Notes on Translog Cost Function

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The translog cost function is a second-order approximation to a cost function. As an example, suppose there are two categories of labor inputs plus the nonlabor input. A general translog cost function specification with provision for non-neutral technological change is given by:

$$\ln(C) = \beta_0 + \sum_{i=1}^{3} \beta_i \ln(p_i) \ + \beta_q \ln(Q) + \frac{1}{2}\beta_{qq} \left[\ln(Q)\right]^2 + \sum_{i=1}^{3} \beta_{qi} \ln(Q) \ln(p_i)$$
$$+ \frac{1}{2}\sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln(p_{i)} \ln(p_j) + \sum_{i=1}^{3} \gamma_i \left(\frac{1}{t}\right) \ln(p_i)$$

where $p_1 = w_1, p_2 = w_2$, and $p_3 = r$.

Consider the case of constant returns to scale, $\beta_q = 1$, and homotheticity, $\beta_{qq} = 0$, $\beta_{qi} = 0$ for $i = 1, 2, 3$:

$$\ln(C) = \beta_0 + \sum_{i=1}^{3} \beta_i \ln(p_i) \ + \ln(Q) + \frac{1}{2}\sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln(p_{i)} \ln(p_j) + \sum_{i=1}^{3} \gamma_i \left(\frac{1}{t}\right) \ln(p_i).$$

We have the parameter restrictions that impose homegeneity of degree 1 in input prices and symmetry:

$$\sum_{i=1}^{3} \beta_i = 1$$
$$\sum_{i=1}^{3} \delta_{ij} = 0$$
$$\sum_{i=j}^{3} \delta_{ij} = 0$$
$$\sum_{i=1}^{3} \gamma_i = 0$$
$$\delta_{ij} = \delta_{ji}$$

From Shepard's lemma we know that $\dfrac{\partial C}{\partial p_i} = X_i$ which is the conditional input demand function for the ith input, i.e. L_1, L_2, and K. Note that

$$\begin{aligned}\frac{\partial \ln(C)}{\partial \ln(p_i)} &= \frac{\partial C}{\partial p_i}\frac{p_i}{C} \\ &= X_i \frac{p_i}{C} \\ &= s_i,\end{aligned}$$

where s_i is the cost share of input i. Therefore, by differentiating the cost function with respect to the $\ln(p_i)'s$, applying Shepard's lemma, and imposing the above parameter restrictions, we obtain the cost share equations:

$$\begin{aligned}s_1 &= \beta_1 + \delta_{11} \ln\left(\frac{w_1}{r}\right) + \delta_{12} \ln\left(\frac{w_2}{r}\right) + \gamma_1\left(\frac{1}{t}\right) \\ s_2 &= \beta_2 + \delta_{21} \ln\left(\frac{w_1}{r}\right) + \delta_{22} \ln\left(\frac{w_2}{r}\right) + \gamma_2\left(\frac{1}{t}\right) \\ s_3 &= \beta_3 + \delta_{31} \ln\left(\frac{w_1}{r}\right) + \delta_{32} \ln\left(\frac{w2}{r}\right) + \gamma_3\left(\frac{1}{t}\right),\end{aligned}$$

where $s_1 = \left(\dfrac{w_1L_1}{w_1L_1 + w_2L_2 + rK}\right)$, $s_2 = \left(\dfrac{w_2L_2}{w_1L_1 + w_2L_2 + rK}\right)$, and $s_3 = \left(\dfrac{rK}{w_1L_1 + w_2L_2 + rK}\right)$.

The empirical equations would be specified as

$$s_{1t} = \beta_1 + \delta_{11} \ln\left(\frac{w_{1t}}{r_t}\right) + \delta_{12} \ln\left(\frac{w_{2t}}{r_t}\right) + \gamma_1\left(\frac{1}{t}\right) + \varepsilon_{1t}$$
$$s_{2t} = \beta_2 + \delta_{21} \ln\left(\frac{w_{1t}}{r_t}\right) + \delta_{22} \ln\left(\frac{w_{2t}}{r_t}\right) + \gamma_2\left(\frac{1}{t}\right) + \varepsilon_{2t}$$
$$s_{3t} = \beta_3 + \delta_{31} \ln\left(\frac{w_{1t}}{r_t}\right) + \delta_{32} \ln\left(\frac{w_{2t}}{r_t}\right) + \gamma_3\left(\frac{1}{t}\right) + \varepsilon_{3t}$$

Since the share equations add to 1, we have $\sum_{i=1}^{3} \varepsilon_{it} = 0$. The variance/covariance matrix for the disturbances is singular. Because there are only two independent equations, we could drop, for example, the third equation. The resulting set of equations to be estimated would therefore be

$$s_{1t} = \beta_1 + \delta_{11} \ln\left(\frac{w_{1t}}{r_t}\right) + \delta_{12} \ln\left(\frac{w_{2t}}{r_t}\right) + \gamma_1\left(\frac{1}{t}\right) + \varepsilon_{1t}$$
$$s_{2t} = \beta_2 + \delta_{21} \ln\left(\frac{w_{1t}}{r_t}\right) + \delta_{22} \ln\left(\frac{w_{2t}}{r_t}\right) + \gamma_2\left(\frac{1}{t}\right) + \varepsilon_{2t}.$$

In this case the resulting variance-covariance matrix of the disturbances is no longer singular. The model could be estimated by SUR with cross-equation restrictions. i.e. $\delta_{12} = \delta_{21}$. If there is endogeneity in the input price ratios, then a non-linear 3SLS method would be appropriate.