The Econometrics of Wage Decompositions

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I. BASIC DECOMPOSITIONS

Background reading:

Blinder, Alan S. (1973), "Wage Discrimination: Reduced Form and Structural Estimates," *Journal of Human Resources* 8: 436-455.

Oaxaca, Ronald L. (1973), "Male-Female Wage Differentials in Urban Labor Markets," *International Economic Review.* 14(3): 693-709.

Greene, William H. (2003), *Econometric Analysis,* Fifth Edition (International Edition), Chapter 3.

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For purposes of illustration only, the decomposition techniques and issues are applied initially to the case of decomposing (log) wage differentials between men and women into explained and unexplained differences. However, it is clear that the approaches described below apply to any attempt to decompose mean sample wage differences between any two categories of observations, e.g. union workers versus nonunion workers, manufacturing firms versus non manufacturing firms, public sector workers versus private sector workers, workers in Luxembourg City versus workers in Milano. Also, the methodology shown below is extended to decompositions of non-wage variables, e.g. prison sentences.

From the properties of OLS we have $\overline{Y} = \bar{X}'\widehat{\beta}$, where $\overline{Y}$ is Tx1, $\bar{X}'$is 1xk, and $\widehat{\beta}$ is kx1. A standard male/female wage decomposition is

$$\overline{Y}_m - \overline{Y}_f = \left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_m + \bar{X}_f' \left(\widehat{\beta}_m - \widehat{\beta}_f\right). \tag{1}$$

This decomposition assumes that the m (male) structure is the norm. Accordingly, the term $\left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_m$ represents the "explained" differential. Therefore, the term $\bar{X}_f' \left(\widehat{\beta}_m - \widehat{\beta}_f\right)$ represents the "unexplained" differential. In some circles and in some contexts this term is interpreted as a measure of discrimination.

We might think of $\overline{Y}_f^0 = \bar{X}_f' \widehat{\beta}_m$ as the mean competitive, nondiscriminatory (log) wage for females. In this case $\overline{Y}_m^0 = \overline{Y}_m = \bar{X}_m' \widehat{\beta}_m$ since the male wage structure is the norm. The decomposition can be equivalently stated as

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f &= \left(\overline{Y}_m^0 - \overline{Y}_f^0\right) + \left(\overline{Y}_f^0 - \overline{Y}_f\right) \\ &= \left(\overline{Y}_m - \overline{Y}_f^0\right) + \left(\overline{Y}_f^0 - \overline{Y}_f\right) \end{aligned}$$

Typically in wage regressions Y is the log of the wage, so that

$$\bar{Y} = \left(\sum_{t=1}^{T} \ell n\left(w_t\right)\right) / T = \ell n\left(\tilde{w}\right),$$

where $\tilde{w}$ is the geometric mean. In this situation

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f &= \ell n\left(\tilde{w}_m\right) - \ell n\left(\tilde{w}_f\right) \\ &= \ell n\left(G + 1\right) \end{aligned}$$

where $G = \dfrac{\tilde{w}_m}{\tilde{w}_f} - 1$ is the gross (unadjusted) wage differential. Along these lines we can view the explained gap as the gap attributable to qualifications differences, i.e.

$$\begin{aligned}
\left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_m &= \overline{Y}_m - \overline{Y}^0_f \\
&= \ell n\left(\tilde{w}_m\right) - \ell n\left(\tilde{w}^0_f\right) \\
&= \ell n\left(Q_m + 1\right)
\end{aligned}$$

where $Q_m = \dfrac{\tilde{w}_m}{\tilde{w}^0_f} - 1$ is the wage differential attributable to differences in qualifications when using the male wage structure as the norm. This leaves the unexplained differential, i.e.

$$\begin{aligned}
\bar{X}'_f \left(\widehat{\beta}_m - \widehat{\beta}_f\right) &= \left(\overline{Y}^0_f - \overline{Y}_f\right) \\
&= \ell n\left(\tilde{w}^0_f\right) - \ell n\left(\tilde{w}_f\right) \\
&= \ell n\left(D_m + 1\right)
\end{aligned}$$

where $D_m = \dfrac{\tilde{w}^0_f}{\tilde{w}_f} - 1$ is the wage differential that is unexplained (discrimination ?) when using the male wage structure as the norm . With this notation in hand, we arrive at the following accounting identity:

$$\ell n\left(G+1\right) = \ell n\left(Q_m + 1\right) + \ell n\left(D_m + 1\right).$$

An alternative decomposition is given by

$$\overline{Y}_m - \overline{Y}_f = \left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_f + \bar{X}'_m \left(\widehat{\beta}_m - \widehat{\beta}_f\right)$$

This decomposition assumes that the f structure is the norm. Now the term $\left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_f$ measures the "explained" differential and $\bar{X}'_m \left(\widehat{\beta}_m - \widehat{\beta}_f\right)$ measures the "unexplained" differential. Here we can think of $\overline{Y}^0_m = \bar{X}'_m \widehat{\beta}_f$ as the mean competitive, nondiscriminatory (log) wage for males. In this case $\overline{Y}^0_f = \overline{Y}_f = \bar{X}'_f \widehat{\beta}_f$ since the female wage structure is the norm. Equivalently

$$\begin{aligned}
\overline{Y}_m - \overline{Y}_f &= \left(\overline{Y}^0_m - \overline{Y}^0_f\right) + \left(\overline{Y}_m - \overline{Y}^0_m\right) \\
& \left(\overline{Y}^0_m - \overline{Y}_f\right) + \left(\overline{Y}_m - \overline{Y}^0_m\right)
\end{aligned}$$

The accounting identity that results from this decomposition is given by

$$\ell n\left(G+1\right)=\ell n\left(Q_f+1\right)+\ell n\left(D_f+1\right)$$

where

$$\begin{aligned}\ell n\left(Q_f+1\right) &= \overline{Y}_m^0-\overline{Y}_f \\ &= \left(\bar{X}_m-\bar{X}_f\right)^{'}\widehat{\beta}_f \\ &= \ell n\left(\tilde{w}_m^0\right)-\ell n\left(\tilde{w}_f\right).\end{aligned}$$

and

$$\begin{aligned}\ell n\left(D_f+1\right) &= \left(\overline{Y}_m-\overline{Y}_m^0\right) \\ &= \bar{X}_m'\left(\widehat{\beta}_m-\widehat{\beta}_f\right) \\ &= \ell n\left(\tilde{w}_m\right)-\ell n\left(\tilde{w}_m^0\right).\end{aligned}$$

It is clear that $Q_f=\dfrac{\tilde{w}_m^0}{\tilde{w}_f}-1$ is the wage differential attributable to differences in qualifications when using the female wage equation as the norm, and D_f is the wage differential that is unexplained.

The two sets of decompositions corresponding to the male wage structure as the norm or the female wage structure as the norm illustrate the index number problem with decompositions. In other words the separately calculated explained and unexplained components will in general differ depending on which structure is assumed to be the norm. Of course their sum will be the same.

II. IDENTIFICATION ISSUES

Background Reading:

Jones, F.L., (1983), "On Decomposing the Wage Gap: A Critical Comment on Blinder's Method," *Journal of Human Resources* 18, 126-130.

Oaxaca, Ronald L. and Michael Ransom (1999) "Identification in Detailed Wage Decompositions"(with Michael R. Ransom), *Review of Economics and Statistics*, vol. 81, no. 1, 154-157.

Javier Gardeazabal and Arantza Ugidos, "More on Identification in Detailed Wage Decompositions," *The Review of Economics and Statistics*, Vol. 86, No. 4, November 2004, 1034-1036.

Myeong-Su Yun, "A Simple Solution to the Identification Problem in Detailed Wage Decompositions," *Economic Inquiry*, vol. 43, Oct 2005, 766 - 772.

Greene, William H. (2003), *Econometric Analysis,* Fifth Edition (International Edition), Chapter 7 (pp.116-120).

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Separately estimated (log) wage equations for males and females evaluated at the sample means are given by

$$\bar{Y}_m = \hat{\beta}_{mo} + \sum_{j=1}^{N} \bar{X}_m^{(j)\prime}\, \hat{\beta}_m^{(j)} \ ,$$

$$\bar{Y}_f = \hat{\beta}_{fo} + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime}\, \hat{\beta}_f^{(j)} \ ,$$

where $\hat{\beta}_{io}$ is the estimated intercept term, $\hat{\beta}_i^{(j)}$ is a column vector of estimated slope coefficients for the set of regressors comprising the jth variable , and $\bar{X}_i^{(j)\prime}$ is a row vector of regressor means for the set of regressors comprising the jth variable. There are N variables defined by N sets of regressors, e.g. experience and experience squared would constitute the experience variable. If we adopt the m wage structure as the norm, the gender wage gap is decomposed according to

$$\bar{Y}_m - \bar{Y}_f = \underbrace{\left(\hat{\beta}_{mo} - \hat{\beta}_{fo}\right) + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime} \Delta\,\hat{\beta}^{(j)}}_{\text{discrimination}} + \underbrace{\sum_{j=1}^{N} \Delta \bar{X}^{(j)\prime} \hat{\beta}_m^{(j)}}_{\text{endowments}}\ , \tag{2}$$

where $\Delta\,\hat{\beta}^{(j)} = \hat{\beta}_m^{(j)} - \hat{\beta}_f^{(j)}$ and $\Delta \bar{X}^{(j)\prime} = \bar{X}_m^{(j)\prime} - \bar{X}_f^{(j)\prime}$. The contributions of the jth variable to discrimination and endowments are $\bar{X}_f^{(j)\prime} \Delta\,\hat{\beta}^{(j)}$ and $\Delta \bar{X}^{(j)\prime} \hat{\beta}_m^{(j)}$, and the contribution of the intercept term to the discrimination component is $\left(\hat{\beta}_{mo} - \hat{\beta}_{fo}\right)$. Given the specification of the $X's$, there is seemingly no ambiguity surrounding the decomposition. However, this is an illusion.

Consider the case in which a variable V defined by a set of dummy variables is added to the wage regressions, e.g. marital status. The set of dummy variable mean values are denoted by $\left\{\bar{V}_{ik} \mid k = 1, ...K_1\right\}$, where $\sum_{k=1}^{K_1} \bar{V}_{ik} = 1,\ i = m, f$. Without loss of generality the first dummy variable category $\left(\bar{V}_{i1}\right)$ will initially serve as the left out reference group, e.g. married, spouse present. The separately estimated wage equations for men and women evaluated at the sample means can now be expressed as

$$\begin{aligned} \bar{Y}_m &= \hat{\beta}_{mo} + \sum_{k=2}^{K_1} \bar{V}_{mk}\,\hat{\delta}_{mk} + \sum_{j=1}^{N} \bar{X}_m^{(j)\prime}\,\hat{\beta}_m^{(j)} \\ &= \sum_{k=1}^{K_1} \bar{V}_{mk}\,\hat{\theta}_{mk} + \sum_{j=1}^{N} \bar{X}_m^{(j)\prime}\,\hat{\beta}_m^{(j)}\ , \end{aligned}$$

$$\begin{aligned} \bar{Y}_f &= \hat{\beta}_{fo} + \sum_{k=2}^{K_1} \bar{V}_{fk}\,\hat{\delta}_{fk} + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime}\,\hat{\beta}_f^{(j)} \\ &= \sum_{k=1}^{K_1} \bar{V}_{fk}\,\hat{\theta}_{fk} + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime}\,\hat{\beta}_f^{(j)}\ , \end{aligned}$$

where $\hat{\delta}_{ik} = \hat{\theta}_{ik} - \hat{\theta}_{i1}$, and our left out reference group choice implies the normalization $\hat{\beta}_{io} = \hat{\theta}_{i1}$. Accordingly, the resulting wage decomposition is given by

$$
\begin{aligned}
\bar{Y}_m - \bar{Y}_f \;\; = \;\; & \underbrace{\left(\hat{\beta}_{mo} - \hat{\beta}_{fo}\right) + \sum_{k=2}^{K_1} \bar{V}_{fk} \left(\hat{\delta}_{mk} - \hat{\delta}_{fk}\right) + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime} \Delta \hat{\beta}^{(j)}}_{\text{discrimination}} \qquad (3) \\
& \underbrace{+ \sum_{k=2}^{K_1} \left(\bar{V}_{mk} - \bar{V}_{fk}\right) \hat{\delta}_{mk} + \sum_{j=1}^{N} \Delta \bar{X}^{(j)\prime} \hat{\beta}_m^{(j)}}_{\text{endowments}} \\
= \;\; & \underbrace{\sum_{k=1}^{K_1} \bar{V}_{fk} \left(\hat{\theta}_{mk} - \hat{\theta}_{fk}\right) + \sum_{j=1}^{N} \bar{X}_f^{(j)\prime} \Delta \hat{\beta}^{(j)}}_{\text{discrimination}} \\
& \underbrace{+ \sum_{k=1}^{K_1} \left(\bar{V}_{mk} - \bar{V}_{fk}\right) \hat{\theta}_{mk} + \sum_{j=1}^{N} \Delta \bar{X}^{(j)\prime} \hat{\beta}_m^{(j)}}_{\text{endowments}} \; .
\end{aligned}
$$

A number of things are immediately apparent from the decompositions described by (3). First, the estimated overall discrimination and the estimated overall endowment effect are invariant to the choice of left out reference group and to the suppression of the constant term in the absence of a left out reference group. That is, the alternative expressions for the estimated overall discrimination and endowment contributions in (3) are the same decompositions. Second, the contribution of the variable V to discrimination as estimated by $\sum_{k=2}^{K_1} \bar{V}_{fk} \left(\hat{\delta}_{mk} - \hat{\delta}_{fk}\right)$ is not invariant with respect to the choice of left out reference group. For example, designating the last dummy variable category $\left(\bar{V}_{iK_1}\right)$ as the left out reference group (e.g. single, never married) would replace the immediately preceding measure of discrimination effects with $\sum_{k=1}^{K_1-1} \bar{V}_{fk} \left(\hat{\phi}_{mk} - \hat{\phi}_{fk}\right)$, where $\hat{\phi}_{ik} = \hat{\theta}_{ik} - \hat{\theta}_{iK_1}$. These two sets of estimates will not be the same because the intercept term is altered by the renormalization, $\hat{\beta}_{io} = \hat{\theta}_{iK_1}$. Third, it is possible to identify the contribution of V to discrimination as $\left(\hat{\beta}_{mo} - \hat{\beta}_{fo}\right) + \sum_{k=2}^{K_1} \bar{V}_{fk}\left(\hat{\delta}_{mk} - \hat{\delta}_{fk}\right) = \sum_{k=1}^{K_1} \bar{V}_{fk}\left(\hat{\theta}_{mk} - \hat{\theta}_{fk}\right)$, i.e. the intercept contribution is part of the contribution of V to discrimination. This interpretation requires a normalizing restriction that holds that in the absence of variable V there would be no constant term. Last, the contribution of V to endowments is invariant with respect to the choice of left out reference group, i.e. $\sum_{k=2}^{K_1}\left(\bar{V}_{mk} - \bar{V}_{fk}\right)\hat{\delta}_{mk} = \sum_{k=1}^{K_1-1}\left(\bar{V}_{mk} - \bar{V}_{fk}\right)\hat{\phi}_{mk} = \sum_{k=1}^{K_1}\left(\bar{V}_{mk} - \bar{V}_{fk}\right)\hat{\theta}_{mk}$.

The analysis can be generalized to multiple sets of dummy variables. It is clear that if the constant term is suppressed in models with more than one set of dummy variables, all but one of the sets of dummy variables must have left out reference categories in order to avoid perfect multicollinearity. A number of implications follow from the generalization. First, alternative decompositions are equivalent in terms of the estimates of overall discrimination and the overall contribution of endowments. Therefore, the overall decomposition is invariant with respect to the choice of left out reference groups. Second, it can be shown that the *combined* estimated contributions of all sets of dummy variables to overall discrimination (inclusive of the constant term) and to overall endowment effects are invariant with respect to the choice of left out reference groups. Third, the separate contributions of sets of dummy variables to discrimination are not invariant with respect to the choice of left out reference groups. Fourth, unlike the case with only one set of dummy variables expressed by (3), there are no unique estimates of the separate contributions of the sets of dummy variables to overall discrimination.

Empirical Examples

The first example is based on gender salary decompositions for a sample of full time, full-year U.S. college and university professors. The model consists of variables to measure total experience as a professor, seniority at current institution, and a set of dummy variables to indicate highest degree held, along with continuous measures of publication activity (journal articles, books, and collections), and dummy variables for a 12 month contract, teaching field, and race/ethnicity. We specify the "highest degree" variable with respect to two different reference groups. In one case, the reference group is those who have no advanced degree; the other reference group is those with a Ph.D. degree. The unadjusted differential is 26.6 log points. That is, women's salaries are about 25 percent less than men's. The decomposition table below illustrates the identification problem.

Wage Decompositions with Alternative Reference Groups

	No ADV. Deg (Ref)		Ph.D. (Ref)	
Variable	Disc	Endow	Disc	Endow
Constant	0.219	0.000	0.037	0.000
Seniority	-0.014	0.007	-0.014	0.007
Experience	0.064	0.074	0.064	0.074
Degree Type	-0.193	0.042	-0.011	0.042
Cont length	0.007	-0.005	0.007	-0.005
Pub Activity	-0.046	0.049	-0.046	0.049
Field	0.053	0.010	0.053	0.010
Race/Ethnic	-0.000	-0.001	-0.000	-0.001
Total	0.090	0.176	0.090	0.176

Differences in average qualifications between men and women explain 17.6 log points, so the estimate of discrimination is 9 log points. When No Advanced Degree is the reference group, the partial contribution of degree type to discrimination is -19.3 log points, and the contribution of constant term differences is 21.9 log points. For Ph.D. as the left out group, these contributions are -1.1 log points and 3.7 log points, respectively. The partial contribution of degree type to the endowment effect is 4.2 log points regardless of the left out reference group. This clearly demonstrates the identification problem–the choice of education category for the reference group is entirely arbitrary, yet the amount of discrimination that is attributed to degree type varies dramatically.

Second example: G is an indicator variable for university graduate (non university graduate is the omitted reference group) and T is work experience

$$\begin{aligned}\bar{Y}_m - \bar{Y}_f &= \left(\bar{G}_m - \bar{G}_f\right)\hat{\beta}_{1m} + \left(\bar{T}_m - \bar{T}_f\right)\hat{\beta}_{2m} \\ &+ \left(\hat{\beta}_{0m} - \hat{\beta}_{0f}\right) + \left(\hat{\beta}_{1m} - \hat{\beta}_{1f}\right)\bar{G}_f + \left(\hat{\beta}_{2m} - \hat{\beta}_{2f}\right)\bar{T}_f,\end{aligned}$$

Suppose instead that the omitted reference group is non university graduate ($S = 1 - G$), the resulting decomposition is

$$\begin{aligned}\bar{Y}_m - \bar{Y}_f &= \left(\bar{S}_m - \bar{S}_f\right)\hat{\theta}_{1m} + \left(\bar{T}_m - \bar{T}_f\right)\hat{\beta}_{2m} \\ &+ \left(\hat{\theta}_{0m} - \hat{\theta}_{0f}\right) + \left(\hat{\theta}_{1m} - \hat{\theta}_{1f}\right)\bar{S}_f + \left(\hat{\beta}_{2m} - \hat{\beta}_{2f}\right)\bar{T}_f.\end{aligned}$$

Unfortunately, $\left(\hat{\beta}_{0m} - \hat{\beta}_{0f}\right) \neq \left(\hat{\theta}_{0m} - \hat{\theta}_{0f}\right)$, and $\left(\hat{\beta}_{1m} - \hat{\beta}_{1f}\right) \neq \left(\hat{\theta}_{1m} - \hat{\theta}_{1f}\right)$ even though $\left(\hat{\beta}_{0m} - \hat{\beta}_{0f}\right) + \left(\hat{\beta}_{1m} - \hat{\beta}_{1f}\right)\bar{G}_f = \left(\hat{\theta}_{0m} - \hat{\theta}_{0f}\right) + \left(\hat{\theta}_{1m} - \hat{\theta}_{1f}\right)\bar{S}_f$.

<u>Solutions: Gardeazabal and Ugidos (2004) and Yun (2005)</u>

Force coefficients on a set of indicator variables to sum to zero

$$\begin{aligned}Y_i &= b_0 + b_1\, G_i + c_1 S_i + \beta_2\, T_i + \varepsilon_i \\ &= b_0 + b_1\, (G_i - S_i) + \beta_2\, T_i + \varepsilon_i\end{aligned}$$

since $b_1 + c_1 = 0$, so that $c_1 = -\, b_1$.

The decomposition is now expressed by

$$\begin{aligned}\bar{Y}_m - \bar{Y}_f &= \left(\hat{b}_{0m} - \hat{b}_{0f}\right) + \left(\hat{b}_{1m} - \hat{b}_{1f}\right)\left(\bar{G}_f - \bar{S}_f\right) + \left(\hat{\beta}_{2m} - \hat{\beta}_{2f}\right)\bar{T}_f \\ &\quad + \hat{b}_{1m}\left[\left(\bar{G}_m - \bar{G}_f\right) - \left(\hat{S}_m - \hat{S}_f\right)\right] + \hat{\beta}_{2m}\left(\bar{T}_m - \bar{T}_f\right).\end{aligned}$$

the choice of omitted reference group no longer matters, i.e. $\left(\hat{b}_{1m} - \hat{b}_{1f}\right)\left(\bar{G}_f - \bar{S}_f\right) = -\left(\hat{c}_{1m} - \hat{c}_{1f}\right)\left(\bar{G}_f - \bar{S}_f\right)$.

III. GENERALIZED WAGE DECOMPOSITIONS

Background Reading:

Cotton, Jeremiah (1988), "On the Decomposition of Wage Differentials," *Review of Economics and Statistics* 70: 236-243.

Reimers, Cordelia (1983), "Labor Market Discrimination against Hispanic and Black Men," *Review of Economics and Statistics* 65: 570-579.

Neumark, David (1988), Employers' "Discriminatory Behavior and the Estimation of Wage Discrimination," *Journal of Human Resources* 23: 279-295.

Oaxaca, Ronald L. and Michael Ransom (1988), "Searching for the Effect of Unionism on the Wages of Union Workers and Nonunion Workers", *Journal of Labor Research*, IX (2): 139-148.

Oaxaca, Ronald L. and Michael Ransom (1994), "On Discrimination and the Decomposition of Wage Differentials," *Journal of Econometrics* 9: 5-21.

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The standard decomposition technique estimates only relative differences. In the case of discrimination estimates, we do not know how much of the unexplained (discriminatory) wage gap arises from favoritism toward one group of workers and how much arises from pure discrimination against the other group. If we let 'o' denote the absence of discrimination in a competitive labor market, the following relationships are implied by the log wage decompositions:

$G = W_m/W_f - 1$ (the unadjusted male/female wage differential)

$Q = W_m^o/W_f^o - 1$ (the male/female wage differential attributable to qualifications)

$D = (W_m/W_f - W_m^o/W_f^o)/\left(W_m^o/W_f^o\right)$ represents the discrimination differential.

In general we could write

$$\begin{aligned}
\ell n(G+1) &= \ell n(D+1) + \ell n(Q+1) \\
&= \ell n(W_m/W_m^o) + \ell n(W_f^o/W_f) + \ell n(W_m^o/W_f^o) \\
&= \ell n(\delta_{mo}+1) + \ell n(\delta_{of}+1) + \ell n(Q+1)
\end{aligned}$$

where $\delta_{mo} = W_m/W_m^o - 1$ (favoritism toward males) and $\delta_{of} = W_f^o/W_f - 1$ (pure discrimination against females).

In log terms the nondiscriminatory wages for men and women could be expressed as

$$\begin{aligned}
\ell n(\tilde{W}_m^o) &= \bar{X}_m' \hat{\beta}^* \\
\ell n(\tilde{W}_f^o) &= \bar{X}_f' \hat{\beta}^*
\end{aligned}$$

where $\hat{\beta}^*$ is the estimated parameter vector in the absence of discrimination. An operational wage decomposition for a sample of workers can be expressed as

$$\begin{aligned}
\ell n(G+1) &= \ell n(\tilde{W}_m/\tilde{W}_m^o) + \ell n(\tilde{W}_f^o/\tilde{W}_f) + \ell n(\tilde{W}_m^o/\tilde{W}_f^o) \\
&= \bar{X}_m' (\hat{\beta}_m - \hat{\beta}^*) + \bar{X}_f' (\hat{\beta}^* - \hat{\beta}_f) + (\bar{X}_m' - \bar{X}_f') \hat{\beta}^* \\
&= \ell n(\hat{\delta}_{mo}+1) + \ell n(\hat{\delta}_{of}+1) + \ell n(Q+1).
\end{aligned}$$

We can quickly narrow down the possibilities for obtaining $\hat{\beta}^*$ to an infinite number. Fortunately, we can confine our attention to a smaller number of plausible possibilities. Assume that in the immediate aftermath of a sudden cessation of labor market discrimination, $\hat{\beta}^*$ would be a function of the currently estimated wage structures for males and females. A simple approximation would be to express $\hat{\beta}^*$ as a matrix weighted average of the vectors $\hat{\beta}_m$ and $\hat{\beta}_f$:

$$\hat{\beta}^* = \Omega \hat{\beta}_m + (I - \Omega) \hat{\beta}_f,$$

where Ω is an arbitrary kxk matrix and I is a kxk identity matrix. This still appears to admit an infinite number of possibilities. A reasonable choice for the weighting matrix is

$$\Omega = (X'X)^{-1}(X_m'X_m)$$

where $X'X = X_m'X_m + X_f'X_f$ is the cross product matrix for the combined sample of males and females. It is easily verified that $\hat{\beta}^*$ in this case is the OLS estimator applied to the combined sample:

$$\begin{aligned} \hat{\beta}^* &= (X'X)^{-1}X'Y \\ &= \hat{\beta}. \end{aligned}$$

There are some interesting special cases to consider:

$\Omega = I \Rightarrow \hat{\beta}^* = \hat{\beta}_m$, $\hat{\delta}_{mo} = 0$, $\hat{D} = \hat{\delta}_{of} = \tilde{W}_f^o/\tilde{W}_f - 1$.

$\Omega = 0 \Rightarrow \hat{\beta}^* = \hat{\beta}_f$, $\hat{\delta}_{of} = 0$, $\hat{D} = \hat{\delta}_{mo} = \tilde{W}_m/\tilde{W}_m^o - 1$.

Cotton (1988): $\Omega = \ell_m I$, where ℓ_m is the male proportion of the labor force.

Suppose the sample proportion of male workers is used, $\ell_m = T_m/T$.

Cotton's weighting scheme is equivalent to that of Oaxaca & Ransom in the following instance:

$$\begin{aligned} \Omega_c &= \Omega_o \Rightarrow T_m/T \cdot I = (X'X)^{-1}(X_m'X_m) \\ &\Rightarrow (T_m/T)\,(X'X) = (X_m'X_m) \\ &\Rightarrow T^{-1}(X'X) = T_m^{-1}(X_m'X_m). \end{aligned}$$

This implies that the first and second sample moments for the regressors are identical for males and females. In particular this means that average characteristics are identical, so that all of the unadjusted differential is attributable to discrimination.

Reimers (1983): $\Omega = \frac{1}{2}I$. This is a special case of Cotton's weighting scheme, i.e. $T_m/T = \frac{1}{2}$.

IV. DECOMPOSITION STANDARD ERRORS

Background reading:

Oaxaca, Ronald L. and Michael Ransom (1998) "Calculation of Approximate Variances for Wage Decomposition Differentials", *Journal of Economic and Social Measurement*, 24(1).

Greene, William H. (2003), *Econometric Analysis,* Fifth Edition (International Edition), Chapter 4 (pp. 53-54), Chapter 5 (p. 70), Appendix D (pp. 913-914).

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We start with the variances of the log wage decomposition when adopting the male wage structure as the norm. Note

$$\begin{aligned} E\left[\ell n\left(Q_m+1\right)\right] &= \left(\bar{X}_m - \bar{X}_f\right)' E\left(\widehat{\beta}_m\right) \\ &= \left(\bar{X}_m - \bar{X}_f\right)' \beta_m \end{aligned}$$

when we condition on $\bar{X}$. The true variance of $\ell n\left(Q_m+1\right)$ is given by

$$\begin{aligned} var\left[\ell n\left(Q_m+1\right)\right] &= E\left\{\left(\bar{X}_m - \bar{X}_f\right)' \left(\widehat{\beta}_m - \beta_m\right)\left(\widehat{\beta}_m - \beta_m\right)' \left(\bar{X}_m - \bar{X}_f\right)\right\} \\ &= \left(\bar{X}_m - \bar{X}_f\right)' E\left[\left(\widehat{\beta}_m - \beta_m\right)\left(\widehat{\beta}_m - \beta_m\right)'\right]\left(\bar{X}_m - \bar{X}_f\right) \\ &= \left(\bar{X}_m - \bar{X}_f\right)' \Sigma_{\hat{\beta}_m}\left(\bar{X}_m - \bar{X}_f\right) \end{aligned}$$

where $\Sigma_{\hat{\beta}_m} = var\left(\hat{\beta}_m\right)$ is the kxk variance/covariance matrix of $\hat{\beta}_m$. Next we seek an expression for the variance of $\ell n\left(D_m+1\right)$. Note

$$\begin{aligned} E\left[\ell n\left(D_m+1\right)\right] &= \bar{X}_f' \left[E\left(\widehat{\beta}_m\right) - E\left(\widehat{\beta}_f\right)\right] \\ &= \bar{X}_f' \left(\beta_m - \beta_f\right). \end{aligned}$$

Accordingly, the true variance of $\ell n\left(D_m+1\right)$ is given by

$$\begin{aligned} var\left[\ell n\left(D_m+1\right)\right] &= E\left\{\left[\bar{X}_f'\left(\left(\widehat{\beta}_m-\beta_m\right)-\left(\widehat{\beta}_f-\beta_f\right)\right)\right]\right. \\ &\quad \cdot\left.\left[\left(\left(\widehat{\beta}_m-\beta_m\right)-\left(\widehat{\beta}_f-\beta_f\right)\right)'\bar{X}_f\right]\right\} \\ &= \bar{X}_f'\left(\Sigma_{\hat{\beta}_m}+\Sigma_{\hat{\beta}_f}\right)\bar{X}_f \end{aligned}$$

where $\Sigma_{\hat{\beta}_f}=var\left(\hat{\beta}_f\right)$ is the kxk variance/covariance matrix of $\hat{\beta}_f$. Note that $cov\left(\widehat{\beta}_m,\widehat{\beta}_f\right)=0$. It is straightforward to show

$$var\left[\ell n\left(G+1\right)\right]=\bar{X}_m'\,\Sigma_{\hat{\beta}_m}\bar{X}_m+\bar{X}_f'\,\Sigma_{\hat{\beta}_f}\bar{X}_f\,.$$

The true standard errors of $\ell n\left(G+1\right)$, $\ell n\left(Q_m+1\right)$, and $\ell n\left(D_m+1\right)$ are simply the square roots of $var\left[\ell n\left(G+1\right)\right]$, $var\left[\ell n\left(Q_m+1\right)\right]$, and $var\left[\ell n\left(D_m+1\right)\right]$. In practice the variance and standard errors are estimated by using the estimated values of $\Sigma_{\hat{\beta}_m}$ and $\Sigma_{\hat{\beta}_f}$.

We next consider the variances and standard errors for the decomposition terms corresponding to the decomposition that assumes that the f structure is the competitive, nondiscriminatory norm. In a parallel fashion to the above expressions, we obtain

$$\begin{aligned} E\left[\ell n\left(Q_f+1\right)\right] &= \left(\bar{X}_m-\bar{X}_f\right)'\beta_f \\ E\left[\ell n\left(D_f+1\right)\right] &= \bar{X}_m'\left(\beta_m-\beta_f\right) \end{aligned}$$

$$\begin{aligned} var\left[\ell n\left(Q_f+1\right)\right] &= \left(\bar{X}_m-\bar{X}_f\right)'\Sigma_{\hat{\beta}_f}\left(\bar{X}_m-\bar{X}_f\right) \\ var\left[\ell n\left(D_f+1\right)\right] &= \bar{X}_m'\left(\Sigma_{\hat{\beta}_m}+\Sigma_{\hat{\beta}_f}\right)\bar{X}_m\,. \end{aligned}$$

The true standard errors of $\ell n\left(Q_f+1\right)$ and $\ell n\left(D_f+1\right)$ are simply the square roots of $var\left[\ell n\left(Q_f+1\right)\right]$ and $var\left[\ell n\left(D_f+1\right)\right]$. In practice the variance and standard errors are estimated by using the estimated values of $\Sigma_{\hat{\beta}_m}$ and $\Sigma_{\hat{\beta}_f}$.

Suppose we are interested in placing standard errors on the differentials Q and D. As an example, consider the simple decomposition in which the m structure is the competitive, nondiscriminatory norm. We know that

$$\ell n\left(Q_m+1\right)=\left(\bar{X}_m-\bar{X}_f\right)^{\prime} \widehat{\beta}_m$$

implies

$$Q_m=e^{\left(\bar{X}_m-\bar{X}_f\right)^{\prime} \widehat{\beta}_m}-1$$

which is a nonlinear function of $\widehat{\beta}_m$. In a simple case like this, one can apply the delta method which is a first order Taylor series approximation. In general $g\left(\hat{\theta}\right) \approx g(\theta)+g^{\prime}(\theta)\left(\hat{\theta}-\theta\right)$, where $g^{\prime}(\theta)$ is the first derivative of $g(\theta)$ with respect to θ. An asymptotic approximation yields

$$\begin{aligned}
& E\left[g\left(\hat{\theta}\right)\right]=g(\theta) \\
& var\left[g\left(\hat{\theta}\right)\right]=\left[g^{\prime}(\theta)\right] var\left(\hat{\theta}\right)\left[g^{\prime}(\theta)\right]^{\prime} .
\end{aligned}$$

Under fairly general conditions

$$g\left(\hat{\theta}\right) \xrightarrow{a} N(g(\theta),\left[g^{\prime}(\theta)\right] var\left(\hat{\theta}\right)\left[g^{\prime}(\theta)\right]^{\prime}).$$

In practice θ is replaced by $\hat{\theta}$ for purposes of calculation. In our example

$$\begin{aligned}
& g\left(\hat{\theta}\right)=Q_m \\
& \hat{\theta}=\widehat{\beta}_m \\
& var\left(\hat{\theta}\right)=var(\widehat{\beta}_m)=\Sigma_{\hat{\beta}_m} \\
& g^{\prime}(\hat{\theta})=\frac{\partial Q_m}{\partial \widehat{\beta}_m}=e^{\left(\bar{X}_m-\bar{X}_f\right)^{\prime} \widehat{\beta}_m}\left(\bar{X}_m-\bar{X}_f\right)^{\prime}=\left(Q_m+1\right)\left(\bar{X}_m-\bar{X}_f\right)^{\prime} .
\end{aligned}$$

Accordingly,

$$var\left(Q_m\right)=\left(Q_m+1\right)^2\left(\bar{X}_m-\bar{X}_f\right)^{\prime} \Sigma_{\hat{\beta}_m}\left(\bar{X}_m-\bar{X}_f\right) .$$

In the case of D_m we have

$$\ell n\left(D_m+1\right)=\bar{X}_f'\left(\widehat{\beta}_m-\widehat{\beta}_f\right)$$

which implies

$$D_m=e^{\bar{X}_f'\left(\widehat{\beta}_m-\widehat{\beta}_f\right)}+1.$$

Application of the delta method yields

$$var\left(D_m\right)=\left(D_m+1\right)^2\bar{X}_f'\left(\Sigma_{\hat{\beta}_m}+\Sigma_{\hat{\beta}_f}\right)\bar{X}_f\,.$$

The construction of the variances for the decomposition differentials under the f structure is quite similar. Note that

$$\ell n\left(Q_f+1\right)=\left(\bar{X}_m-\bar{X}_f\right)'\widehat{\beta}_f$$

implies

$$Q_f=e^{\left(\bar{X}_m-\bar{X}_f\right)'\widehat{\beta}_f}-1$$

and

$$\ell n\left(D_f+1\right)=\bar{X}_m'\left(\widehat{\beta}_m-\widehat{\beta}_f\right)$$

implies

$$D_f=e^{\bar{X}_m'\left(\widehat{\beta}_m-\widehat{\beta}_f\right)}-1.$$

It is straightforward to show

$$var\left(Q_f\right)=\left(Q_f+1\right)^2\left(\bar{X}_m-\bar{X}_f\right)'\Sigma_{\hat{\beta}_f}\left(\bar{X}_m-\bar{X}_f\right)$$

and

$$var\left(D_f\right)=\left(D_f+1\right)^2\bar{X}_m'\left(\Sigma_{\hat{\beta}_m}+\Sigma_{\hat{\beta}_f}\right)\bar{X}_m\,.$$

The estimated standard errors for these various differentials are obtained by taking the square roots of the variances and replacing the variance/covariance matrices for $\hat{\beta}_m$ and $\widehat{\beta}_f$ with their estimated values. One can also obtain variances and standard errors for the favoritism and pure discrimination differentials for the generalized decomposition in an analogous fashion.

V. DECOMPOSITIONS WITH SELECTIVITY CORRECTIONS

Background reading:

Ronald L. Oaxaca and Shoshana Neuman, "Wage Decompositions with Selectivity-Corrected Wage Equations: A Methodological Note," *Journal of Economic Inequality* vol. 2, No. 1, April 2004, 3-10..

Neuman, Shoshana and Ronald L. Oaxaca. (April 2001), "Gender Versus Ethnic Wage Differentials Among Professionals: Evidence from Israel," *Annales D'Économie et de Statistique.*

Greene, William H. (2003), *Econometric Analysis,* Fifth Edition (International Edition), Chapter 22 (pp. 780-787).

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In this section we consider how decompositions are affected by corrections for sample selection. For example, working men and women may not be a random sample of the working age population. This sample selectivity may impart biases in wage equations unless the sample selection effects are taken into account when estimating the wage equation. The simplest approach is to first model the probability of employment as a probit. A two equation model arises for each gender group:

$$E_{ij}^* = Z_{ij}'\gamma_j + \varepsilon_{ij}$$

$$Y_{ij} = X_{ij}'\beta_j + u_{ij},$$

where for individual 'i'in the jth gender group, E_{ij}^* is a latent variable associated with employment, Z_{ij}' is a vector of the determinants of employment, Y_{ij} is the market wage (in logs), X_{ij}' is a vector of determinants of market wages, γ_j and β_j are the associated parameter vectors, and ε_{ji} and u_{ji} are $i.i.d$ error terms that follow a bivariate normal distribution $\left(0, 0, \sigma_{\varepsilon j}, \sigma_{uj}, \rho_j\right)$. For identification purposes, the variance of ε_{ji} is normalized to 1.

While E_{ij}^* is unobserved as a continuous variable, market wages (Y_{ij}) are observed when $E_{ij}^* > 0$. The probability of being employed is given by

$$\begin{aligned} \Pr\text{ob}(E_{ij}^* &> 0) = \Pr\text{ob}\left(\varepsilon_{ij} > -Z_{ij}'\gamma_j\right) \\ &= \Phi(Z_{ij}'\,\gamma_j), \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal C.D.F. (the variance of ε is normalized to 1). The market wage equation is estimated for $\left\{i \mid \varepsilon_{ij} > -Z_{ij}'\gamma_j\right\}$.

We have the familiar result that the expected wage of an employed worker is

$$\begin{aligned} E\left(Y_{ij} \mid E_{ij}^* > 0\right) &= X_{ij}'\,\beta_j + E\left(u_{ij} \mid \varepsilon_{ij} > -Z_{ij}'\gamma_j\right). \\ &= X_{ij}'\,\beta_j + \rho_j\sigma_{u_j}\lambda_{ij} \\ &= X_{ij}'\,\beta_j + \theta_j\,\lambda_{ij}, \end{aligned}$$

where $\theta_j = \rho_j\sigma_{u_j}$, $\lambda ij = \phi(Z_{ij}'\;\gamma_j)/\Phi(Z_{ij}'\;\gamma_j)$, and $\phi(\cdot)$ is the standard normal density function.

It is clear that correction for selectivity bias when comparing two demographic groups m and f requires a wage decomposition of the following sort (assuming the m wage structure is the norm):

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f &= \left(\bar{X}_m'\,\widehat{\beta}_m + \,\widehat{\theta}_m\,\widehat{\lambda}_m\right) - \left(\bar{X}_f'\,\widehat{\beta}_f \,+\,\widehat{\theta}_f\,\widehat{\lambda}_f\right) \\ &= \bar{X}_f'\,\left(\widehat{\beta}_m - \widehat{\beta}_f\right) + \left(\bar{X}_m - \bar{X}_f\right)'\,\widehat{\beta}_m \\ &\quad + \left(\widehat{\theta}_m\,\widehat{\lambda}_m - \widehat{\theta}_f\,\widehat{\lambda}_f\right) \end{aligned}$$

The first two terms in the above decomposition are the familiar discrimination and endowment components, and the last term measures gender differences in the selection effects. A potentially critical issue is how to analyze and interpret this last term. One way to finesse the problem of what to do with the term $\left(\widehat{\theta}_m\,\widehat{\lambda}_m - \widehat{\theta}_f\,\widehat{\lambda}_f\right)$ is to simply net out the estimated differences in conditional means from the overall wage differential so that one is left with the familiar decomposition terms:

$$\begin{aligned}\left(\overline{Y}_m - \overline{Y}_f\right) - \left(\widehat{\theta}_m \widehat{\lambda}_m - \widehat{\theta}_k \widehat{\lambda}_k\right) &= \bar{X}_f' \left(\widehat{\beta}_m - \widehat{\beta}_f\right) \\ &+ \left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_m.\end{aligned}$$

On issue that arises is that this approach does not provide a decomposition of the *observed* wage differential $\overline{Y}_m - \overline{Y}_f$.

Of particular interest is the question of whether or not the term $\left(\widehat{\theta}_m \widehat{\lambda}_m - \widehat{\theta}_f \widehat{\lambda}_f\right)$ should be subject to further decomposition into discrimination and endowment components, and if so, how should this be done? It is important to understand what gives rise to gender differences in the selection terms. Consider the following decomposition of the gender difference in the conditional mean error terms for the wage equations for the employed :

$$\begin{aligned}&\bar{E}\left(u_m \mid \varepsilon_m > -Z_m' \hat{\gamma}_m\right) - \bar{E}\left(u_f \mid \varepsilon_f > -Z_f' \widehat{\gamma}_f\right) \\ =\; &\widehat{\theta}_m \widehat{\lambda}_m - \widehat{\theta}_f \widehat{\lambda}_f \\ =\; &\widehat{\theta}_m \,(\widehat{\lambda}_f^0 - \widehat{\lambda}_f) + \widehat{\theta}_m \,(\widehat{\lambda}_m - \widehat{\lambda}_f^0) + (\widehat{\theta}_m - \widehat{\theta}_f)\, \widehat{\lambda}_f,\end{aligned}$$

where $\widehat{\lambda}_j = \sum_{i=}^{N_j} \widehat{\lambda}_{ij}/N_j$ and $\widehat{\lambda}_{ij} = \phi(Z_{ij}' \,\widehat{\gamma}_j)/\Phi(Z_{ij}' \,\widehat{\gamma}_j)$ for $j = m, f,$ $\widehat{\lambda}_f^0 = \sum_{i=}^{N_k} \widehat{\lambda}_{if}^0/N_f$, and $\widehat{\lambda}_{if}^0 = \phi(Z_{if}' \;\widehat{\gamma}_m)/\Phi(Z_{if}' \;\widehat{\gamma_m})$. The term $\widehat{\lambda}_f^0$ is the mean value of the Inverse Mills Ratio (IMR) if females faced the same selection equation that the men face. The term $\widehat{\theta}_m \,(\widehat{\lambda}_f^0 - \widehat{\lambda}_f)$ measures the effects of gender differences in the parameters of the probit selectivity equation on the male/female wage differential. The effects of gender differences in the variables that determine employment are measured by the term $\widehat{\theta}_m \,(\widehat{\lambda}_m - \widehat{\lambda}_f^0)$. Finally, the effects of gender differences in the observed wage response to selection are captured by the term $(\widehat{\theta}_m - \widehat{\theta}_f)\, \widehat{\lambda}_f$. Given that $\hat{\theta}_j = \hat{\rho}_j \hat{\sigma}_{u_j}$

and the parameters $\hat{\rho}_j$ and $\hat{\sigma}_{u_j}$ are identified, further decomposition of $\widehat{\theta}_m - \widehat{\theta}_f$ is possible:

$$\widehat{\theta}_m - \widehat{\theta}_f = \hat{\rho}_m \left(\hat{\sigma}_{u_m} - \hat{\sigma}_{u_f}\right) + \left(\hat{\rho}_m - \hat{\rho}_f\right) \hat{\sigma}_{u_f} \tag{4}$$

$$= \left(\hat{\rho}_m - \hat{\rho}_f\right) \hat{\sigma}_{u_m} + \hat{\rho}_f \left(\hat{\sigma}_{u_m} - \hat{\sigma}_{u_f}\right). \tag{5}$$

The decompositions derived from (4) and (5) measure the effects of gender differences in wage error term variances and correlations between unobserved errors in the selection and wage equations. Decompositions (4) and (5) correspond to standardizing on the male correlation coefficient (female wage error variance) or on the female correlation coefficient (male wage error variance).

We can consider four alternative decompositions that in effect define labor market inequity with respect to how sample selection varies across demographic groups.

The most straightforward approach is imply to identify the overall selection component as a category apart from discrimination and endowment effects:

$$\overline{Y}_m - \overline{Y}_f = \underbrace{\bar{X}_f' \left(\widehat{\beta}_m - \widehat{\beta}_f\right)}_{\text{discrimination}} + \underbrace{\left(\bar{X}_m - \bar{X}_f\right)' \widehat{\beta}_m}_{\text{endowments}} + \underbrace{\left(\widehat{\theta}_m \widehat{\lambda}_m - \widehat{\theta}_f \widehat{\lambda}_f\right)}_{\text{selectivity}}. \tag{6}$$

If one believed that gender differences in the probit selection parameters for employment represented discrimination and that gender differences in personal attributes that determine the probability of employment are simply endowment differences, the resulting decomposition would be:

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f = {} & \underbrace{\overline{X}_f' (\widehat{\beta}_m - \widehat{\beta}_f) + \widehat{\theta}_m (\widehat{\lambda}_f^0 - \widehat{\lambda}_f)}_{\text{discrimination}} \\ & + \underbrace{(\overline{X}_m - \overline{X}_f)' \, \widehat{\beta}_m + \widehat{\theta}_m (\widehat{\lambda}_m - \widehat{\lambda}_f^0)}_{\text{endowments}} \\ & + \underbrace{(\widehat{\theta}_m - \widehat{\theta}_f) \widehat{\lambda}_f}_{\text{selectivity}} . \end{aligned} \tag{7}$$

A second alternative is to add the effects of gender differences in ρ to the estimated endowment (human capital) effects on the grounds that the gender difference in the error correlation coefficient is a justifiable structural source of gender wage gaps. It is difficult to know where to assign the wage gap effects of gender differences in the wage error variances. Differences in wage dispersion might or might not reflect direct labor market discrimination. Consequently, we include wage dispersion effects in the neutral category of selection effects. Upon standardizing on the male wage error variance, the overall wage decomposition becomes

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f &= \underbrace{\overline{X}_f^{\prime} (\widehat{\beta}_m - \widehat{\beta}_f) + \widehat{\theta}_m (\widehat{\lambda}_f^0 - \widehat{\lambda}_f)}_{\text{discrimination}} \\ &+ \underbrace{(\overline{X}_m - \overline{X}_f)' \widehat{\beta}_m + \widehat{\theta}_m (\widehat{\lambda}_m - \widehat{\lambda}_f^0) + \left(\hat{\rho}_m - \hat{\rho}_f\right) \hat{\sigma}_{u_f}}_{\text{endowments}} \\ &+ \underbrace{\hat{\rho}_m \left(\hat{\sigma}_{u_m} - \hat{\sigma}_{u_f}\right)}_{\text{selectivity}}. \end{aligned} \tag{8}$$

The most encompassing view of discrimination is to regard both gender differences in the estimated γ parameters from the probit selection equation for employment and gender differences in the wage effects of selectivity (θ) as manifestations of discrimination. Gender differences in the values of the employment determining variables (H') continue be treated as nondiscriminatory endowment effects. These assumptions lead to

$$\begin{aligned} \overline{Y}_m - \overline{Y}_f &= \underbrace{\overline{X}_f^{\prime} (\widehat{\beta}_m - \widehat{\beta}_f) + \widehat{\theta}_m (\widehat{\lambda}_f^0 - \widehat{\lambda}_f) + (\widehat{\theta}_m - \widehat{\theta}_f) \widehat{\lambda}_f}_{\text{discrimination}} \\ &+ \underbrace{(\overline{X}_m - \overline{X}_f)' \widehat{\beta}_m + \widehat{\theta}_m (\widehat{\lambda}_m - \widehat{\lambda}_f^0)}_{\text{endowments}} . \\ &= \underbrace{\overline{X}_f^{\prime} (\widehat{\beta}_m - \widehat{\beta}_f) + \widehat{\theta}_m \widehat{\lambda}_f^0 - \widehat{\theta}_f \widehat{\lambda}_f}_{\text{discrimination}} \\ &+ \underbrace{(\overline{X}_m - \overline{X}_f)' \widehat{\beta}_m + \widehat{\theta}_m (\widehat{\lambda}_m - \widehat{\lambda}_f^0)}_{\text{endowments}}. \end{aligned} \tag{9}$$

An example taken from the Israeli 1995 Census illustrates how much difference it makes when selectivity is not taken into account and when it is taken into account, how much difference the chosen decomposition method makes.

Wage Decomposition with Selectivity Correction

(log) wage gap = 0.2567

Decomposition Method	Endow	Disc	Select
Standard (1)	0.0916	0.1651	0.0000
	(35.68%)	(64.32%)	(0.00%)
Selection (6)	0.0976	0.1730	-0.0139
	(38.02%)	(67.39%)	(-5.41%)
Selection (7)	0.1595	0.1305	-0.0333
	(62.13%)	(50.84%)	(-12.97%)
Selection (9)	0.1595	0.0972	0.0000
	(62.13%)	(37.87%)	(0.00%)

The decompositions are applied to a sample of male and female professional workers. The unadjusted (log) wage differential is 0.2567. The endowment effect accounts from 35.68% to 62.13% of the wage gap, and the discrimination effect accounts from 37.87% to 67.39% of the wage gap.

VI. ECONOMETRICS AND EQUITY SALARY ADJUSTMENTS

Background reading:

"Using Econometric Models for Intrafirm Equity Salary Adjustments" (with Michael R. Ransom), *Journal of Economic Inequality*, vol. 1, No. 1, December 2003, 221-249.

Ashenfelter, Orley and Ronald L. Oaxaca (1987), "The Economics of Discrimination: Economists Enter the Courtroom," *American Economic Review*, 77 (2): 321-325.

Becker,William and Rebecca Goodman 1991), "The Semilogarithmic Earnings Equation and Its Use in Assessing Salary Discrimination in Academe," *Economics of Education Review,* 10(4): 323-32.

Koch, J.V. and Chizmar, J.F. (1975), "Sex discrimination and affirmative action in faculty salaries," *Economic Inquiry*, 14: 16-24.

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<u>The Linear Salary Model</u>

Let $\hat{D} = \bar{X}_f' \left(\hat{\beta}_m - \hat{\beta}_f\right)$ represent an unbiased estimator of average discrimination or inequity.

Let N_f denotes the number of female workers in the relevant job unit.

Then $N_f \hat{D}$ represents an unbiased estimator of the aggregate amount of salary inequity.

The salary predicted for the ith female according to the estimated male salary model is $\hat{Y}_{f_i}^m = X_{f_i}' \hat{\beta}_m$.

Therefore, the predicted salary gap for the ith female can be calculated as
$e_{f_i}^m = Y_{f_i} - \hat{Y}_{f_i}^m$.

Method 1

A seemingly straightforward approach would be to let $A_{f_i}^{(1)} = -e_{f_i}^m$ define the salary adjustment algorithm.

The aggregate adjustment is given by

$$\begin{aligned} A_f^{(1)} &= \sum_{i=1}^{N_f} A_{f_i}^{(1)} \\ &= -\sum_{i=1}^{N_f} e_{f_i}^m \\ &= N_f \, \hat{D}. \end{aligned}$$

The mean equity-adjusted salary for females is simply

$$\begin{aligned} \hat{Y}_f^m &= \frac{\sum_{i=1}^{N_f} \hat{Y}_{f_i}^m}{N_f} \\ &= \bar{Y}_f + \hat{D}. \end{aligned}$$

Note that salary adjustment $A_{f_i}^{(1)}$ can be expressed equivalently as

$$\begin{aligned} A_{f_i}^{(1)} &= X_{f_i}' \, \hat{\beta}_m - \left(X_{f_i}' \, \hat{\beta}_f + e_{f_i}^f \right) \\ &= X_{f_i}' \left(\hat{\beta}_m - \hat{\beta}_f \right) - e_{f_i}^f, \end{aligned}$$

where $e_{f_i}^f$ is the in-sample prediction error (residual) for the ith female:

$$\begin{aligned} e_{f_i}^f &= Y_{f_i} - \hat{Y}_{f_i}^f. \\ &= Y_{f_i} - X_{f_i}' \, \hat{\beta}_f. \end{aligned}$$

Method 2

An alternative salary adjustment is one in which the own in-sample prediction error is added to the salary gap estimated on the basis of the male salary regression:

$$\begin{aligned} A_{f_i}^{(2)} &= A_{f_i}^{(1)} + e_{f_i}^{f} \\ &= -e_{f_i}^{m} + e_{f_i}^{f} \\ &= \hat{Y}_{f_i}^{m} - \hat{Y}_{f_i}^{f} \\ &= X_{fi}' \left(\hat{\beta}_m - \hat{\beta}_f \right). \end{aligned}$$

The equity-adjusted salary implied by ($A_{f_i}^{(2)}$) is

$$\begin{aligned} \hat{Y}_{f_i}^{(2)} &= A_{f_i}^{(2)} + Y_{f_i} \\ &= X_{fi}' \left(\hat{\beta}_m - \hat{\beta}_f \right) + X_{fi}' \hat{\beta}_f + e_{f_i}^{f} \\ &= X_{fi}' \hat{\beta}_m + e_{f_i}^{f} \\ &= \hat{Y}_{f_i}^{m} + e_{f_i}^{f}. \end{aligned}$$

The aggregate salary adjustment $A_f^{(2)}$ is identical to that yielded by $A_f^{(1)}$:

$$\begin{aligned} A_f^{(2)} &= \sum_{i=1}^{N_f} A_{f_i}^{(2)} \\ &= -\sum_{i=1}^{N_f} e_{f_i}^{m} + \sum_{i=1}^{N_f} e_{f_i}^{f} \\ &= A_f^{(1)} = N_f \hat{D} \end{aligned}$$

since $\sum_{i=1}^{N_f} e_{f_i}^{f} = 0$ by the property of OLS.

The method 2 adjustment implies that the mean adjusted salary for females is identical to that of $A_f^{(1)}$, i.e.

$$\begin{aligned}\hat{Y}_f^{(2)} &= \frac{\sum_{i=1}^{N_f} \hat{Y}_{f_i}^{(2)}}{N_f} \\ &= \frac{\sum_{i=1}^{N_f} \hat{Y}_{f_i}^{m}}{N_f} + \frac{\sum_{i=1}^{N_f} e_{f_i}^{f}}{N_f} \\ &= \hat{Y}_f^{m}.\end{aligned}$$

There is an invariance property for the effects of the adjustment algorithm on the salaries of the firm's male employees when the algorithm is symmetrically applied to males.

Method 3

To avoid nominal wage cuts for women, salary adjustments could be implemented only for those women for whom $A_{f_i}^{(2)} = -e_{f_i}^{m} + e_{f_i}^{f} > 0$:

$$A_{f_i}^{(3)} = \phi_{f_i}\ A_{f_i}^{(2)},$$

$$\begin{aligned}\phi_{f_i} &= 1 \text{ if } A_{f_i}^{(2)} = -e_{f_i}^{m} + e_{f_i}^{f} > 0 \\ &= 0 \text{ otherwise.}\end{aligned}$$

The adjusted salary implied by $A_{f_i}^{(3)}$ is

$$\begin{aligned}\hat{Y}_{f_i}^{(3)} &= Y_{f_i} + \phi_{f_i}\ A_{f_i}^{(2)} \\ &= X_{f_i}'\hat{\beta}_f + e_{f_i}^{f} + \phi_{f_i}\, X_{f_i}' \left(\hat{\beta}_m - \hat{\beta}_f\right) \\ &= X_{f_i}'\hat{\beta}_{f_i}^{(3)} + e_{f_i}^{f},\end{aligned}$$

where $\hat{\beta}_{f_i}^{(3)} = \left[\phi_{f_i}\ \hat{\beta}_m + \left(1 - \phi_{f_i}\right)\ \hat{\beta}_f\right]$ is a constrained nondiscriminatory wage structure for females and is a special case of the matrix weighted average wage structures developed in Oaxaca and Ransom (1994).

One potential difficulty with algorithm #3 is that if female salary adjustments are ruled out for those women for whom $A_{f_i}^{(2)} = -e_{f_i}^m + e_{f_i}^f < 0$, there will be an upward bias to the total equity adjustment.

The aggregate salary adjustment under algorithm #3 is given by

$$\begin{aligned} A_f^{(3)} &= \sum_{i=1}^{N_f} A_{f_i}^{(3)} \\ &= \sum_{i=1}^{N_f} \phi_{f_i}\ A_{f_i}^{(2)} \\ &= \sum_{\phi_{f_i}=1} A_{f_i}^{(2)} \geq \sum_{i=1}^{N_f} A_{f_i}^{(2)} = A_f^{(2)}. \end{aligned}$$

Method 4

Award only positive adjustments in proportion to each individual's shadow contribution to the sum of the positive adjustments, $A_f^{(3)}$.

Each individual's allotted share of the original adjustment is given by $\lambda_{f_i}\ A_f^{(2)}$, where

$$\lambda_{f_i} = \frac{A_{f_i}^{(3)}}{A_f^{(3)}} = \frac{\phi_{f_i}\ A_{f_i}^{(2)}}{A_f^{(3)}}$$

for $0 \leq \lambda_{f_i} \leq 1$ and $\sum_{i=1}^{N_f} \lambda_{f_i} = 1$.

The resulting constrained equity salary adjustment is given by

$$A_{f_i}^{(4)} = \lambda_{f_i}\ A_f^{(2)}.$$

The constrained equity adjusted salary is

$$\begin{aligned}
\hat{Y}_{f_i}^{(4)} &= Y_{f_i} + \lambda_{f_i}\ A_f^{(2)} \\
&= X'_{f_i}\hat{\beta}_f + e_{f_i}^f + \frac{\phi_{f_i}\ A_{f_i}^{(2)}}{A_f^{(3)}} A_f^{(2)} \\
&= X'_{f_i}\hat{\beta}_{f_i}^{(4)} + e_{f_i}^f,
\end{aligned}$$

where $\hat{\beta}_{f_i}^{(4)} = \left[\left(\frac{\phi_{f_i} A_f^{(2)}}{A_f^{(3)}}\right) \hat{\beta}_m + \left(1 - \frac{\phi_{f_i} A_f^{(2)}}{A_f^{(3)}}\right) \hat{\beta}_f \right]$ is a constrained nondiscriminatory wage structure for females and is another special case of the matrix weighted average wage structures developed in Oaxaca and Ransom (1994).

Algorithm #4 constrains the total salary adjustment to equal the original discrimination estimate :

$$\begin{aligned} A_f^{(4)} &= \sum_{i=1}^{N_f} A_{f_i}^{(4)} \\ &= \sum_{i=1}^{N_f} \lambda_{f_i}\ A_f^{(2)} \\ &= A_f^{(2)} = N_f\, \hat{D}. \end{aligned}$$

Hence, there is no aggregate under- or over-compensation.

The mean constrained salary adjustment is calculated as

$$\begin{aligned} \bar{Y}_f^{(4)} &= \frac{\sum_{i=1}^{N_f} Y_{fi}^{(4)}}{N_f} \\ &= \bar{Y}_f + \hat{D} = \hat{Y}_f^m. \end{aligned}$$

Satisfaction of all of our constraints is not without cost. Anyone who receives an adjustment can do no better than she would under the previous procedures and may possibly do worse.

Those entitled to an adjustment $\left(A_{f_i}^{(2)} > 0\right)$ would receive

$$A_{f_i}^{(4)} = \frac{A_{f_i}^{(2)} A_f^{(2)}}{\sum\limits_{\phi_{f_i}=1} A_{f_i}^{(2)}} \leq A_{f_i}^{(2)}.$$

since $A_f^{(2)} \leq \sum_{\phi_{f_i}=1} A_{f_i}^{(2)}$.

Method 5

For legal or other reasons it is sometimes the case that all females must receive a positive salary adjustment regardless of whether or not some are already overcompensated relative to the standard adopted.

Consider the following ordering of all of the provisional salary adjustment amounts from salary adjustment algorithm #2:

$A_{f_h}^{(2)} \geq ... \geq A_{f_\ell}^{(2)}$, where $A_{f_h}^{(2)}$ is the highest provisional award ($A_{f_h}^{(2)} > 0$), and $A_{f_\ell}^{(2)}$ is the lowest provisional award.

For simplicity we consider the case in which $A_{f_\ell}^{(2)} < 0$.

Suppose $A_f^{(2)} = N_f \, \hat{D}$ is allocated according to $\psi_{f_i} \; A_f^{(2)}$ where

$$\psi_{f_i} = \frac{A_{f_i}^{(2)} - A_{f_\ell}^{(2)} + \gamma}{\sum_{i=1}^{N_f} \left(A_{f_i}^{(2)} - A_{f_\ell}^{(2)} + \gamma \right)}$$

for $0 \leq \psi_{f_i} \leq 1$, $\sum_{i=1}^{N_f} \psi_{f_i} = 1$,and $\gamma \geq 0$.

In general if $A_{\min}^{(5)} < \hat{D}$ is the minimum allowed adjustment, then setting $A_{f_i}^{(2)} = A_{f_\ell}^{(2)}$ implies $\left[\frac{\gamma}{\hat{D} - A_{fl}^{(2)} + \gamma} \right] \hat{D} = A_{\min}^{(5)}$. Therefore, the supplementary adjustment factor is calculated according to $\gamma = \frac{A_{\min}^{(5)} \left(\hat{D} - A_{fl}^{(2)} \right)}{\hat{D} - A_{\min}^{(5)}}$.

The implied post-adjustment salary is given by

$$\begin{aligned} \hat{Y}_{f_i}^{(5)} &= Y_{f_i} + A_{f_i}^{(5)} \\ &= X'_{f_i} \hat{\beta}_f^{(5)} + \frac{\gamma - A_{f_\ell}^{(2)}}{\hat{D} - A_{f_\ell}^{(2)} + \gamma} + e_{f_i}^f, \end{aligned}$$

where $\hat{\beta}_f^{(5)} = (\frac{\hat{D}}{\hat{D} - A_{f_\ell}^{(2)} + \gamma}) \hat{\beta}_m + \left(1 - \frac{\hat{D}}{\hat{D} - A_{f_\ell}^{(2)} + \gamma} \right) \hat{\beta}_f$ is a constrained nondiscriminatory wage structure for females, and is yet another special case of the Oaxaca/Ransom matrix weighted average wage structures.

The Log Salary Model

$$\ell n \left(Y_{j_i} \right) = X'_{j_i} \beta_j + \varepsilon_{j_i}, \quad j = m, f.$$

For log normal models $E \left(Y \mid X' \beta \right) = \exp \left(X' \beta + 0.5 \sigma_\varepsilon^2 \right)$.

One's actual and predicted salaries may be expressed in terms of the own estimated logarithmic salary regression by

$$\begin{aligned} Y_{j_i} &= \exp(X'_{j_i} \hat{\beta}_j + \hat{\varepsilon}_{j_i}) \\ \hat{Y}^j_{j_i} &= \exp(X'_{j_i} \hat{\beta}_j + \hat{\theta}_j), \quad j = m, f, \end{aligned}$$

where $\hat{\varepsilon}_{j_i}$ is the log salary residual for the ith individual in the jth gender group, and $\hat{\theta}_j$ is an estimate of $0.5 \sigma^2_{\varepsilon_j}$.

The variance parameter θ_j can be estimated from the residual variance.

An alternative estimation method for θ_j is to impose the restriction that the predicted mean salary equal the sample mean salary:

$$\begin{aligned} \frac{\sum_{i=1}^{N_j} \hat{Y}^j_{j_i}}{N_j} &= \bar{Y}_j \\ &\Rightarrow \frac{\sum_{i=1}^{N_j} \left[\exp(X'_{j_i} \hat{\beta}_j + \hat{\theta}_j) \right]}{N_j} = \bar{Y}_j \\ &\Rightarrow \hat{\theta}_j = \ell n \left(N_j \bar{Y}_j \right) - \ell n \left\{ \sum_{i=1}^{N_j} \left[\exp(X'_{j_i} \hat{\beta}_j) \right] \right\} \end{aligned}$$

Define the own residual between the actual and predicted salaries of the ith individual as

$$e^j_{j_i} = Y_{j_i} - \hat{Y}^j_{j_i}.$$

It is clear that $\sum_{i=1}^{N_j} e^j_{j_i} = 0$.

Log Salary Adjustment Algorithms

The predicted salary for the ith female from the estimated logarithmic salary regression for males is given by

$$\tilde{Y}^m_{f_i} = \exp(X\prime_{f_i}\ \hat{\beta}_m + \hat{\theta}_f).$$

By the same reasoning used for adjustment #2 in the linear model, we add one's own salary residual to the individual conditional mean salary prediction to arrive at an equity adjusted salary.

Average and total inequity are estimated by $\hat{D} = \frac{\sum_{i=1}^{N_f} A^{(2)}_{f_i}}{N_f}$ and $N_f \hat{D} = \sum_{i=1}^{N_f} A^{(2)}_{f_i}$.

Post-Adjustment Salary Orderings

It follows from $A^{(2)}_f = \sum_{\phi_{f_i}=0} A^{(2)}_{f_i} + \sum_{\phi_{f_i}=1} A^{(2)}_{f_i}$ that a cet. par. rise in the amount of overpayment of some women will reduce the aggregate amount of inequity. Under algorithm #4, this means that those receiving positive adjustments will receive smaller increases.

It is obvious that anyone for whom $A^{(2)}_{f_i} \leq 0$ is no worse off under salary adjustment algorithm #5 than under algorithm #4 since $A^{(5)}_{f_i} \geq A^{(4)}_{f_i} = 0$.

Those with relatively higher provisional awards must subsidize those with lower provisional awards in order to hold the total adjustment constant at the best estimated value of $N_f\, \hat{D}$.

It is easily shown that

$$\frac{\partial \psi_{fi}}{\partial \gamma} \gtreqless 0 \text{ as } A^{(2)}_{f_i} \lesseqgtr \hat{D}.$$

Post Adjustment Convergence

Post-adjustment salary inequity for the kth adjustment algorithm would be measured by

$$N_f \hat{D}^{(k)} = \sum_{i=1}^{N_f} \left(\hat{Y}^m_{f_i} - Y^{(k)}_{f_i} \right),\ k = 1, ..., 5.$$

$\hat{D}^{(k)} = 0$ for algorithms $k = 1, 2, 4$, and 5 because $\sum_{i=1}^{N_f} Y^{(k)}_{f_i} = \sum_{i=1}^{N_f} \hat{Y}^m_{f_i}$.

For method #3, post-adjustment salary inequity would be estimated by

$$
\begin{aligned}
N_f \hat{D}^{(3)} &= \sum_{i=1}^{N_f} \left(\hat{Y}^m_{f_i} - Y^{(3)}_{f_i} \right) \\
&= \sum_{i=1}^{N_f} \left(\hat{Y}^m_{f_i} - Y_{f_i} - \phi_{f_i} \ A^{(2)}_{f_i} \right) \\
&= A^{(2)}_f - \sum_{i=1}^{N_f} \phi_{f_i} \ A^{(2)}_{f_i} \leq 0,
\end{aligned}
$$

therefore, $\hat{D}^{(3)} \leq 0$.

Since overcompensated women cannot be subjected to nominal salary reductions, algorithm #3 will produce negative discrimination (favoritism) for women.

Log salary error variance

Let $\varepsilon_i = \alpha v_i$, where $v_i \sim i.i.d. \left(0, \sigma^2_v\right)$.

Then $\varepsilon_i \sim i.i.d. \left(0, \alpha^2 \sigma^2_v\right)$, where $\alpha^2 \sigma^2_v = 2\theta$.

Use θ_f if $\alpha_f = \alpha_m = \alpha$ and $\sigma^2_{v_f} \neq \sigma^2_{v_m}$.

Use θ_m if $\alpha_f \neq \alpha_m$ and $\sigma^2_{v_f} = \sigma^2_{v_m} = \sigma^2_v$.

If $\alpha_f \neq \alpha_m$ and $\sigma^2_{v_f} \neq \sigma^2_{v_m}$, the adjustment algorithms are not identified.

VII. SPECIFICATION BIAS IN WORK EXPERIENCE MEASURES

Background reading:

Regan, T. and R.L. Oaxaca, “Work Experience as a Source of Specification Error in Earnings Models: Implications for Gender Wage Decompositions.” *Journal of Population Economics*, Vol. 22, No.2 (2009), pp. 463-499.

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Our discussion of specification error will be framed in the simplest of models—a traditional (Mincerian) log wage equation,

$$Y_i = \beta_0 + \beta_1 S_i + \beta_2 X_i^* + \beta_3 X_i^{*2} + \sum_{i=1}^{K} \alpha_i H_i + \varepsilon_i,\ i = 1, ..., N, \tag{10}$$

where Y is the natural log of the hourly wage, S is the schooling level, X^* is actual work experience, H is a set of K other control variables, ε is a random error term, i indexes the individual, and N represents the sample size. More compactly, we can express (10) as,

$$Y = W^* \gamma + \varepsilon,$$

where Y and ε are $(N \times 1)$ vectors, W^* is the $(N \times (K + 4))$ observation matrix, and γ is the $((K + 4) \times 1)$ coefficient vector. Taking the probability limit of the OLS estimator yields,

$$plim(\widehat{\gamma}) = \gamma + \Sigma_{W^* W^*}^{-1} \Sigma_{W^* \varepsilon},$$

which is consistent only if $plim(N^{-1} W^{*\prime} \varepsilon) = \Sigma_{W^* \varepsilon} = 0$. Thus, the regressors, specifically schooling and experience, must be exogenously determined (i.e. uncorrelated with ε).

Now suppose that actual work experience, X^*, is unobserved. Instead one observes X, which can be thought of as potential work experience (e.g age - education - 6). For simplicity we can express the relationship between potential and actual work experience as,

$$X_i = X_i^* + v_i, \tag{11}$$

where v is the discrepancy between the experience measures. At this point we will allow that v may be correlated with X^* and that the mean of v may not be, and most probably is not, zero. As is traditionally the case we will, however, assume that there is no correlation between v and ε.

The nature of the model misspecification problem we are considering can be seen by substituting (11) into (10) yielding,

$$Y_i = \beta_0 + \beta_1 S_i + \beta_2 X_i + \beta_3 X_i^2 + \sum_{i=1}^{K} \alpha_i H_i + \varepsilon_i^*, \tag{12}$$

where $\varepsilon_i^* = \varepsilon_i - \beta_2 v_i - 2\beta_3 X_i^* v_i - \beta_3 v_i^2$.

More compactly, (12) can be expressed as,

$$Y = W\gamma + \varepsilon^*,$$

where W is the $(N \times (K+4))$ new observation matrix, and ε^* is the new $(N \times 1)$ error vector. The error vector ε^* may be expressed as,

$$\varepsilon^* = \varepsilon - v\beta_2 - 2\left[X^* \odot v\right]\beta_3 - \left[v \odot v\right]\beta_3,$$

where $X^* \odot v$ and $v \odot v$ are Hadamard products (i.e. element by element multiplication between X^* and v and between v and v, respectively).

The probability limit of the OLS estimates is,

$$\begin{aligned} plim(\widehat{\gamma}) &= \gamma + \Sigma_{WW}^{-1}\Sigma_{W\varepsilon} - \Sigma_{WW}^{-1}\Sigma_{Wv}\beta_2 - 2\Sigma_{WW}^{-1}\Sigma_{W,X^*\odot v}\beta_3 \\ &\quad -\Sigma_{WW}^{-1}\Sigma_{W,v\odot v}\beta_3 \\ &= \gamma - \Sigma_{WW}^{-1}\Sigma_{Wv}\beta_2 - 2\Sigma_{WW}^{-1}\Sigma_{W,X^*\odot v}\beta_3 - \Sigma_{WW}^{-1}\Sigma_{W,v\odot v}\beta_3, \end{aligned}$$

assuming $\Sigma_{WW}^{-1}\Sigma_{W\varepsilon} = 0$. Now, with specification error associated with substitution of X for X^*, the asymptotic bias in $\widehat{\gamma}$ consists of three distinct terms.

Our approach to correcting for specification error consists of modeling actual experience as a stochastic regressor generated from a semi-log model:

$$\ell n\,(X_i^*) = Z_i\gamma_1 + \psi_{1i}, \tag{13}$$

where Z is a set of regressors that includes the regressors in (10) (i.e. S, H) and a set of identifying variables (i.e. a respondent's age, a set of occupational dummy variables, and the number of children for females), and ψ_{1i} satisfies the standard assumptions without any particular distributional assumption.

The semi-log specification bounds X_i^* away from zero. Our proposed correction procedure uses a predicted measure of actual work experience constructed in the following fashion:

$$\widehat{X_i^*} = \hat{\delta}_1 \exp\left(Z_i\hat{\gamma}_1\right),$$

where $\widehat{\gamma}_1$ is obtained from OLS estimation of (13), and $\hat{\delta}_1$ is a scale factor that forces the predicted mean to match the sample mean: (see Oaxaca and Ransom, 2003 and Sarnikar et al., 2007)

$$\hat{\delta}_1 = \frac{\sum_i X_i^*}{\sum_i \exp\left(Z_i\hat{\gamma}_1\right)}.$$

While our procedure for predicting experience resembles instrumental variables, its motivation does not depend on endogeneity issues. Our motivation is simply to apply the correction model to data sets lacking information on actual experience.

Our empirical implementation of (12) includes completed schooling, marital status, industry dummies, regional dummies, and SMSA (Standard Metropolitan Statistical Area) dummies as the set of control variables, H.

Of particular interest are the implications of misspecification of work experience for gender wage gap decomposition. Without loss of generality, we will adopt the estimated wage structure for males as the comparison standard. Accordingly, the standard decomposition is expressed as,

$$\begin{aligned}\bar{Y}_m - \bar{Y}_f &= \left(\bar{X}^{m,a} - \bar{X}^{f,a}\right)\widehat{\beta}^{m,a} + \bar{X}^{f,a}\left(\widehat{\beta}^{m,a} - \widehat{\beta}^{f,a}\right) \\ &= \left(\bar{X}^{m,j} - \bar{X}^{f,j}\right)\widehat{\beta}^{m,j} + \bar{X}^{f,j}\left(\widehat{\beta}^{m,j} - \widehat{\beta}^{f,j}\right),\end{aligned}$$

where m, and f denote males and females, a denotes actual experience, j denotes predicted or potential experience, $\bar{Y}$ is the mean log wage, $\bar{X}$ is the mean characteristics vector, and $\widehat{\beta}$ is the estimated parameter vector. The effects of experience specification bias on the endowment (explained) component of the wage decomposition can be decomposed into parameter bias and mean experience measure bias:

$$\begin{aligned}\left(\bar{X}^{m,a} - \bar{X}^{f,a}\right)\widehat{\beta}^{m,a} - \left(\bar{X}^{m,j} - \bar{X}^{f,j}\right)\widehat{\beta}^{m,j} &= \left(\bar{X}^{m,a} - \bar{X}^{f,a}\right)\left(\widehat{\beta}^{m,a} - \widehat{\beta}^{m,j}\right) \\ &\quad + \left[\left(\bar{X}^{m,a} - \bar{X}^{f,a}\right) - \left(\bar{X}^{m,j} - \bar{X}^{f,j}\right)\right]\widehat{\beta}^{m,j}.\end{aligned} \tag{14}$$

The first term on the rhs of (14) represents the difference in the explained wage gap component that arises because of differences in the estimated parameters. The second term on the rhs of (14) represents the difference in the explained wage gap component that arises because of mean differences in the measures of experience. The effects of experience specification bias on the discrimination (unexplained) component of the wage decomposition can also be decomposed into parameter bias and mean experience measure bias:

$$\begin{aligned}\bar{X}^{f,a}\left(\widehat{\beta}^{m,a} - \widehat{\beta}^{f,a}\right) - \bar{X}^{f,j}\left(\widehat{\beta}^{m,j} - \widehat{\beta}^{f,j}\right) &= \bar{X}^{f,j}\left[\left(\widehat{\beta}^{m,a} - \widehat{\beta}^{f,a}\right) - \left(\widehat{\beta}^{m,j} - \widehat{\beta}^{f,j}\right)\right] \\ &\quad + \left(\bar{X}^{f,a} - \bar{X}^{f,j}\right)\left(\widehat{\beta}^{m,a} - \widehat{\beta}^{f,a}\right).\end{aligned} \tag{15}$$

The first term on the rhs of (15) represents the difference in the unexplained wage gap component that arises because of differences in the estimated parameters. The second term on the rhs of (15) represents the difference in the unexplained wage gap component that arises because of mean differences in the measures of experience. Note that the only differences in the mean characteristics vectors between actual and potential or predicted experience stem from the differences between mean actual experience and its square and mean potential or predicted experience and its square. On the other hand, all of the parameter estimates can differ between specifications that use actual experience and those using either potential or predicted experience.

VIII. JUHN-MURPHY-PIERCE DECOMPOSITION

Background Reading:

Juhn, Chinhui; Murphy, Kevin M.; Pierce, Brooks (1991). "Accounting for the Slowdown in Black-White Wage Convergence." AEI Studies, no. 520. Washington, D.C.: AEI Press; distributed by University Press of America, Lanham, Md.

Suen, W. , "Decomposing Wage Residuals: Unmeasured Skill or Statistical Artifact?," *Journal of Labor Economics*, 15, July 1997, 555-566.

Datta Gupta et. al, "Swimming Upstream, Floating Downstream: Comparing Women's Relative Wage Progress in the United States and Denmark," *Industrial and Labor Relations Review*, 59, no. 2, January 2006, 243-266.

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The original intent of the JMP decomposition was to account for changes in the unobserved prices and quantities that comprise the change in the unexplained wage gap over time.

The wage equation for a typical worker in period t would be written as

$Y_{it} = X'_{it}\beta_t + \sigma_{\varepsilon t}v_{it}$,where $\sigma_{\varepsilon t}v_{it} = \varepsilon_{it}$ and v_{it} is a standardized residual with mean 0 and variance 1.

Adopting the male wage structure as the standard, the gender wage gap is decomposed as $\Delta\bar{Y}_t = \Delta\bar{X}'_t\hat{\beta}_{mt} + \hat{\sigma}_{\varepsilon mt}\Delta\hat{v}_t$,where $\Delta\bar{Y}_t = \bar{Y}_{mt} - \bar{Y}_{ft}$, $\Delta\bar{X}'_t = \left(\bar{X}'_{mt} - \bar{X}'_{ft}\right)$, and $\hat{\sigma}_{\varepsilon mt}\Delta\hat{v}_t$ represents the gender difference in standardized residuals (unobserved components).

It is easily seen that $\hat{\sigma}_{\varepsilon mt}\Delta\hat{v}_t = \bar{X}'_{ft}\left(\hat{\beta}_{mt} - \hat{\beta}_{ft}\right)$.

Datta Gupta et. al (2006) relates the JMP decomposition to the Oaxaca and Ransom (2004) generalized decomposition.

$\hat{\sigma}_{\varepsilon t}\bar{v}_{mt} = \bar{X}'_{mt}\left(\hat{\beta}_{mt} - \hat{\beta}^*_t\right)$ and $\hat{\sigma}_{\varepsilon t}\bar{v}_{ft} = -\bar{X}'_{ft}\left(\hat{\beta}^*_t - \hat{\beta}_{ft}\right)$ so that

$$
\begin{aligned}
\hat{\sigma}_{\varepsilon t}\Delta\bar{v}_t &= \hat{\sigma}_{\varepsilon t}\bar{v}_{mt} - \hat{\sigma}_{\varepsilon t}\bar{v}_{ft} \\
&= \bar{X}'_{mt}\left(\hat{\beta}_{mt} - \hat{\beta}^*_t\right) + \bar{X}'_{ft}\left(\hat{\beta}^*_t - \hat{\beta}_{ft}\right).
\end{aligned}
$$

JMP decomposition of changes in the gender wage gap between period t and period t_0,

$$
\begin{aligned}
\Delta\bar{Y}_t - \Delta\bar{Y}_{t_0} &= \left(\Delta\bar{X}'_t - \Delta\bar{X}'_{t_0}\right)\hat{\beta}_{mt_0} + \Delta\bar{X}'_t\left(\hat{\beta}_{mt} - \hat{\beta}_{mt_0}\right) \\
&\quad + \left(\Delta\hat{v}_t - \Delta\hat{v}_{t_0}\right)\hat{\sigma}_{\varepsilon mt_0} + \Delta\hat{v}_t\left(\hat{\sigma}_{\varepsilon mt} - \hat{\sigma}_{\varepsilon mt_0}\right). \qquad (16)
\end{aligned}
$$

$\left(\Delta\hat{v}_t - \Delta\hat{v}_{t_0}\right)\hat{\sigma}_{\varepsilon mt_0} + \Delta\hat{v}_t\left(\hat{\sigma}_{\varepsilon mt} - \hat{\sigma}_{\varepsilon mt_0}\right)$ is the sum of the effects of changes in unobserved quantities and unobserved prices but can also be interpreted as the change in the unexplained or discriminatory gap.

IX. DECOMPOSITIONS WITH A "PARTIALLY CENSORED TOBIT MODEL"

Background Reading: Sarnikar, et. al. "Do You Receive a Lighter Prison Sentence BecauseYou are a Woman or a White? An Economic Analysis of Federal Criminal Sentencing Guidelines", November 4, 2010

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Theoretical Utility Model

A judge seeks to maximize their utility over the ideal sentence for a convicted defendant subject to costs from departures from statuatory sentencing guidelines. Assume a quadratic utility function

$$U_i = \frac{-1}{2}\left(S_i - S_i^*\right)^2 - \theta_h(S_i - G_i^h)(D_i^+) - \theta_l\left(G_i^l - S_i\right)(D_i^-)$$

where for the ith convicted defendant, U_i is the sentencing judge's utility, S_i is the sentence awarded, S_i^* is the ideal sentence in the absence of costs from departures from the sentencing guidelines (sentencing bliss point), G_i^h is the maximum sentence specified by the guidelines, G_i^l is the minimum sentence specified by the guidelines, $0 \le G_i^l \le G_i^h$, D_i^+ and D_i^- are indicator variables for upward and downward departures from the guidelines and are defined by $D_i^+ = 1\left[S_i > G_i^h\right]$ and $D_i^- = 1\left[S_i < G_i^l\right]$. The parameter restrictions are $\theta_h, \theta_l > 0$.

Utility maximization implies the FOC:

$$\frac{\partial U_i}{\partial S_i} = -\left(S_i - S_i^*\right) - \theta_h D_i^+ + \theta_l D_i^- = 0$$

which yields the sentencing function

$$\dot{S}_i = S_i^* - \theta_h\ D_i^+ + \theta_l D_i^-,$$

where $\dot{S}_i$ is the constrained utility maximizing sentence. Note that for a judge for whom $D_i^+ = 1$ for a defendant, it is the case that $\dot{S}_i - S_i^* = -\theta_h < 0$. In other words, the utility maximizing sentence is below the ideal sentence. The judge would depart upwards from the guidelines but not as much as would have been preferred. Similarly for a judge for whom $D_i^- = 1$ for a defendant, it is the case that $\dot{S}_i - S_i^* = \theta_l \ > 0$. In other words, the constrained utility maximizing sentence is above the ideal sentence. The judge would depart downwards from the guidelines but not as much as would have been preferred. Actual sentences deviate from ideal sentences whenever the guidelines are binding.

Empirical Model

Assume that the ideal sentence is specified by the stochastic function

$$S_i^* = X_i'\beta + \varepsilon_i \tag{17}$$

where X_i' is a vector of the defendant's characteristics and facts of the case that determines the judge's preferences for the ideal sentence, β is a vector of parameters, and ε_i represents random utility and is $i.i.d.N(0, \sigma_\varepsilon^2)$.

Given the threshold nature of the guidelines, the actual sentence awarded is based on a utility maximization problem which spans 6 regions: $S_i = 0, 0 < S_i < G_i^l$, $S_i = G_i^l, G_i^l < S_i < G_i^h, S_i = G_i^h$, and $G_i^h < S_i$. From the utility model, one obtains a partially censored Tobit model that allows for mass points at G_i^l and G_i^h as well as at 0. The first set of boundary constraints on S_i arises from a downward departure from the guidelines:

$$\begin{aligned} U'(S_i \quad &| \quad -\infty < \dot{S}_i < G_i^l) \leq 0 \\ S_i \cdot U'(S_i \quad &| \quad -\infty < \dot{S}_i < G_i^l) = 0 \end{aligned}$$

It follows that if the constrained utility maximizing value $\dot{S}_i \; \epsilon(-\infty, G_i^l)$, the actual sentence awarded is determined according to

$$\begin{aligned} S_i &= \max\left[0, \; \dot{S}_i = S_i^* + \theta_l \;\right] \\ &= \max\left[0, \; X_i'\beta + \theta_l + \varepsilon_i\right]. \end{aligned}$$

Thus, the empirical sentencing function is described by:

$$\begin{aligned} S_i &= X_i'\beta + \theta_l + \varepsilon_i \text{ if } 0 < RHS < G_i^l \\ &= 0 \text{ if } RHS \leq 0. \end{aligned}$$

The next set of boundary constraints occur in the interior region that encompasses non-departures from the guidelines.

$$U'(S_i \mid G_i^l < \dot{S}_i < G_i^h) = 0.$$

If the utility maximizing value $\dot{S}_i \; \epsilon(G_i^l, \; G_i^h \;)$, the empirical sentencing function is described by

$$\begin{aligned} S_i &= \dot{S}_i \\ &= S_i^* \\ &= X_i'\beta + \varepsilon_i \end{aligned}$$

Consider now the case for upward departures from the guidelines. If the utility maximizing value $\dot{S}_i > G_i^h$, it follows that

$$U'(S_i \mid G_i^h < \dot{S}_i \; < \infty) = 0.$$

In this case the empirical sentencing function is given by

$$\begin{aligned} S_i &= \dot{S}_i \\ &= S_i^* - \theta_h \\ &= X_i'\beta - \theta_h + \varepsilon_i. \end{aligned}$$

In order to accommodate mass points at G_i^l and G_i^h, we first need to determine the probabilities that the utility maximizing values $\dot{S}_i$ yield sentences that fall in the six regions already considered. From the assumption of a normal distribution on random utilities, it is easily shown that

$$\begin{aligned} prob(S_i &= 0) = prob(\varepsilon_i < -(X_i'\beta + \theta_l)) \\ &= 1 - \Phi\left(\frac{X_i'\beta + \theta_l}{\sigma_\varepsilon}\right) \end{aligned}$$

$$\begin{aligned} prob(0 &< S_i < G_i^l) = prob(S_i < G_i^l) - prob(S_i < 0) \\ &= \Phi\left(\frac{G_i^l - X_i'\beta - \theta_l}{\sigma_\varepsilon}\right) - \left[1 - \Phi\left(\frac{X_i'\beta + \theta_l}{\sigma_\varepsilon}\right)\right] \end{aligned}$$

$$\begin{aligned} prob(G_i^l &< S_i < G_i^h) = prob(S_i^* < G_i^h) - prob(S_i^* < G_i^l) \\ &= \Phi\left(\frac{G_i^h - X_i'\beta}{\sigma_\varepsilon}\right) - \Phi\left(\frac{G_i^l - X_i'\beta}{\sigma_\varepsilon}\right) \end{aligned}$$

$$\begin{aligned} prob(S_i &> G_i^h) = prob(\varepsilon_i > G_i^h - X_i'\beta + \theta_h) \\ &= 1 - \Phi\left(\frac{G_i^h - X_i'\beta + \theta_h}{\sigma_\varepsilon}\right). \end{aligned}$$

To determine the probability of a mass point at $S_i = G_i^l$, note

$$\begin{aligned} prob(S_i &= G_i^l) = prob(S_i^* < G_i^l) - prob(S_i < G_i^l) \\ &= \Phi\left(\frac{G_i^l - X_i'\beta}{\sigma_\varepsilon}\right) - \Phi\left(\frac{G_i^l - X_i'\beta - \theta_l}{\sigma_\varepsilon}\right). \end{aligned}$$

Similarly, the probability of a mass point at $S_i = G_i^h$ is determined according to

$$\begin{aligned} prob(S_i &= G_i^h) = prob(S_i^* > G_i^h) - prob(S_i > G_i^h) \\ &= \left[1 - prob(S_i^* < G_i^h\right] - \left[1 - prob(S_i < G_i^h)\right] \\ &= prob(S_i < G_i^h) - prob(S_i^* < G_i^h) \\ &= \Phi\left(\frac{G_i^h - X_i'\beta + \theta_h}{\sigma_\varepsilon}\right) - \Phi\left(\frac{G_i^h - X_i'\beta}{\sigma_\varepsilon}\right). \end{aligned}$$

It is readily verified that the probabilities over all regions sum to 1. We can summarize the six regions according to

$$
\begin{aligned}
&\text{Region 1:} & S_i &= 0 \\
&\text{Region 2:} & 0 < S_i &< G_i^l \\
&\text{Region 3:} & S_i &= G_i^l \\
&\text{Region 4:} & G_i^l < S_i &< G_i^h \\
&\text{Region 5:} & S_i &= G_i^h \\
&\text{Region 6:} & G_i^h &< S_i.
\end{aligned}
$$

The corresponding log likelihood function for the sentencing model is specified by

$$
\begin{aligned}
ln\,(L) \;=\; & \sum_{S_i=0} ln\left[1-\Phi\left(\frac{X_i'\beta+\theta_l}{\sigma_\varepsilon}\right)\right] \qquad (18)\\
& + \sum_{S_i=G_i^l} ln\left[\Phi\left(\frac{G_i^l-X_i'\beta}{\sigma_\varepsilon}\right)-\Phi\left(\frac{G_i^l-X_i'\beta-\theta_l}{\sigma_\varepsilon}\right)\right]\\
& + \sum_{S_i=G_i^h} ln\left[\Phi\left(\frac{G_i^h-X_i'\beta+\theta_h}{\sigma_\varepsilon}\right)-\Phi\left(\frac{G_i^h-X_i'\beta}{\sigma_\varepsilon}\right)\right]\\
& + \sum_{0<S_i<G_i^l} ln\left[\phi\left(\frac{S_i-X_i'\beta-\theta_l}{\sigma_\varepsilon}\right)\right] + \sum_{G_i^l<S_i<G_i^h} ln\left[\phi\left(\frac{S_i-X_i'\beta}{\sigma_\varepsilon}\right)\right]\\
& + \sum_{G_i^h<S_i} ln\left[\phi\left(\frac{S_i-X_i'\beta+\theta_h}{\sigma_\varepsilon}\right)\right] - n\cdot ln\,(\sigma_\varepsilon)
\end{aligned}
$$

where n = the number of observations for which $0 < S_i < G_i^l, G_i^l < S_i < G_i^h$, or $G_i^h < S_i$.

For each sentencing case there are six conditional sentences corresponding to each possible sentencing region:

$$
\begin{aligned}
E(S_i \quad &| \quad S_i = 0) = 0 \\
E(S_i \quad &| \quad 0 < S_i < G_i^l) = X_i'\beta + \theta_l + \sigma_\varepsilon \left[\frac{\phi\left(\frac{-X_i'\beta - \theta_l}{\sigma_\varepsilon}\right) - \phi\left(\frac{G_i^l - X_i'\beta - \theta_l}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{G_i^l - X_i'\beta - \theta_l}{\sigma_\varepsilon}\right) - \Phi\left(\frac{-X_i'\beta - \theta_l}{\sigma_\varepsilon}\right)} \right] \\
E(S_i \quad &| \quad S_i = G_i^l) = G_i^l \\
E(S_i \quad &| \quad G_i^l < S_i < G_i^h) = X_i'\beta + \sigma_\varepsilon \left[\frac{\phi\left(\frac{G_i^l - X_i'\beta}{\sigma_\varepsilon}\right) - \phi\left(\frac{G_i^h - X_i'\beta}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{G_i^h - X_i'\beta}{\sigma_\varepsilon}\right) - \Phi\left(\frac{G_i^l - X_i'\beta}{\sigma_\varepsilon}\right)} \right] \\
E(S_i \quad &| \quad S_i = G_i^h) = G_i^h \\
E(S_i \quad &| \quad S_i > G_i^h) = X_i'\beta - \theta_h + \sigma_\varepsilon \left[\frac{\phi\left(\frac{G_i^h - X_i'\beta + \phi_h}{\sigma_\varepsilon}\right)}{1 - \Phi\left(\frac{G_i^h - X_i'\beta + \phi_h}{\sigma_\varepsilon}\right)} \right].
\end{aligned}
$$

The expected sentence for the ith case is calculated as

$$
\begin{aligned}
E(S_i) \quad &= \quad prob(S_i = 0) \cdot E(S_i \mid S_i = 0) + prob(0 < S_i < G_i^l) \cdot E(S_i \mid 0 < S_i < G_i^l) \\
+prob(S_i \quad &= \quad G_i^l) \cdot E(S_i \mid S_i = G_i^l) + prob(G_i^l < S_i < G_i^h) \cdot E(S_i \mid G_i^l < S_i < G_i^h) \\
+prob(S_i \quad &= \quad G_i^h) \cdot E(S_i \mid S_i = G_i^h) + prob(S_i > G_i^h) \cdot E(S_i \mid S_i > G_i^h) \\
&= \quad prob(0 < S_i < G_i^l) \cdot E(S_i \mid 0 < S_i < G_i^l) + prob(S_i = G_i^l) \cdot \ G_i^l \\
+prob(G_i^l \quad &< \quad S_i < G_i^h) \cdot E(S_i \mid G_i^l < S_i < G_i^h) + prob(S_i = G_i^h) \cdot G_i^h \\
+prob(S_i \quad &> \quad G_i^h) \cdot E(S_i \mid S_i > G_i^h). \qquad (19)
\end{aligned}
$$

The estimated sentence for the ith individual ($\hat{S}_i$) is calculated be evaluating eq(19) at the estimated parameter values.

<u>Decompositon Methodology</u>

To examine how much of the gender/race differences in sentences can be ascribed to leniency toward one group or the other, one may use empirical methods developed in the labor economics literature to estimate gender/race preferences in criminal sentencing outcomes. These methods have the advantage of decomposing group differences in sentencing outcomes into three different components – one due to differences in the observable circumstances of the convictions and the other two pertaining to judicial preferences for each group in a binary comparison.

A generalized decomposition methodology exists that permits one to apportion the unexplained outcome sentencing gap to a positive preference for one group and a negative preference for the other group, e.g. Neumark (1988), Oaxaca and Ransom (1988,1994). A natural norm for the generalized decomposition is the estimated model obtained from pooling the two groups being compared. In the present case of the partially censored Tobit model, the model is estimated with the pooled samples and also separately for each demographic group. The predicted mean sentences are obtained from

$$
\begin{aligned}
\hat{S}^0_{wm} &= \frac{\sum_{i=1}^{N_{wm}} \hat{S}^0_{wm_i}}{N_{wm}}, \text{ (pooled model)} \\
\hat{S}_{wm} &= \frac{\sum_{i=1}^{N_{wm}} \hat{S}_{wm_i}}{N_{wm}}, \text{ (own model)} \\
\hat{S}^0_{j} &= \frac{\sum_{i=1}^{N_{j}} \hat{S}^0_{j_i}}{N_{j}}, \text{ (pooled model)} \\
\hat{S}_{j} &= \frac{\sum_{i=1}^{N_{f}} \hat{S}_{j_i}}{N_{j}}, \text{ (own model).}
\end{aligned}
$$

The decomposition of observed sample mean sentences proceeds according to

$$
\begin{aligned}
\bar{S}_{wm} - \bar{S}_j &= \left(\hat{S}_{wm} - \hat{S}^0_{wm\_j}\right) + \left(\hat{S}^0_j - \hat{S}_j\right) + \left(\hat{S}^0_{wm\_j} - \hat{S}^0_j\right) + \hat{\delta}_{wm\_j} \qquad (20) \\
&= \left(\hat{S}_{wm\_j} - \hat{S}_j\right) + \hat{\delta}_{wm\_j}
\end{aligned}
$$

where $\bar{S}_{wm} = \frac{\sum_{i=1}^{N_{wm}} S_{wm_i}}{N_{wm}}$, $\bar{S}_j = \frac{\sum_{i=1}^{N_j} S_{j_i}}{N_j}$, wm represents white males, and $j = wf, bm$ for white females and black males, and $\hat{\delta}_{wm\_j}$ is the difference between the sample mean sentencing gap $\bar{S}_{wm} - \bar{S}_j$ and the predicted mean sentencing gap $\hat{S}_{wm\_j} - \hat{S}_j = \left(\hat{S}_{wm} - \hat{S}^0_{wm\_j}\right) + \left(\hat{S}^0_j - \hat{S}_j\right) + \left(\hat{S}^0_{wm\_j} - \hat{S}^0_j\right)$. With respect to eq(20), the term $\hat{S}_{wm} - \hat{S}^0_{wm\_j}$ is an estimate judges' sentencing preferences toward white males (when compared with group j), the term $\left(\hat{S}^0_j - \hat{S}_j\right)$ measures sentencing preferences toward group j (when compared with group wm), and the term $\left(\hat{S}^0_{wm\_j} - \hat{S}^0_j\right)$ estimates the portion of the predicted sentencing gap attributable to differences in the case circumstances.

Empirical Example

Decomposition of Mean Sentences (months)	
Preferences toward males	1.0
Preferences toward females	5.5
Differences in Case Circumstances	13.7
Statistical Adjustment	0.1
Total Gap	20.3