Multi-Period Bargaining: Asymmetric Information and Risk Aversion*

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Abstract: A two period bargaining model with asymmetric information is considered. An uninformed seller charges a uniform price to two buyers. A risk averse seller offers a larger price cut in period two when one buyer remains in the market than when two buyers remain. The price in period one is sensitive to the number of buyers and the seller's degree of risk aversion. The initial price charged to a single buyer may be higher or lower than the price charged to two buyers, depending on the degree of seller risk aversion.

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1. Introduction

Many bargaining situations involve negotiations that take place over a period of time. Sobel and Takahashi (1983) and Fudenberg, Levine and Tirole (1985) analyze bargaining environments in which an uninformed seller can make a series of offers to a buyer who is privately informed about their value of the good. This type of model is called a screening model; actions taken by the uninformed party (the seller) lead to self-selection by different types of the informed party (the buyer). A typical result is that in equilibrium the seller makes a series of price offers that decline over time. After each rejected offer, the seller revises her belief about the buyer's value downward.

Experimental evidence suggests some difficulties with equilibrium predictions. Reynolds (2000) reports on 1-buyer and 5-buyer experiments with time discounting and a finite time horizon. In experiments with a single buyer the initial price offers are

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higher in experiments with a larger number of time periods - just the opposite of the predictions for equilibrium price offers.[1] Reynolds (2000) also reports that average initial prices are higher in 1-buyer experiments than in 5-buyer experiments, holding the time horizon fixed. The equilibrium prediction (under risk neutrality) is that a change in the number of buyers has no effect on the initial price.

This paper investigates the role of risk aversion in multi-period bargaining. The focus is on how seller risk aversion interacts with the number of buyers to influence seller pricing in equilibrium. Proposition 1 states that the equilibrium prices are independent of the number of buyers when agents are risk neutral (assuming buyers' values are independent). Proposition 2 characterizes equilibrium prices for the case of a risk averse seller facing two buyers. A numerical example demonstrates that increasing the number of buyers may either increase or decrease the initial price, depending on the level of risk aversion. This suggests that seller risk aversion could account for the effect of changes in the number of buyers on observed initial prices in experiments.[2]

2. Asymmetric Information Model

This formulation of Sobel and Takahashi (1983) is extended to allow for multiple buyers with independent values. Values are drawn from a common cumulative distribution function F with support $[0, \bar{v}]$. Each buyer value is private information. The seller has

[1] Rapoport, Erev, and Zwick (1995) report on asymmetric bargaining experiments with time discounting and an unlimited time horizon. They find that increases in the discount factor lead to *higher* initial price offers - again, just the opposite of the predictions for equilibrium price offers.

[2] See Reynolds (2000) for more on this point. He argues that a promising path toward reconciling the theory with a variety of experimental results involves a combination of risk aversion and bounded rationality.

constant marginal cost, which is normalized to zero. Trading takes place over a finite number T of time periods. All agents have a common discount factor, δ. The seller makes a uniform price offer in each time period. Buyers who have not yet accepted an offer may either accept or reject the seller's current offer; a buyer may purchase at most one unit during the T periods. A buyer who has not yet made a purchase is referred to as an active buyer.

Sobel and Takahashi show that if the distribution function takes the form, $F(v) = (v/\overline{v})^a$ with $a > 0$, then there is a unique perfect Bayesian equilibrium (PBE) for the single buyer case with risk neutrality. When there are multiple buyers, the informational asymmetries are more complex. The seller is uninformed about values for all buyers. A PBE requires the seller to have beliefs about the distributions of values for all active buyers. Moreover, buyers are uninformed about values for other buyers; a PBE requires each active buyer to have beliefs about distributions of values for other active buyers. However, under some conditions this complexity has a simple resolution.

Proposition 1. *If buyers' values are independent and agents are risk neutral then there is a symmetric perfect Bayesian equilibrium in which the seller offers the same sequence of prices that she would offer in the single-buyer case. Symmetric PBE price offers are invariant with respect to the number of active buyers.*

Sketch of proof. The proposed PBE requires that each buyer adopt the PBE strategy of the buyer in the single-buyer case. The seller's belief about values for active buyers is summarized by a single cut-off value in each period. This is a perfect Bayesian equilibrium because: (1) each buyer's strategy is a best response to the seller's pricing strategy; the behavior of other buyers does not influence a buyer since the seller's strategy does not depend on the number of active buyers, and (2) since each buyer's strategy is independent of the choices of other buyers and the seller is risk neutral, the seller chooses prices as if she is playing *n* separate games against single buyers.

3. Risk Aversion

The analysis will focus on risk aversion of the seller. Introducing risk aversion for buyers has only a minor effect on the results. Buyer risk aversion turns out to be equivalent to assigning a smaller discount factor for a buyer, so that a buyer acts as if they are more impatient. Buyer risk aversion does not affect the invariance result of Proposition 1. Buyers are assumed to be risk neutral.

Seller risk aversion changes the level of price offers and causes price offers to vary with the number of active buyers. Risk aversion is introduced by using a time-separable utility function for the seller that has the constant relative risk aversion (CRRA) form for each time period. The utility function for a monetary payoff of π in a single period is $u(\pi) = \pi^{1-\alpha}/(1-\alpha)$, where $\alpha \in [0,1)$ is the index of relative risk aversion. $\alpha = 0$ corresponds to risk neutrality. A higher value of α corresponds to a greater degree of risk aversion. By using this particular form for the utility function it is possible to solve for the seller's value function in period two in closed form. This greatly simplifies the analysis. The model is further simplified so that there are two time periods ($T = 2$), two buyers, and a uniform distribution of buyer values ($a = 1$).

Proposition 2. *There is a symmetric Perfect Bayesian Equilibrium for the two period model with two buyers and seller risk aversion; this is the only symmetric equilibrium. The seller* *offers a smaller price cut in the second period when both buyers reject the first offer than when one of the buyers accepts the first offer.*

Proof. See Appendix.

Figures 1 and 2 illustrate equilibrium prices for different indices of seller risk aversion, given that the discount factor is equal to 0.9. Figure 1 shows that the period one price can be either increasing or decreasing in the total number of buyers.[3] When the seller is risk neutral the initial price is the same regardless of the number of buyers; this illustrates Proposition 1. For positive but low indices of risk aversion, the initial price is lower with two buyers than with one buyer. The impact of the number of buyers on price is quite different in this multi-period setting than in a static setting (in which the seller makes a single take-it-or-leave-it offer). In a static setting, adding more buyers always induces a risk averse seller to set a higher (uniform) price. For higher indices of risk aversion (e.g., $\alpha > .3$) the ranking of initial prices for one buyer and two buyers is reversed.

Figure 2 shows period two prices for three situations. Prices for the 1-buyer model are the period two equilibrium prices following a rejection of the initial offer. There are two price curves for the 2-buyer model. Prices for the "2-active" case are prices that would be set if both buyers are active in the second period. Prices for the "1-active" case are prices that would be set if one buyer accepted the initial offer and the other buyer is active in the second period. There is a spread between prices for the "2-active" and "1-active" cases, as predicted in Proposition 2. The spread increases as the seller becomes more risk averse. Prices for the

[3] Equilibrium conditions for the 2-buyer model with seller risk aversion appear in the Appendix. Equilibrium conditions for the 1-buyer model may be derived by using the value function, $w_2(1,b)$, defined in the Appendix.

1-buyer case lie between the two prices for the 2-buyer case. There is a common equilibrium price in the second period if the seller is risk neutral ($\alpha = 0$).

4. Conclusion

Seller risk aversion and the number of buyers are shown to interact to produce an effect on price offers in a multi-period bargaining setting. The price in the terminal period depends on the number of remaining buyers when the seller is risk averse. The seller offers a larger price cut when one buyer remains in the market than when two buyers remain. The price in period one is sensitive to the number of buyers and the seller's degree of risk aversion. The price charged to a single buyer may be higher or lower than the price charged to two buyers, depending on the degree of seller risk aversion. This is in contrast to results for a static model where adding buyers provides an incentive for a risk averse seller to raise her price.

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Figure 1
Period One Prices

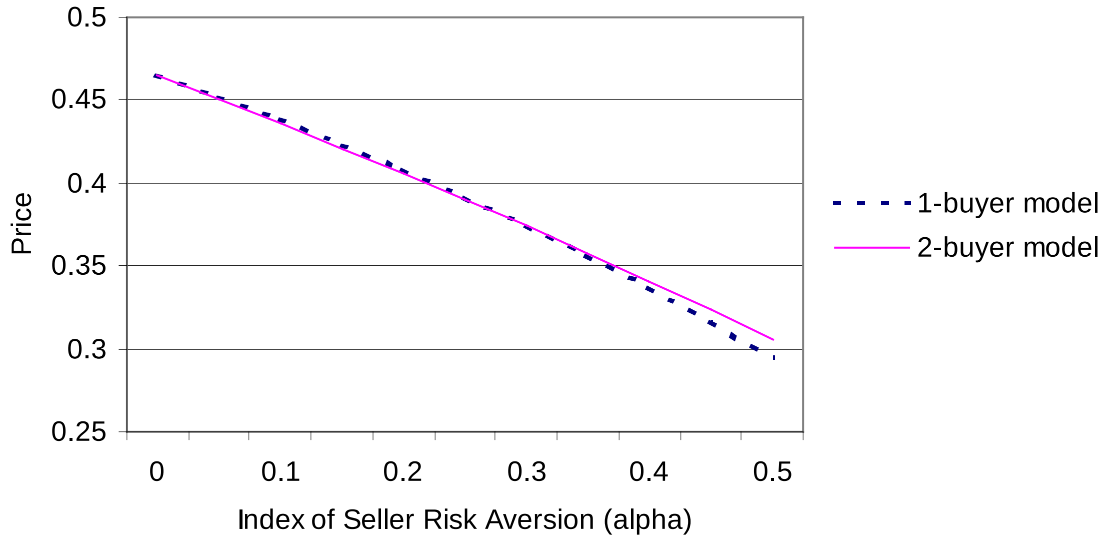
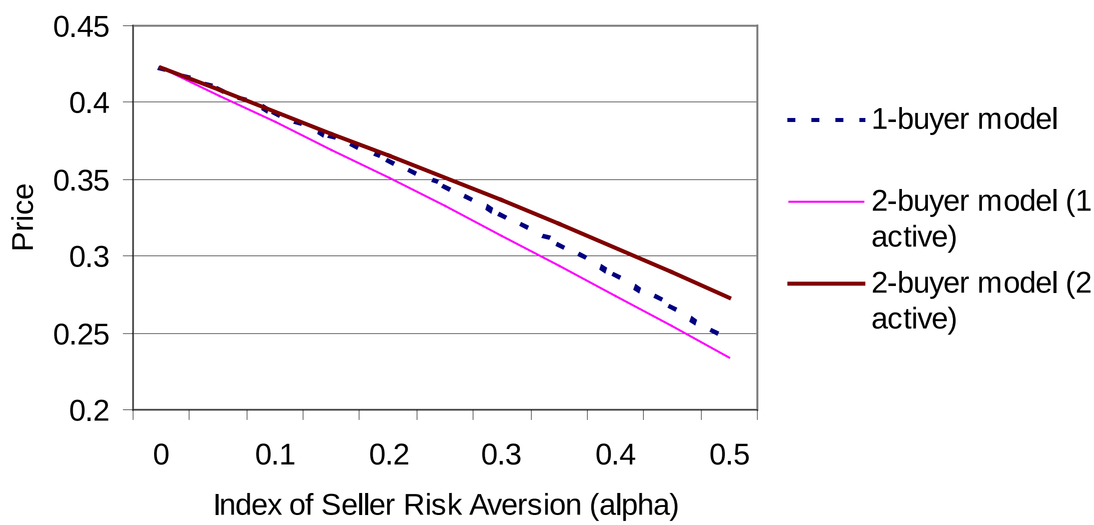


Figure 2
Period Two Prices



Appendix

Proof of Proposition 2: A symmetric Perfect Bayesian Equilibrium is derived by using backward induction. Let *b* be the belief about the cut-off value for active buyers in period 2. The seller has identical beliefs for the two buyers (if there are two active buyers) because, by symmetry, the two buyers employ identical strategies. An active buyer will accept any price offer less than or equal to their value in period 2.

If there is a single active buyer in the second period then the seller will set her price to solve the following maximization problem:

$$w_2(1,b) = \max_{p \in [0,b]} \left\{ \left(\frac{b-p}{b} \right) u(p) + \left(\frac{p}{b} \right) u(0) \right\}$$

The optimal price is the fraction $(1-\alpha)/(2-\alpha)$ times the belief, *b*. For a risk averse seller, this fraction is smaller than the pricing coefficient of one-half used by a risk neutral seller.

If there are two active buyers in the second period then the seller solves the following maximization problem:

$$w_2(2,b) = \max_{p \in [0,b]} \left\{ 2\left(\frac{p}{b} \right)\left(\frac{b-p}{b} \right) u(p) + \left(\frac{b-p}{b} \right)^2 u(2p) \right\}$$

The first order condition for this maximization problem is:

(1) $(b-2p)p^{1-\alpha} - p(b-p)(1-\alpha)p^{-\alpha} - (b-p)(2p)^{1-\alpha} + (b-p)^2(1-\alpha)(2p)^{-\alpha} = 0$

Suppose that the price *p* is equal to a fraction γ times *b*. Then the first order condition (1) is equivalent to, $0 = \gamma^{-\alpha} b^{2-\alpha} f(\gamma)$, where,

(2) $$f(\gamma) \equiv (3-\alpha)(2^{1-\alpha} - 2)\gamma^2 + (2-\alpha)(2 - 2^{2-\alpha})\gamma + (1-\alpha)2^{1-\alpha}.$$

The function $f(\cdot)$ is continuous and decreasing for $\gamma \in (0,1)$ and crosses the horizontal axis between $(1-\alpha)/(2-\alpha)$ and 1/2. Let $\tilde{\gamma}$ be the value of γ that satisfies, $f(\gamma)=0$. This implies that the optimal price charged by a risk averse seller when there are two active buyers exceeds the optimal price charged to a single active buyer. This price is still less than the price charged by a risk neutral seller, given a particular belief, *b*.

The pricing strategy, $r_2(\ )$, and the value function, $w_2(\ )$, for the seller in period two have the number of active buyers and the belief, *b*, as arguments. These functions are summarized as follows.

$$r_2(1,b)=\left(\frac{1-\alpha}{2-\alpha}\right)b \qquad r_2(2,b)=\tilde{\gamma}\,b,\ \ \tilde{\gamma}\in((1-\alpha)/(2-\alpha),1/2)$$

$$w_2(1,b)=\frac{b^{1-\alpha}}{(1-\alpha)^{\alpha}(2-\alpha)^{2-\alpha}} \qquad w_2(2,b)=\frac{(1-\tilde{\gamma})\tilde{\gamma}^{1-\alpha}}{(1-\alpha)}(2\tilde{\gamma}+(1-\tilde{\gamma})2^{1-\alpha})b^{1-\alpha}$$

In period one the seller sets a price to maximize the expected discounted present value of utility, given the strategies employed by the buyers. When the seller is risk neutral, an individual buyer is unconcerned about rejected price offers by other buyers, since the seller's future prices are not affected by the future number of active buyers. With a risk averse seller, the period two price offer will differ depending on whether both buyers reject in period one or only one buyer rejects. If *b* is the value for a marginal buyer, *p* is the price in period one, and both buyers adopt the same strategy (by symmetry) then the buyer incentive constraint may be written as follows:

(3) $$b-p=\delta\left[\frac{b}{\bar{v}}(b-r_2(2,b))+\left(\frac{\bar{v}-b}{\bar{v}}\right)(b-r_2(1,b))\right]$$

The period two expectation of buyer surplus on the right hand side of (3) is based on a probability $b/\bar{v}$ that the other buyer will reject *p*, so that the seller charges $r_2(2,b)$ in period 2, and a probability $(\bar{v}-b)/\bar{v}$ that the other buyer will accept *p*, so that the seller charges $r_2(1,b)$ next period. The incentive constraint for buyers may be rewritten as,

(4) $p = g(b) \equiv b[1-\delta((1-\tilde{\gamma})b/\bar{v}+(\bar{v}-b)/(\bar{v}(2-\alpha)))]$.

The constraint function *g* is non-linear in *b*, in contrast to the linear buyer incentive constraint for the risk neutral case. The constraint function is strictly increasing, strictly convex, and twice differentiable in *b*, with $g(0)=0$.

The seller's problem in period one may now be expressed as,

$$w_1(2,\bar{v}) = \max_{b\in[0,\bar{v}]}\left\{\left(\frac{b}{\bar{v}}\right)^2 \delta w_2(2,b) + 2\frac{b(\bar{v}-b)}{\bar{v}^2}(u(p)+\delta w_2(1,b)) + \left(\frac{\bar{v}-b}{\bar{v}}\right)^2 u(2p)\right\},$$

subject to the price *p* satisfying the buyer incentive constraint, (4).

The proof proceeds by establishing uniqueness of the optimal period two belief. Once this is established, uniqueness of the period one price follows because there is a single price associated with each belief, according to the incentive constraint, $p=g(b)$. After substituting $g(b)$ for *p*, the objective for the seller may be written as the following function of the belief, *b*:

$$W(b) = [b^2\delta A b^{1-\alpha} + 2b(\bar{v}-b)(g(b)^{1-\alpha}/(1-\alpha)+\delta B b^{1-\alpha}) + (\bar{v}-b)^2 2^{1-\alpha} g(b)^{1-\alpha}/(1-\alpha)]/\bar{v}^2,$$

where, $A \equiv \dfrac{(1-\tilde{\gamma})\tilde{\gamma}^{1-\alpha}}{(1-\alpha)}(2\tilde{\gamma}+(1-\tilde{\gamma})2^{1-\alpha})$ and $B \equiv \dfrac{1}{(1-\alpha)^{\alpha}(2-\alpha)^{2-\alpha}}$.

W is non-negative, continuous and twice differentiable for $b\in[0,\bar{v}]$. $W(0)=0$, since $g(0)=0$, and $W(\bar{v}) = \delta A\bar{v}^{1-\alpha} > 0$.

Since W is a continuous function defined on the compact set, $[0,\bar{v}]$, it will attain its maximum value for some $b \in [0,\bar{v}]$. Since $W(0) = 0$, W will attain its maximum either at $b = \bar{v}$ or at an interior b. It can be shown that the maximum must be attained at an interior b by showing that $W'(\bar{v}) < 0$.

$$W'(b) = [(3-\alpha)\delta(A-2B)b^{2-\alpha} + 2(2-\alpha)\delta\bar{v}Bb^{1-\alpha} + \\ (2(\bar{v}-2b) - 2(\bar{v}-b)2^{1-\alpha})g(b)^{1-\alpha}/(1-\alpha) + (2b(\bar{v}-b) + (\bar{v}-b)^2 2^{1-\alpha})g(b)^{-\alpha}g'(b)]/\bar{v}^2. \tag{5}$$

Now evaluate the derivative at $b = \bar{v}$:

$$W'(\bar{v}) = [(3-\alpha)\delta(A-2B)\bar{v}^{2-\alpha} + 2(2-\alpha)\delta B\bar{v}^{2-\alpha} - 2\bar{v}g(\bar{v})^{1-\alpha}/(1-\alpha)]/\bar{v}^2. \tag{6}$$

The expression in square brackets has three terms. The first term is negative because it can be shown that $A < 2B$. The sum of the second and third terms can be shown to be negative. So, W must attain a maximum at an interior value of b.

It can be shown that if $W'(b) = 0$ then $W''(b) < 0$ (this is a tedious algebraic exercise). This establishes that W attains its maximum at a unique interior belief b^*. There is a unique optimal price, p^*, for the seller in period one, given by $p^* = g(b^*)$.