Explicit Mapping of Acoustic Regimes for a Wind Instrument

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Wind (and many other!) instruments are nonlinear systems for which we would like to:

- Understand the coupling between various factors
- Propagate the effect of uncertainties
- Optimize their design for various criteria such as playability
**Proposed approach:** *explicit* identification of regions of the parameter space where given acoustic regimes are reached.

Example of region identification
Objectives
Physics. Simplified model.

Construction of Maps with SVM

Classification Criteria
Sound Producing vs. Soundless Configurations
Frequency-based Criterion

Examples
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Conclusion

Simplified clarinet model
Simplified clarinet model

- Ignore reed dynamics (considered as static spring)

\[ y = -H \frac{\Delta p}{p_M} \]

where \( \Delta p = p_m - p \), \( p_M \) is the closing pressure, and \( p_m \) is the pressure in the mouth.

Flow is given by a nonlinear function of pressure:

\[ u \propto (1 + y_H) \sqrt{|\Delta p|} \frac{p_M}{\text{sgn}(\Delta p)} \]

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Introducing adimensional quantities, the relation between flow and pressure becomes:

\[
\begin{align*}
    u &= \zeta (1 - \gamma + p) \sqrt{|\gamma - p|} \sgn(\gamma - p) \quad \text{if} \quad \gamma - p \leq 1 \quad (1) \\
    u &= 0 \quad \text{if} \quad \gamma - p \geq 1 \quad (2)
\end{align*}
\]

where \( \zeta \) is the adimensional reed opening and \( \gamma \) is the adimentional mouth pressure (two very important parameters!)

Notice these equations account for the beating of the reed (opening switching from closed to open).
Simplified clarinet model

Input impedance governs the behavior of the instrument:

\[ Z(\omega) = j\omega \sum_n \frac{F_n}{\omega_n^2 - \omega^2 + j\omega\omega_n/Q_n} \]  

where \( F_n = \frac{2c}{L} \); \( \omega_n = (2n - 1)\frac{2\pi c}{4L} \)

In the time domain, using \( m \) first modes:

\[ \frac{d^2 p_1(t)}{dt^2} + \frac{\omega_1}{Q_1} \frac{dp_1(t)}{dt} + \omega_1^2 p_1(t) = F_1 \frac{du(t)}{dt} \]  

\[ \frac{d^2 p_m(t)}{dt^2} + \frac{\omega_m}{Q_m} \frac{dp_m(t)}{dt} + \omega_m^2 p_m(t) = F_m \frac{du(t)}{dt} \]

Pressure approximation: \( p(t) = \sum_{n=1}^{m} p_n(t) \)
Support Vector Machines (SVMs)

In order to build the boundaries: use an SVM.

Schematic depiction of an SVM boundary classifying sound producing configurations and soundless ones.
Support Vector Machines (SVMs)

Given $N$ samples $x_i$ (e.g., from a design of experiments) with classes $y_i = +1$ or $y_i = -1$, an SVM $s(x)$ splits the space into a positive and a negative region:

$$s(x) = b + \sum_{i=1}^{N} \lambda_i y_i K(x_i, x)$$

(5)

where $K$ is the Kernel. The Gaussian kernel is:

$$K(x_i, x_j) = \exp\left(-\frac{||x_i - x_j||^2}{2\sigma^2}\right)$$

(6)
Adaptive Sampling

The SVM boundary might not be accurate after a simple design of experiments. An adaptive sampling scheme was developed.

Schematic representation of the EDSD adaptive sampling scheme.

\[ \max \left\| \mathbf{x} - \mathbf{x}_{\text{nearest}} \right\| \]  

(7)
Classification criteria. Sound vs. no sound.

Soundless configuration. Static regime.

Sound through self-sustained oscillations.

Compare the mean of pressure over the last third of time serie:

\[
\frac{1}{N_{2/3}} \sum_{N_{2/3}}^{} p(t_i) > \epsilon_1 \quad \text{Oscillations (i.e., sound)} \quad (8)
\]
Classification criteria. Frequency comparison.

Typically based on number of cents:

\[ N_{cents} = 1200 \log_2 \left( \frac{f_{act}}{f_{ref}} \right) \]  (9)

Notice the jumps (discontinuities) in playing frequencies.

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Sound/No sound criterion. 2D problem.

One mode approximation. Construction in the \((\zeta, \gamma)\) (i.e., opening, mouth pressure) space.

nosound_20_30_1mode_JBModif.pdf

20 DOE + 30 adaptive samples. Comparison to theory (red line)
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Sound/No sound criterion. 3D problem.

One mode approximation. Construction in the $(\zeta, \gamma, Q)$ (i.e., opening, mouth pressure, quality factor) space.

Q_zeta_gamma_2.pdf
Frequency criterion (Intonation)

For a tube of length $L=1$ m (modeled with 2 modes), find region of the space within 5 cents of first resonator mode.

100_180adapt_5_cents_mode1.pdf
Frequency criterion (Intonation)

Find region of the space within 5 cents of note F2 (closest note from the equal temperament scale to the first resonator frequency for a L=1 m resonator).

len_g_z_100_400_5cents_temp.pdf
Influence of initial conditions

Model with two modes. Introduction of a parameter $\beta$ dictating the initial pressure components:

$$p_1(t = 0) = \beta; \quad p_2(t = 0) = 1 - \beta$$

Find the region of the space within 5 cents of the second register as a function of $\beta$. Built with 100 DOE samples and 80 adaptive samples.

adapt_100_80_ini_weight_beta.pdf
Example of probability estimate

An accurate SVM $s$ with Monte-Carlo simulations provide an efficient tool for the computation of probabilities.

$$P_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} I_s(p_i)$$

$$I_s = \begin{cases} 
0 & \text{if } s(p_i) \leq 0 \\
1 & \text{if } s(p_i) > 0 
\end{cases}$$

Example: 6.5% probability of playing within 5 cents of the first resonator frequency. Normal distributions $N(0.5, 0.05)$. 
Design Optimization Example

Find the optimal length of a resonator so as to maximize the probability of having a playing frequency within 5 cents of note F2. Random variables are ζ and γ.
Conclusion

Summary:

- SVM-based approach used to generate maps of acoustic regimes. Demonstrated on a simplified clarinet model.
- Provide insight into the relationship between the factors.
- Facilitates propagation of uncertainties and optimization.

Future work:

- Apply the approach to a more sophisticated clarinet model (available) (e.g., for design optimization)
- Increase the dimensionality of the problems
- Include experimental data with artificial mouth