Phase Retrieval for the Hubble Space Telescope and other Applications

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Abstract: James R. Fienup used phase retrieval algorithms to determine the aberrations in the Hubble Space Telescope. The theory of phase retrieval is described in this paper. Other applications of phase retrieval, such as undersampled broadband images, lensless coherent imaging, and optical metrology, are also discussed.

Introduction:

James R. Fienup established the use of phase retrieval in the early 1980's. Phase retrieval determines the phase error or aberrations of an optical system. Phase retrieval algorithms use computer modeling that often start with an educated guess. This model is run several times and a pattern is determined. It is from the comparison of this pattern to the known data of the actual optical system that the correct parameters can usually be determined and used to accurately model the system. From this model, the aberration coefficients can be determined. These aberration coefficients are used to calibrate an already working optical system. This is especially useful for optical systems in space, where direct testing cannot be used [1]. Phase retrieval was used to help correct the blurred images from the uncorrected Hubble Space Telescope [2]. There are other more recent applications for phase retrieval, such as undersampled broadband images [3], lensless coherent imaging [4], optical metrology [5], and 3-D locator sets of opaque objects [6].

Theory:

In phase retrieval, the phase error can be found from some a priori information about the Fourier transform of the function [1]. For the case of mathematical modeling, a priori means to analyze data that is collected and look for patterns that are created from that model [7]. Phase retrieval creates a model of the point spread function of the optical system [1]. A point spread function shows the propagation of a light wave from a point source through an optical system [8]. In a perfect optical system, the point spread function from the object should be the same at the detector array. However, when aberrations are present in the optical system, the point-spread function at the entrance pupil of the system is disorganized at the detector array, and the image appears blurred. The phase retrieval algorithm calculates the point spread function of the system so the calibration aberration coefficient can be used to deblur the images.

Many different kinds of aberrations can appear in optical systems. The most common are polynomial aberrations, such as astigmatism, chromatic, coma, and spherical (Fig.(1)). Spherical aberrations are the only on-axis aberrations and cause the focal point to actually be spread out along the optical axis. In other words, the rays closest to the optical axis cross the optical axis before the rays towards the edge of the lens do [9]. Astigmatism, coma, and chromatic aberrations are all off-axis aberrations. Astigmatism occurs when an off-axis bundle of light travels through an optical system. There will be a point where a vertical focus line and a horizontal focus line will appear, instead of circles of focused light [9]. Coma aberrations are similar, but occur when an off-axis bundle of light is not perfectly re-imaged, or not all the rays cross at the same off-axis point [9]. Finally, chromatic aberrations occur because the index of refraction is not the same for all wavelengths of light; therefore, the different wavelength rays focus at different points [9]. The phase-retrieval algorithm determines the Zernike coefficients, which come from the even Zernike polynomial (Eq.[1]) and the odd Zernike polynomial (Eq.[2]) that are typically used to describe aberrations [10].

$$Z_n^m(\rho,\phi) = R_n^m \cos(m\phi) \tag{1}$$

$$Z_n^{-m}(\rho,\phi) = R_n^m \sin(m\phi), \tag{2}$$

where m and n are integers ($n \geq m$), ρ is the normalized radial distance, ϕ is the azimuthal angle (radians), and R_n^m are the radial polynomials.

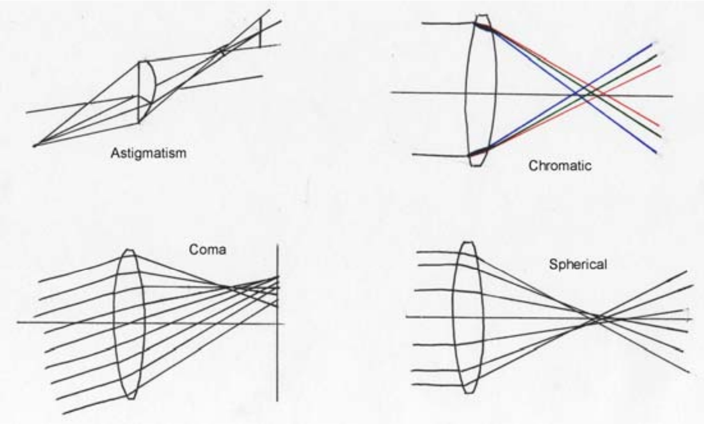

Figure 1. Polynomial aberrations

There are two approaches to determining the phase retrieval: gradient-search algorithms and iterative transform algorithms [1]. Both these algorithms are very complex, with many uses of multiple Fourier transforms. The models that are created from the algorithms are computer generated. The gradient-search algorithm starts with a weighted error metric,

$$E = \sum_{u} W(u)\left[\left|G(u)\right| - \left|F(u)\right|\right]^2 \tag{3}$$

where F(u) is the magnitude of the optical field at the detector, W(u) is a weighting function, and G(u) is a computer generated model of the magnitude of an optical wave front, including the phase error [1]. A weighting function gives a chosen element more "weight" over other elements in the equation [11].

When these algorithms were applied to the Hubble Space Telescope, the spherical aberrations were far greater than the scientists originally thought [1]. The other polynomial aberrations were present but much smaller. The gradient-search algorithm for calculating the single-plane phase retrieval was used to find the Zernike coefficients. The normalized root mean square error (Eq.[4]) was computed to be within 0.001 waves of the correct Zernike coefficients for the optical telescope assembly, which is very accurate [2].

$$\mathrm{err} = \left(\frac{\sum_{\mathrm{u}} \mathrm{W(u)}\left[\left|\mathrm{G(u)}\right| - \left|\mathrm{F(u)}\right|\right]^2}{\sum_{\mathrm{u}} \mathrm{W(u)}\left|\mathrm{F(u)}\right|^2} \right)^{1/2} \tag{4}$$

When the single-plane phase-retrieval algorithm was used for both the optical telescope assembly and the wide-field/planetary camera, the Zernike coefficients were off by as much as –1.977 waves for one coefficient and by –0.2545 waves for a_{11} (the coefficient for spherical aberrations) [2]. These errors were much too large. Because of this, a multi-plane phase retrieval algorithm with a quadratic coefficient had to be used to better represent the model and obtain smaller errors.

Using the multi-plane phase retrieval algorithm, it was determined that not only were there aberrations in Hubble's system, but that the optical axis of the planetary camera was not aligned with the optical telescope assembly. Furthermore, it was determined that the more planes were used in the algorithm, the closer the Zernike coefficients were to the actual coefficients, as seen in Fig.(2). Also, coefficient 11 (spherical aberrations) had a large value.

This showed that the dominant problem with the Hubble Space Telescope was spherical aberrations.

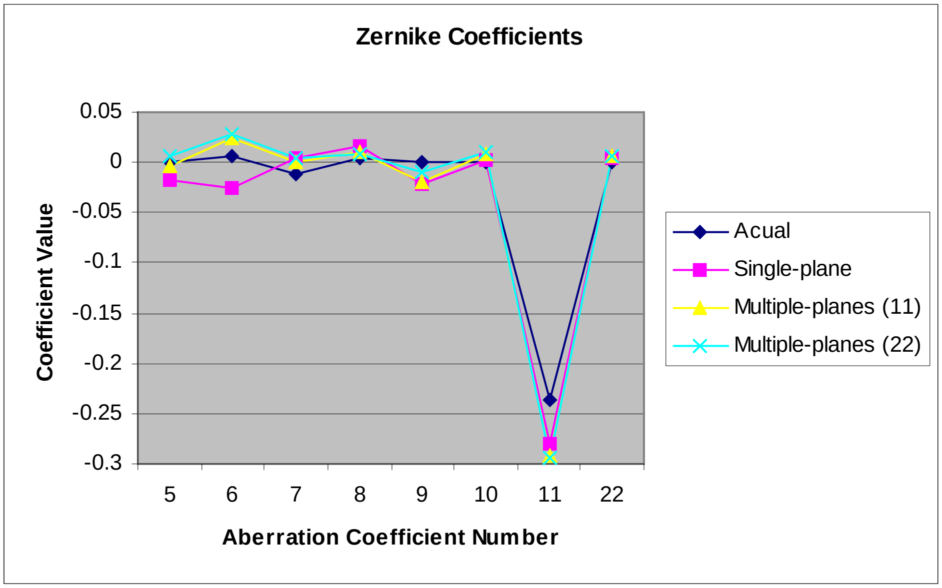

Figure 2. This graph shows the aberration coefficient calculated from the phase-retrieval algorithm compared to the later known actual values of the coefficients. Coefficient 11 (spherical aberrations) is dominant.

Other applications for phase retrieval are undersampled broadband images, improved optical metrology, and lensless coherent imaging. Undersampled broadband images are very small sets of images that are composed of multiple wavelengths. For the case of the Hubble Space Telescope, only the large data sets that have narrow-band filters can be used. When using broadband images, the algorithm becomes much more complicated because it reconstructs broadband images using only a few wavelengths. However, the algorithm still produces the same model of the aberrations in the optical system and uses an error metric that shows the difference between the model and the actual aberrations seen. The gradient of the error metric with unknown parameters is then computed. This new algorithm was tested on the Hubble Space Telescope images. When the algorithm was run using five wavelengths, the largest standard deviation was only 0.005 waves. This is much smaller than the initial estimates or when one wavelength was used as seen in Fig.(3) and Fig. (4) [3].

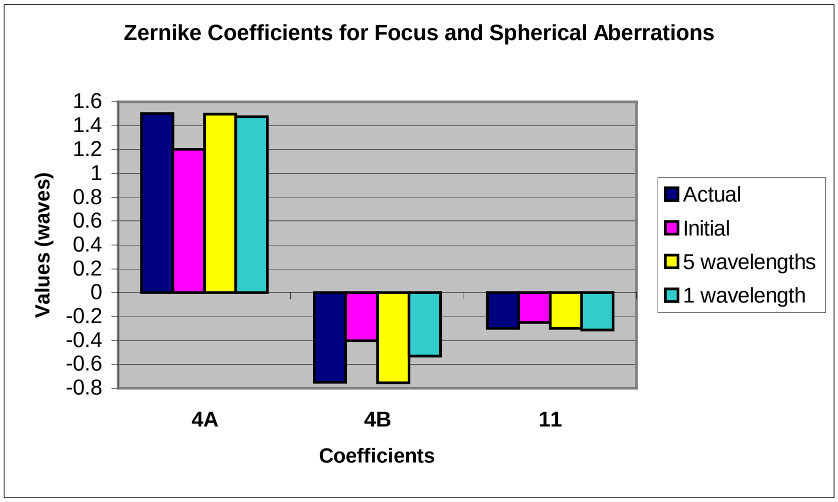

Figure 3. The graph shows the actual, estimated, and reconstructed phase retrieval values for the focus coefficients (4A and AB) and the spherical coefficient (11).

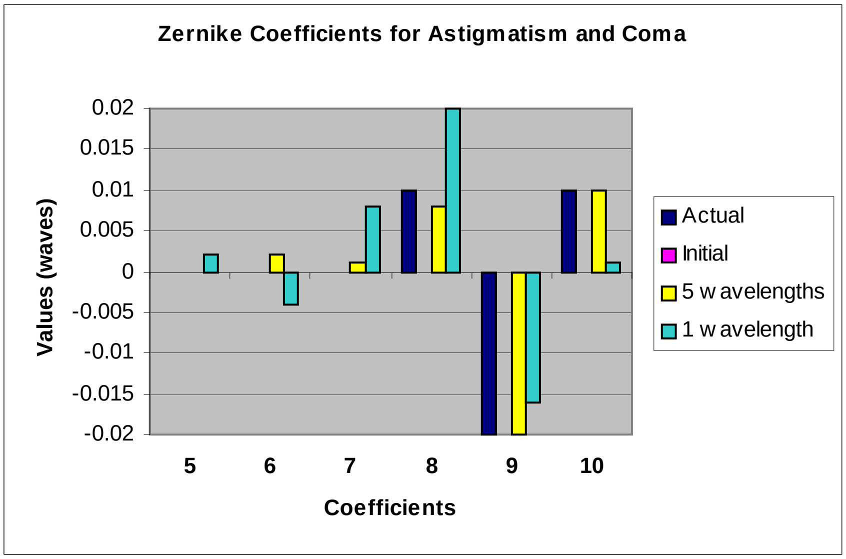

Figure 4. The graph shows the actual, estimated, and reconstructed phase retrieval values for the astigmatism coefficients (5 and 6) and coma coefficients (7, 8, 9, and 10).

Lensless coherent imaging and optical metrology are recent applications for using phase retrieval. The application of lensless coherent imaging using lasers to create speckle patterns is quite limited. The objects have to be asymmetric and have clear defined edges. This system has no imaging optics and must use coherent light (light that has the same wavelength and a defined phase relation so it can interfere constructively or destructively). Again, the Fourier magnitude is multiplied by a weighted function. However, this weighted function starts small and is slowly increased, summing all the phases from each weighted function until the image can be reconstructed [4]. For the improved optical metrology application, phase retrieval will be used to measure the surfaces of lenses, mainly aspherical. It is difficult to test the surface of aspherical lenses to determine if they are the correct shape. Usually, an interferometer is used to test the surface and can be complicated. Using phase retrieval, all that is need is a light source, a CCD array, and a computer with the phase retrieval algorithm, as seen in Fig.(5). However, this technique has only been computer simulated thus far, but the error of the Zernike coefficients were off by only 10^{-7} radians of phase [5].

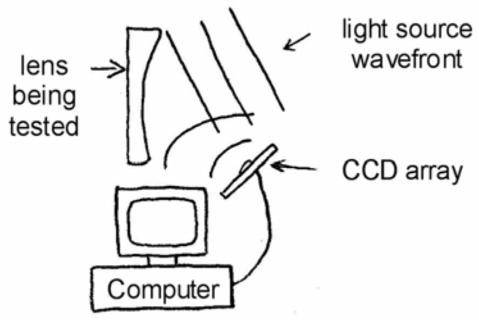

Figure 5. The simple setup for testing the shape of aspherical lenses.

Biography:

James R. Fienup is currently the senior scientist at the Laboratory for Laser Energetics and a Robert E. Hopkins Professor of Optics at the University of Rochester in Rochester, New York. His main area of study is imaging science. This includes imaging with sparse and segmented-aperture systems, synthetic-aperture and pupil-plane active imaging systems, wavefront sensing, image reconstruction, and phase retrieval. Dr. Fienup received his undergraduate degree at Holy Cross College in Physics and Mathematics, and his MS and PhD at Stanford University in Applied Physics. Before becoming a professor, Dr. Fienup was a senior scientist at Environmental Research Institute of Michigan, which later became part of the Advanced Systems Division of General Dynamics, where he began his work on phase retrieval [12].

Conclusion:

Phase retrieval has become a useful tool in many different areas of optics. Its use in determining the aberrations present in the Hubble Space Telescope helped deblur the images already taken and created corrective optics to repair the Hubble in space. Furthermore, NASA is planning on using phase retrieval to align the James Webb Space Telescope [5]. Recent applications may help to change optical metrology and remote sensing fields. With today's computer capabilities, phase retrieval algorithms can be more complex, allowing for these other applications to be possible.

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