Hierarchical Segregation: Issues and Analysis

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Anatomy of a Wage Decomposition

- Endowment effects
- Pure wage discrimination
- Job/occupational segregation
- Example of a conventional model
    1. $\bar{Y}_j = \bar{X}_j \hat{\beta}_j + \sum_{k=2}^{K} \bar{O}_{jk} \hat{\gamma}_{jk}, j = m, f$
    2. $\bar{O}_{jk}$ is the sample proportion of j workers in the *kth* job title. Without loss of generality, the left-out reference job is occupation 1.
    3. $\bar{X}_j$ includes the constant term.

Conventional Decomposition

- $\bar{Y}_m - \bar{Y}_f = (\bar{X}_m - \bar{X}_f)\,\hat{\beta}_m + \sum_{k=2}^{K} (\bar{O}_{mk} - \bar{O}_{fk})\,\hat{\gamma}_{mk} +$
  $\bar{X}_f\,(\hat{\beta}_m - \hat{\beta}_f) + \sum_{k=2}^{K} \bar{O}_{fk}\,(\hat{\gamma}_{mk} - \hat{\gamma}_{fk})$
- Unexplained/discrimination effect $=$
  $\bar{X}_f\,(\hat{\beta}_m - \hat{\beta}_f) + \sum_{k=2}^{K} \bar{O}_{fk}\,(\hat{\gamma}_{mk} - \hat{\gamma}_{fk})$
- Total endowment effect $=$
  $(\bar{X}_m - \bar{X}_f)\,\hat{\beta}_m + \sum_{k=2}^{K} (\bar{O}_{mk} - \bar{O}_{fk})\,\hat{\gamma}_{mk}$, where
  $\sum_{k=2}^{K} (\bar{O}_{mk} - \bar{O}_{fk})\,\hat{\gamma}_{mk}$ is a measure of the segregation effect on the wage gap.

Conventional Decomposition

- Some problems
  1. When the occupational/job title category is discrete, often the occupational aggregation is arbitrary e.g. 2 digit level, 3 digit level, etc.
  2. Depending on the degree of job title aggregation, there may be a lack of overlap in the occupational support, i.e. some occupational categories may contain no observations for one group.
  3. What constitutes the effect of pure wage discrimination?

     $$\bar{X}_f\left(\hat{\beta}_m - \hat{\beta}_f\right) + \sum_{k=2}^{K} \bar{O}_{fk}\left(\hat{\gamma}_{mk} - \hat{\gamma}_{fk}\right)? \quad \sum_{k=2}^{K} \bar{O}_{fk}\left(\hat{\gamma}_{mk} - \hat{\gamma}_{fk}\right)?$$

     Ideally, one would like a measure of gender wage gaps within jobs after conditioning on personal productivity measures. The problem with $\sum_{k=2}^{K} \bar{O}_{fk}\left(\hat{\gamma}_{mk} - \hat{\gamma}_{fk}\right)$ is that this measure is not invariant with respect to the omitted occupation reference group (Oaxaca and Ransom, 1999).

Hierarchical Segregation

- Baldwin et. al (2001)
  - Occupational segregation arises from (male) worker distaste for supervision by women
  - The proportion of women in a given job title relative to the proportion of men in a given job title declines exponentially as one ascends the job ladder.
  - The model dispenses with the need to commit to any degree of job title aggregation. With continuous wage distributions assumed, in effect every wage constitutes a “job”.

- For convenience let us assume that wages within a firm follow a lognormal distribution.
- Assume a wage distribution for males given by

$$f_m(w) = \frac{1}{w\sigma\sqrt{2\pi}} exp\left(\frac{-(\ell n(w) - \mu)^2}{2\sigma^2}\right) \tag{1}$$

- Identifying assumption: the female wage density comes from the same class of distributions as the male wage density.
- Female wage density = male wage density x sorting function
- The sorting function is the ratio of the proportion of females in a given job level to the ratio of males in the same job level In the limit each wage constitutes a job level.

- For the log normal distribution, which is a special case of the class of exponential distributions, the sorting function turns out to be

$$g(w) = \gamma \cdot exp\left\{ \frac{-(\gamma^2 - 1)\ell n(w)}{2\sigma^2} \left[\ell n \left(\frac{w}{exp\left(\frac{2\mu}{\gamma + 1}\right)} \right) \right] \right\} \tag{2}$$

- The female wage density is given by

$$f_f(w) = \frac{1}{w\left(\frac{\sigma}{\gamma}\right)\sqrt{2\pi}} exp\left[\frac{-\left(\ell n(w) - \frac{\mu}{\gamma}\right)^2}{2\left(\frac{\sigma}{\gamma}\right)^2}\right] \tag{3}$$

where $\gamma \geq 1$ is the job segregation parameter. In the absence of job segregation, $\gamma = 1$.

- Worker heterogeneity within job titles can be incorporated into the wage density functions:

$$f_m(w) = \frac{1}{w\sigma\sqrt{2\pi}} exp\left(\frac{-(\ell n(w) - \mu_m)^2}{2\sigma^2}\right) \quad (4)$$

and

$$f_f(z) = \frac{1}{z\left(\frac{\sigma}{\gamma}\right)\sqrt{2\pi}} exp\left[\frac{-(\ell n(z) - \mu_f)^2}{2\left(\frac{\sigma}{\gamma}\right)^2}\right] \quad (5)$$

where z represents the female wage in the presence of worker heterogeneity.

- Conditioning on worker characteristics (x) introduces additional parameters (α): $\mu_m = x_m \alpha_m$, $\mu_f = x_f \alpha_f^*$, and $\alpha_f^* = \dfrac{\alpha_f}{\gamma}$.
- Worker heterogeneity and wage discrimination imply

$$E\left[\ell n\left(z\right)\right] = \left(\frac{1}{\gamma}\right)\left[x_f \alpha_m - \ell n\left(\phi\right)\right]. \tag{6}$$

where ϕ is a measure of wage discrimination.
- The female wage density (5) implies

$$\begin{aligned} E\left[\ell n\left(z\right)\right] &= \mu_f \\ &= x_f \alpha_f^* \\ &= \frac{x_f \alpha_f}{\gamma}. \end{aligned} \tag{7}$$

- Upon equating (6) to (7), one can solve for ϕ from $\ell n(\phi) = x_f\,(\alpha_m - \alpha_f)$:
- $$\phi = exp\left[x_f\,(\alpha_m - \alpha_f)\right]. \tag{8}$$
- Pure wage discrimination (in logs): $\ell n(\phi) = x_f\,(\alpha_m - \alpha_f)$
- Clearly in the absence of wage discrimination, $\alpha_m - \alpha_f = 0 \Rightarrow \phi = 1$.

- Ideally, one would want to jointly estimate α_f ,γ, x_f, and α_m from the likelihood function for the combined sample of males and females.
- The estimated value of ϕ is backed out from (8) using the MLE estimates of the parameters and the conditioning values of x_f and x_m.
- Alternatively, one could separately estimate α_m and α_f^*,
  - In this case additional identifying restrictions are necessary to recover the parameter vector α_f and the segregation parameter γ.
  - Again, the estimated value of ϕ is backed out from (8).

- Identification strategy adopted in Shatnawi et al. (2010a) in the case of worker heterogeneity and pure wage discrimination.
  - Set $\sigma = \sigma_m$ and $\frac{\sigma}{\gamma} = \frac{\sigma_m}{\gamma} = \sigma_f$ and separately estimate the log wage equations for males and females
  - The segregation parameter is recovered from

$$\gamma = \frac{\sigma_m^2}{\sigma_f^2}$$

- Without covariates (homogenous workers) the log wage gap is entirely the result of job segregation:

$$E\left[\ell n(w_m)\right] - E\left[\ell n(w_f)\right] = \mu\left(\frac{\gamma - 1}{\gamma}\right)$$

- Decomposition of wage levels

$$E(w_m) - E(w_f) = exp\left(\mu + \frac{\sigma^2}{2}\right) - exp\left(\frac{\mu}{\gamma} + \frac{\sigma^2}{2\gamma^2}\right).$$

Empirically, the decomposition of wage levels is given by

$$\begin{aligned}\overline{w}_m - \overline{w}_f &= \left[exp\left(\hat{\theta}_m\right)\right]\left[exp(\hat{\mu} + 0.5\hat{\sigma}^2) - exp\left(\frac{\hat{\mu}}{\hat{\gamma}} + 0.5\frac{\hat{\sigma}^2}{\hat{\gamma}^2}\right)\right] \\ &+ exp\left(\frac{\hat{\mu}}{\hat{\gamma}} + 0.5\frac{\hat{\sigma}^2}{\hat{\gamma}^2}\right)\left[exp(\widehat{\theta}_m) - exp(\widehat{\theta}_f)\right].\end{aligned}$$

Decompositions

Absence of pure wage discrimination

- $\widehat{\theta}_m$ and $\widehat{\theta}_f$ are remainder terms that equate the means of the predicted wages for males and females to their sample means, i.e.

$$\widehat{\theta}_m = \ell n(\overline{w}_m) - (\hat{\mu} + 0.5\hat{\sigma}^2)$$

$$\widehat{\theta}_f = \ell n(\overline{w}_f) - \left(\frac{\hat{\mu}}{\hat{\gamma}} + 0.5\frac{\hat{\sigma}^2}{\hat{\gamma}^2}\right)$$

Decompositions

Worker heterogeneity

- Conditioning on sample mean characteristics $(\bar{x})$, the expected log wage decomposition can be written as

$$\begin{aligned} E\left[\ell n(w_m|\overline{x}_m)\right] - E\left[\ell n(w_F|\overline{x}_f)\right] &= \left(\overline{x}_m - \overline{x}_f\right)\alpha_m + \overline{x}_f\left(\alpha_m - \alpha_f\right) \\ &\quad + \overline{x}_f\alpha_f\left(\frac{\gamma - 1}{\gamma}\right). \end{aligned}$$

- Endowments $= \left(\overline{x}_m - \overline{x}_f\right)\alpha_m$
- Pure wage discrimination $= \overline{x}_f\left(\alpha_m - \alpha_f\right)$
- Segregation $= \overline{x}_f\alpha_f\left(\frac{\gamma-1}{\gamma}\right)$

- Note that $\overline{x}_f \alpha_f \left(\frac{\gamma-1}{\gamma}\right)$ reflects both discrimination and segregation when $\alpha_f \neq \alpha_m$.
- A better measure of segregation (and pure wage discrimination) would include an interaction term

$$\overline{x}_f \alpha_f \left(\frac{\gamma - 1}{\gamma}\right) = \overline{x}_f \alpha_m \left(\frac{\gamma - 1}{\gamma}\right) + \overline{x}_f \left(\alpha_f - \alpha_m\right) \left(\frac{\gamma - 1}{\gamma}\right).$$

1. $\overline{x}_f \alpha_m \left(\frac{\gamma-1}{\gamma}\right)$ measures segregation effects on wages assuming no pure wage discrimination.
2. $\overline{x}_f \left(\alpha_f - \alpha_m\right) \left(\frac{\gamma-1}{\gamma}\right)$ measures the interaction effect of pure wage discrimination and segregation.

- In wage levels (conditioning on mean characteristics) the wage gap is given by

$$E(w_m|\overline{x}_m) - E(w_f|\overline{x}_f) = exp\left(\bar{x}_m\alpha_m + \frac{\sigma^2}{2}\right) - exp\left(\frac{\bar{x}_f\alpha_f}{\gamma} + \frac{\sigma^2}{2}\right).$$

- The decomposition of the wage level gap can be expressed as

$$\begin{aligned} E(w_m|\overline{x}_m) - E(w_f|\overline{x}_f) &= \left[exp\left(\frac{\sigma^2}{2}\right)\right]\left[exp\left(\bar{x}_m\alpha_m\right) - exp\left(\bar{x}_f\alpha_m\right)\right] \\ &+ \left[exp\left(\frac{\sigma^2}{2}\right)\right]\left[exp\left(\bar{x}_f\alpha_m\right) - exp\left(\bar{x}_f\alpha_f\right)\right] \\ &+ exp\left(\overline{x}_f\alpha_f + \frac{\sigma^2}{2}\right) - exp\left(\frac{\overline{x}_f\alpha_f}{\gamma} + \frac{\sigma^2}{2}\right) \end{aligned}$$

Decompositions

Worker heterogeneity

- Endowments $= \left[exp\left(\frac{\sigma^2}{2}\right)\right] \left[exp\left(\bar{x}_m \alpha_m\right) - exp\left(\bar{x}_f \alpha_m\right)\right]$
- Pure wage discrimination
  $= \left[exp\left(\frac{\sigma^2}{2}\right)\right] \ \left[exp\left(\bar{x}_f \alpha_m\right) - exp\left(\bar{x}_f \alpha_f\right)\right]$
- Segregation $= exp\left(\overline{x}_f \alpha_f + \frac{\sigma^2}{2}\right) - exp\left(\frac{\overline{x}_f \alpha_f}{\gamma} + \frac{\sigma^2}{2}\right)$

- The empirical wage level decomposition can be expressed as $\overline{w}_m - \overline{w}_f =$

$$exp(0.5\hat{\sigma}_m^2 + \widehat{\theta}_m)\left[N_m^{-1}\sum_{i=1}^{N_m} exp(x_{mi}\hat{\alpha}_m)\right.$$

$$\left. - N_f^{-1}\sum_{i=1}^{N_f} exp(x_{fi}\hat{\alpha}_m)\right]$$

$$+ N_f^{-1} exp(0.5\hat{\sigma}_m^2 + \widehat{\theta}_m)\left\{\sum_{i=1}^{N_f}\left[exp(x_{fi}\hat{\alpha}_m) - exp(x_{fi}\hat{\alpha}_f)\right]\right\}$$

$$+ N_f^{-1} exp(\widehat{\theta}_f)\left\{\sum_{i=1}^{N_f}\left[exp(x_{fi}\hat{\alpha}_f + 0.5\hat{\sigma}_m^2) - exp\left(\frac{x_{fi}\widehat{\alpha}_f}{\hat{\gamma}} + 0.5\hat{\sigma}_f^2\right)\right]\right\}$$

$$+ N_f^{-1}\left[\sum_{i=1}^{N_f} exp(x_{fi}\hat{\alpha}_f + 0.5\hat{\sigma}_m^2)\right]\left[exp(\widehat{\theta}_m) - exp(\widehat{\theta}_f)\right].$$

- Again, $\widehat{\theta}_m$ and $\widehat{\theta}_f$ are remainder terms that equate the means of the predicted wages for males and females to their sample means.
- Endowments =
$$exp(0.5\hat{\sigma}_m^2 + \widehat{\theta}_m)\left[N_m^{-1}\sum_{i=1}^{N_m} exp(x_{mi}\hat{\alpha}_m) - N_f^{-1}\sum_{i=1}^{N_f} exp(x_{fi}\hat{\alpha}_m)\right]$$
- Pure wage discrimination
$$= N_f^{-1} exp(0.5\hat{\sigma}_m^2 + \widehat{\theta}_m)\left\{\sum_{i=1}^{N_f}\left[exp(x_{fi}\hat{\alpha}_m) - exp(x_{fi}\hat{\alpha}_f)\right]\right\}$$
- Segregation =
$$N_f^{-1} exp(\widehat{\theta}_f)\left\{\sum_{i=1}^{N_f}\left[exp(x_{fi}\hat{\alpha}_f + 0.5\hat{\sigma}_m^2) - exp\left(\frac{x_{fi}\widehat{\alpha}_f}{\hat{\gamma}} + 0.5\hat{\sigma}_f^2\right)\right]\right\}$$
- Statistical adjustment
$$= N_f^{-1}\left[\sum_{i=1}^{N_f} exp(x_{fi}\hat{\alpha}_f + 0.5\hat{\sigma}_m^2)\right]\left[exp(\widehat{\theta}_m) - exp(\widehat{\theta}_f)\right]$$

- Numerical example from Shatnawi, et. al (2010a)
- No wage discrimination because of union contract.

Table 6

Heterogenous Case-No Discrimination

Lognormal Results and Decomposition

	Pooled Men and Women
constant	0.0622
age	0.1034
age-squared	-0.0012
tenure	0.0460
ten2	-0.0015
σ^2	0.2584
γ	1.0789
N	1976

based off of joint estimation of the male and female likelihood functions

Decomposition	Wage Difference	Percent Difference
Endowment	-0.9334	-1.4071
Discrimination	0.0000	0.0000
Segregation	1.4462	2.1802
Non-linear	0.1505	0.2269
Total	0.6633	1

- Conventional log wage model with occupational controls:

$$\overline{\ell n(w_j)} = \bar{X}_j\hat{\beta}_j + \sum_{k=2}^{K} \bar{O}_{jk}\hat{\gamma}_{jk}, j = m, f$$

- Conventional type wage level decomposition with occupational controls: $\overline{w}_m - \overline{w}_f$
- Endowments + Segregation

$= exp(0.5\tilde{\sigma}^2_m + \tilde{\theta}_m)[N_m^{-1} \sum_{i=1}^{N_m} exp(X_{mi}\hat{\beta}_m + O_{mi}\hat{\gamma}_m) -$

$N_f^{-1} \sum_{i=1}^{N_f} exp(X_{fi}\hat{\beta}_m + O_{fi}\hat{\gamma}_m)]$

- Note that the endowment and segregation effects cannot be separately identified in wage levels from a log wage model.

- Pure wage discrimination $= N_f^{-1} exp(0.5\tilde{\sigma}_m^2 + \tilde{\theta}_m) \left\{ \sum_{i=1}^{N_f} \left[exp(X_{fi}\hat{\beta}_m + O_{fi}\tilde{\beta}_m) - exp(X_{fi}\hat{\beta}_f + O_{fi}\hat{\gamma}_f) \right] \right\}$
- Statistical adjustment $= N_f^{-1} \left[\sum_{i=1}^{N_f} exp(X_{fi}\hat{\beta}_f + O_{fi}\ \hat{\gamma}_f) \right] \cdot \left[exp(0.5\tilde{\sigma}_m^2 + \tilde{\theta}_m) - exp(0.5\tilde{\sigma}_f^2 + \tilde{\theta}_f) \right]$

Fixed Effects Models

- Shatnawi, et. al (2010b)
- Balanced design

$$\begin{aligned} \ell n\left(w_{mit}\right) &= x_{mit}\beta_m + \alpha_{mi} - \bar{\alpha}_m + \varepsilon_{mit}, \; t = 1, ..., T \\ \ell n\left(w_{fit}\right) &= x_{fit}\beta_f^* + \alpha_{fi}^* - \bar{\alpha}_f^* + \varepsilon_{fit}, \; t = 1, ..., T \end{aligned}$$

- $\beta_f^* = \dfrac{\beta_f}{\gamma}, \; \alpha_{fi}^* = \dfrac{\alpha_{fi}}{\gamma}$
- Segregation parameter: $\gamma = \dfrac{\sigma^2_{m\varepsilon}}{\sigma^2_{f\varepsilon}}$
- $\bar{\alpha}_j = \dfrac{\sum_{i=1}^{N_j} \alpha_{ji}}{N_j}, \; j = m, f$ from which the normalization $\sum_{i=1}^{N_j} \left(\alpha_{ji} - \bar{\alpha}_j\right) = 0$ yields a constant term of $\bar{\alpha}_j$ at the overall sample mean.

- Expected log wage decomp at overall sample mean:

$$\begin{aligned} E\left[\ell n(w_m|\overline{\overline{x}}_m)\right] - E\left[\ell n(w_F|\overline{\overline{x}}_f)\right] &= \left(\overline{\overline{x}}_m - \overline{\overline{x}}_f\right)\beta_m \\ &+ [\overline{\overline{x}}_f(\beta_m - \beta_f) + (\overline{\alpha}_m - \overline{\alpha}_f)] \\ &+ \overline{\overline{x}}_f \beta_f \left(\frac{\gamma - 1}{\gamma}\right) \end{aligned}$$

- Overall sample mean: $\overline{\overline{x}} = \dfrac{\sum\limits_{i=1}^{n}\sum\limits_{t=1}^{T} x_{it}}{nT}$

- Expected wage level gap

$$\begin{aligned} E(w_m|\overline{\overline{x}}_m) - E(w_f|\overline{\overline{x}}_f) &= exp\left(\overline{\overline{x}}_m\beta_m + \overline{\alpha}_m + \frac{\sigma^2_{\varepsilon m}}{2}\right) \\ &\quad - exp\left(\frac{\overline{\overline{x}}_f\beta_f + \overline{\alpha}_f}{\gamma} + \frac{\sigma^2_{\varepsilon f}}{2}\right). \end{aligned}$$

$$
\begin{aligned}
&\overline{\overline{w}}_m - \overline{\overline{w}}_f = \\
&\exp\left(\frac{\widehat{\sigma}^2_{\varepsilon m}}{2} + \widehat{\theta}_m\right)\left[\exp\left(\overline{\overline{x}}_m \widehat{\beta}_m + \overline{\alpha}_m\right) - \exp\left(\overline{\overline{x}}_f \widehat{\beta}_m + \overline{\alpha}_m\right)\right] \\
&+ \exp\left(\frac{\widehat{\sigma}^2_{\varepsilon m}}{2} + \widehat{\theta}_m\right)\ \left[\exp\left(\overline{\overline{x}}_f \widehat{\beta}_m + \overline{\alpha}_m\right) - \exp\left(\overline{\overline{x}}_f \widehat{\beta}_f + \overline{\alpha}_f\right)\right] \\
&+ \exp(\widehat{\theta}_f)\left[exp\left(\overline{\overline{x}}_f \widehat{\beta}_f + \overline{\alpha}_f + \frac{\widehat{\sigma}^2_{\varepsilon m}}{2}\right) - \exp\left(\frac{\overline{\overline{x}}_f \widehat{\beta}_f + \overline{\alpha}_f}{\gamma} + \frac{\frac{\widehat{\sigma}^2_{\varepsilon m}}{\widehat{\gamma}}}{2}\right)\right] \\
&+ \left[\exp\left(\overline{\overline{x}}_f \widehat{\beta}_f + \overline{\alpha}_f + \frac{\widehat{\sigma}^2_{\varepsilon m}}{2}\right)\right]\left[\exp(\widehat{\theta}_m) - \exp(\widehat{\theta}_f)\right]
\end{aligned}
$$

Endowments

$$= exp(0.5\hat{\sigma}^2_{\varepsilon m} + \widehat{\theta}_m)\left[(\overline{T}_m n_m)^{-1}\sum_{i=1}^{n_m}\sum_{t=1}^{T_{im}} exp(x_{mit}\widehat{\beta}_m + \widehat{\alpha}_{mi})\right.$$

$$\left.-(\overline{T}_f n_f)^{-1}\sum_{i=1}^{n_f}\sum_{t=1}^{T_{if}} exp(x_{fit}\widehat{\beta}_m + \widehat{\alpha}_{mi})\right]$$

Pure wage discrimination $= (\overline{T}_f n_f)^{-1} exp(0.5\hat{\sigma}^2_{\varepsilon m} + \widehat{\theta}_m)$

$$\cdot\sum_{i=1}^{n_f}\sum_{t=1}^{T_{if}}\left[exp(x_{fit}\widehat{\beta}_m + \widehat{\alpha}_{mi}) - exp(x_{fit}\widehat{\beta}_f + \widehat{\alpha}_{fi})\right]$$

Segregation

$$= (\overline{T}_f n_f)^{-1} exp(\widehat{\theta}_f)\{\sum_{i=1}^{n_f}\sum_{t=1}^{T_{if}}[exp(x_{fit}\widehat{\beta}_f + \widehat{\alpha}_{fi} + 0.5\hat{\sigma}^2_{\varepsilon m})$$

$$-exp\left(\frac{x_{fit}\widehat{\beta}_f + \widehat{\alpha}_{fi}}{\hat{\gamma}} + 0.5\frac{\hat{\sigma}^2_{\varepsilon m}}{\hat{\gamma}}\right)]\}$$

Statistical adjustment

$$= (\overline{T}_f n_f)^{-1} \left[\sum_{i=1}^{n_f} \sum_{t=1}^{T_{if}} exp(x_{fit}\widehat{\beta}_f + \widehat{\alpha}_{fi} + 0.5\hat{\sigma}^2_{\varepsilon m}) \right]$$

$$\cdot \left[exp(\widehat{\theta}_m) - exp(\widehat{\theta}_f) \right].$$

Random Effects Models

- Shatnawi et.al (2010b)

$$\ell n\left(w_{mit}\right) = x_{mit}\beta_m + v_{mit},\ t = 1, ..., T$$
$$\ell n\left(w_{fit}\right) = x_{fit}\beta_f^* + v_{fit},\ t = 1, ..., T$$

$$v_{it}^m = u_i^m + \varepsilon_{it}^m$$
$$v_{it}^f = u_i^f + \varepsilon_{it}^f$$

$$E\left[v_{it}^2\right] = \sigma_u^2 + \sigma_\varepsilon^2 = \sigma_v^2$$

- Segregation parameter: $\gamma = \dfrac{\sigma_{vm}^2}{\sigma_{vf}^2} = \dfrac{\sigma_{um}^2 + \sigma_{\varepsilon m}^2}{\sigma_{\varepsilon m}^2 + \sigma_{\varepsilon m}^2}$

Random Effects Models

Decompositions

- Note

$$\overline{\overline{\ln w}} = \overline{\overline{x}}\widehat{\beta} + \overline{u}, \text{ where } \overline{u} = \frac{\sum_{i=1}^{n} u_i}{n}$$

$$\overline{\overline{\ln w}} \neq \overline{\overline{x}}\widehat{\beta}.$$

- Log wage decompositions

$$\overline{\overline{\ln w_m}} - \overline{\overline{\ln w_f}} = \left(\overline{\overline{x}}_m - \overline{\overline{x}}_f\right)\beta_m + \overline{\overline{x}}_f\left(\beta_m - \beta_f\right) + \overline{\overline{x}}_f\beta_f\left(\frac{\gamma - 1}{\gamma}\right) + \left(\overline{u}_m - \overline{u}_f\right)$$

- The empirical wage level decompositions are identical in form to the FE decompositions with the RE constant term appearing in the place of the individual fixed effects.