmax/2,nint(8*freq*nt*dt))
do jf=1,min(nf,nt,fmax)
input = sqrt(data(2*jf-1)**2 + data(2*jf)**2)
if(input > 0.0d0) trans = spectrum(jf)/input
write(14,'(4e13.4)')
(jf1)/(nt*dt),trans,spectrum(jf),input
end do
close(unit=14)
stop
end program FDTD_gate

subroutine slab_geometry(dx,dy,yin,you,nx,ny,nxmax,nymax,material, &
nm,nmmax,cl,ct,rho,check,comment)

implicit none
real (kind(0D0)), intent(IN) :: dx, dy, yin, you
integer, intent(IN) :: nxmax, nymax, nmmax,
check
integer, intent(OUT) :: nx, ny,
material(nxmax,nymax), nm
real (kind(0D0)), intent(OUT) :: cl(nmmax), ct(nmmax), rho(nmmax)
character(len=*) ,intent(OUT) :: comment

character(len=12) :: lattice
integer :: nper, nin, nou, i, j, k, l, cr, nytot,
real (kind(0D0)) :: alatx, alaty, dnn, y0, x0, re2,
ri1, ri2, ri3, ri4, ri5, ri6, ri7, ri8, ri9, ri10, rii2, r2
real (kind(0D0)) :: limg, as, wl, theta
real (kind(0D0)) :: xin, xout
!!
!! Material constants: longitudinal and transverse sound velocities and density

nm = 3
if(nm > nmmax) stop
!! Air
cl(1) = 0.34d0
ct(1) = 0.0d0
rho(1) = 1.30d-3
!! PVC
cl(2) = 2.23d0
ct(2) = 1.00d0
rho(2) = 1.364d0

!!
!! Dimension of a unit cell, number of periods along y
!!
nper = 50
lattice = 'primitive2'
if(lattice == 'primitive2') then
    alatx = 27.0d-03
    alaty = 27.0d-03
    xin = 100.0d-03
    xout = 100.0d-03
end if

!! number of nodes
nx = nint(nper*alatx/dx + (xin+xout)/dx)
nin = nint(yin/dy)
nou = nint(you/dy)
ny = nint((46*alaty/dy))+ nin + nou

material(1:nx,1:ny) = 1  !!! material matrix
re2 = (12.9d-03)**2  !!! Radius of cylinders

if(lattice == 'primitive2')then
    comment = 'Square lattice of PVC solid cylinders'
    do j=1,ny
        y0 = (j-nin-0.5d0)*dy
        do i=1,nx
            x0 = (i-(xin/dx)-0.5d0)*dx
            !! one cylinder at the center of the unit cell
            do kv=0,nper-1
                do k=0,46
                    r2 = (x0-(2.0d0*kv+1)*alatx/2.0d0)**2 + (y0-
                        alaty/2.0d0-k*alaty)**2
                    if(r2 < re2) material(i,j) = 2
                    end do
                end do
            end do
        end do
    end do
end if

end subroutine slab_geometry

SUBROUTINE four1(data,nn,isign)
    implicit none
    INTEGER, intent(IN) :: isign,nn
    REAL, intent(IN OUT) :: data(2*nn)
    INTEGER :: i,istep,j,m,mmax,n
    REAL :: tempi,tempr
    real (kind(0D0)) :: theta,wi,wpi,wpr,wr,wtemp
    n=2*nn
    j=1
    do i=1,n,2
        if(j.gt.i)then
            tempr=data(j)
            tempi=data(j+1)
            data(j)=data(i)
            data(j+1)=data(i+1)
data(i)=tempr
data(i+1)=tempi
end if
m=n/2
1 if ((m.ge.2).and.(j.gt.m)) then
   j=j-m
   m=m/2
   goto 1
end if
j=j+m
end do
mmax=2
2 if (n.gt.mmax) then
   istep=2*mmax
   theta=6.28318530717959d0/(isign*mmax)
   wpr=-2.d0*sin(0.5d0*theta)**2
   wpi=sin(theta)
   wr=1.d0
   wi=0.d0
   do m=1,mmax,2
      do i=m,n,istep
         j=i+mmax
         tempr=sngl(wr)*data(j)-sngl(wi)*data(j+1)
         tempi=sngl(wr)*data(j+1)+sngl(wi)*data(j)
         data(j)=data(i)-tempr
         data(j+1)=data(i+1)-tempi
         data(i)=data(i)+tempr
         data(i+1)=data(i+1)+tempi
      end do
      wtemp=wr
      wr=wr*wpr-wi*wpi+wr
      wi=wi*wpr+wtemp*wpi+wi
   end do
   mmax=istep
   goto 2
end if
return
end SUBROUTINE four1
A.2 PWE code to generate band structure and Equifrequency diagrams.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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%%
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%%
%% Date: August 12th 2011
%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% This Program is designed to calculate the band structure of a Phononic/Sonic Crystal using the Plane wave expansion method. The code plots Z-modes (longitudinal waves) results are displayed in reduced wave. This code was written with the help from Nicholas Swinteck provided me

clear all;

%% USER ENTERED DATA %%

%% Define Crystal
%!! Enter lattice parameter and radius of inclusions
%!! use standered SI units (meters, kilograms, seconds)
lat_par=.027;
radius=.0129;
fillfrac=0.7171;
latticekey=1; % Square: 1 ,Triangle :2
size=10; % size of space

%!! Enter Material Properties (default: Matrix-Air, Inclusions-PVC)

%!! Matrix density and long sound of speed
rho_matr=1.2929;
speed_matr=343;

%!! Inclusion density and long sound of speed
rho_inc=1364;
speed_inc=2330;

%%% Select number of passbands to show
number_passbands=6;

%%% END OF USER ENTERED DATA

%%% Material property calc
C44_matr=(speed_matr)^2*rho_matr;
C44_inc=(speed_inc)^2*rho_inc;
sp22=(C44_matr/C44_inc-1)/(fillfract*C44_matr/C44_inc+1-fillfract);
denvar=(rho_matr/rho_inc-1)/(fillfract*rho_matr/rho_inc+1-fillfract);
Co=340.395337; %to calculate freq

%%% Call function to generate reciprocal lattice
[CompGxGy nx ny nxrg nyrg]=reciplat(size,lat_par,radius,fillfract,latticekey);

%%% Calculation of Eigenvalues
%%% Flag for directions Gamma-X direction - 1, X-M -2, M-Gamma 3
for dirflg=1:3;toggle=1;
for Kxx=0.05:.05:.5  %% wave vectors
    Kyy=0;Kadj=Kxx;

    %%% set up kxx and kyy depending on flag
    if dirflg==2 ,Kyy=.5 ;end
    if dirflg==3,Kxx=.55-Kxx; Kyy=Kxx ;end

    %%% variable names have no meaning, see PWE method text to follow
    mgk= (Kxx+nx).^2+(Kyy+ny).^2;
    mult= (Kxx+nx).*(Kxx+nxrg)+(Kyy+ny).*(Kyy+nyrg);
    dr= (nxrg==nx) & (nyrg==ny);

    %%% form matrix
matrix1 = mgk.*dr + denvar.*CompGxGy.*mult.*(1-dr);
matrix2 = dr + CompGxGy.*sp22.*(1-dr);

%%% calculate eigenvalues and frequency
matrix3 = matrix1*(matrix2^(-1));
eval = eig(matrix3);
eval2 = find(eval~=0);
eval3 = sort(eval(eval2));
normfreq = eval3;
klength = repmat(Kadj,1,number_passbands);

if toggle==1,  %% to initialize array for each
direction

frequency = [(real(sqrt(normfreq(1:number_passbands))))']* (Co/lat_par)];
    klength_e = [klength];
    toggle = 0;
else

frequency = [frequency, (real(sqrt(normfreq(1:number_passbands ))))]' * (Co/lat_par)];
    klength_e = [klength_e, repmat(Kadj,1,number_passbands)];
end

%% Plotting band structure
figure = figure(1);

if dirflg==1;  %% Plots Gamma-X
    axis1 = subplot(1,3,1);
      GammaXX=plot(klength_e,frequency,'sq','MarkerFaceColor','k')
    ;
    position1 = get(axis1,'Position');
    set(axis1,'YLim',[0,max(frequency)]);
    set(axis1,'XLim',[0,0.5]);
    set(axis1,'nextplot','add');
    set(axis1,'xtick',[]);
    set(axis1,'xticklabel',{});
    ylabel('Frequency (kHz)');
yaxislim=max(frequency); % need to maintain proper y axis

elseif dirflg==2; %%% Plots X-M

    axis2= subplot(1,3,2);
    XM2=plot(klenth_e,frequency,'sq','MarkerFaceColor','k');
    line(klenth_e,frequency,'LineStyle','-','LineWidth',8,'Color','k');
    position2=get(axis2,'Position');
    set(axis2,'nextplot','add');
    position2(1)=position1(1)+position1(3); % Width
    position2(4)=position1(4); %
    set(axis2,'position',position2);
    set(axis2,'YLim',[0,yaxislim]);
    set(axis2,'XLim',[0,.5]);
    set(axis2,'ytick',[]);
    set(axis2,'xticklabel',{});

else % Plots M-Gamma

    axis3= subplot(1,3,3);
    MGamma=plot(klenth_e,frequency,'sq','MarkerFaceColor','k');

    set(axis3,'nextplot','add');
    position3=get(axis3,'Position');
    position3(1)=position2(1)+position2(3);

    position3(4)=position1(4);
    set(axis3,'position',position3);

    set(axis3,'YLim',[0,yaxislim]);
    set(axis3,'XLim',[0,.5]);

    %% Make reduced wave vector lables
    text(-.5,-.05*max(frequency),'
    text(-.5,-.05*max(frequency),'
    text(0, -.05*max(frequency), '
    text(.5, -.05*max(frequency), '
    text(-.3, -.1*max(frequency), 'Reduced Wave
    text(-.3, 1.075*max(frequency), 'Dispersion Relation
    'HorizontalAlignment','center');
    end
    end
REFERENCES


