Technological Change and Gender Wage Gaps in the U.S. Service Industry

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- $$Q_t = A_t \left[\sum_{j=1}^{J} \alpha_{jt} L_{jt}^{\rho} + \left(1 - \sum_{j=1}^{J} \alpha_{jt} \right) K_t^{\rho} \right]^{\frac{\phi}{\rho}}$$

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    - ϕ is the returns to scale parameter
    - $\rho = \frac{\sigma - 1}{\sigma}$, where σ is the elasticity of substitution among inputs

Cost Minimization

- $$MP_{L_{jt}} = \phi A_t^{\frac{\rho}{\phi}} \alpha_{jt} L_{jt}^{\rho-1} Q_t^{1-\frac{\rho}{\phi}} \tag{1}$$

  $$MP_{Kt} = \phi A_t^{\frac{\rho}{\phi}} \left[1 - \sum_{j=1}^{J} \alpha_{jt} \right] K_t^{\rho-1} Q_t^{1-\frac{\rho}{\phi}}. \tag{2}$$

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- $$\frac{MP_{L_{jt}}}{MP_{L_{ht}}} = \frac{w_{jt}}{w_{ht}}, \quad j \neq h \tag{3}$$

  $$\frac{MP_{Kt}}{MP_{Ljt}} = \frac{r_t}{w_{jt}}. \tag{4}$$

Inverse Relative Input Demand Functions

- Upon substituting (1) and (2) into (3) and (4) and normalizing relative to the h^{th} labor input (i.e. L_{ht}, and w_{ht}), one obtains

$$\frac{\alpha_{jt} L_{jt}^{\rho-1}}{\alpha_{ht} L_{ht}^{\rho-1}} = \frac{w_{jt}}{w_{ht}}, \; j \neq h.$$

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- Taking the log of the above relations yields the following set of inverse relative input demand functions:

$$\ln\left(\frac{w_{jt}}{w_{ht}}\right) = \ln\left(\frac{\alpha_{jt}}{\alpha_{ht}}\right) + (\rho - 1)\ln\left(\frac{L_{jt}}{L_{ht}}\right), \; j \neq h \qquad (5)$$

$$\ln\left(\frac{r_t}{w_{ht}}\right) = \ln\left(\frac{\left[1 - \sum_{j=1}^{J} \alpha_{jt}\right]}{\alpha_{ht}}\right) + (\rho - 1)\ln\left(\frac{K_t}{L_{ht}}\right). \qquad (6)$$

Specification of non-neutral technological change

- $$\alpha_{jt} = \frac{e^{\alpha_{j0}+\alpha_{j1}\left(\frac{1}{t}\right)+\epsilon_{jt}}}{1+\sum_{j=1}^{J} e^{\alpha_{j0}+\alpha_{j1}\left(\frac{1}{t}\right)+\epsilon_{jt}}},\ j = 1, ..., J \tag{7}$$

$$\alpha_{J+1,t} = 1 - \sum_{j=1}^{J} \alpha_{jt} = \frac{1}{1+\sum_{j=1}^{J} e^{\alpha_{j0}+\alpha_{j1}\left(\frac{1}{t}\right)+\epsilon_{jt}}}, \tag{8}$$

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- $0 < \alpha_{jt} < 1,\ \sum_{j=1}^{J+1} \alpha_{jt} = 1$
- ϵ_{jt} is a random error term distributed $(0,\ \sigma_{\epsilon}^{2})$

Stochastic inverse relative input demand functions

- $$\ln\left(\frac{w_{jt}}{w_{ht}}\right) = \beta_{jh0} + \beta_{jh1}\frac{1}{t} + (\rho - 1)\ln\left(\frac{L_{jt}}{L_{ht}}\right) + \epsilon_{jht}, \; j \neq h, \qquad (9)$$
  $$\ln\left(\frac{r_t}{w_{ht}}\right) = \beta_{kh0} + \beta_{kh1}\frac{1}{t} + (\rho - 1)\ln\left(\frac{K_t}{L_{ht}}\right) + \epsilon_{kht}, \qquad (10)$$

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- $\beta_{jh0} = \alpha_{j0} - \alpha_{h0}$, $\beta_{jh1} = \alpha_{j1} - \alpha_{h1}$, $\epsilon_{jht} = \epsilon_{jt} - \epsilon_{ht}$ with $j \neq h$, and $j = 1, ..., J$ for equation (9)

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- $\beta_{kh0} = -\alpha_{h0}$, $\beta_{kh1} = -\alpha_{h1}$, $\epsilon_{kht} = -\epsilon_{ht}$ for equation (10).
- non-neutral technological change is captured by the coefficients on $\frac{1}{t}$. A direct measure might be $1/RD_t$.

Unexplained wage gaps

- Let the wage w_{jt}^m of male workers in quarter t, occupation j be equated to the corresponding marginal revenue product:

$$w_{jt}^m = MR_t \cdot MP_{L_{jt}}^m.$$

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- Let the wage w_{jt}^{m} of male workers in quarter t, occupation j be equated to the corresponding marginal revenue product:

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- Let the wage w_{jt}^{f} of female workers in quarter t, occupation j be equated to the corresponding marginal product, discounted by an unexplained wage differences index (d_{jt}):

$$w_{jt}^{f} = \frac{MR_t \cdot MP_{Ljt}^{f}}{(1 + d_{jt})}.$$

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- The relative wage equations for female workers in occupation j, (relative to male workers in occupation h) will reflect potential unexplained wage differences, and can be written as

$$\ln\left(\frac{w_{jt}^{f}}{w_{ht}^{m}}\right) = \ln\left(\frac{MP_{jt}^{f}}{MP_{ht}^{m}}\right) - \ln(1 + d_{jt}).$$

Unexplained wage gaps

- It follows that

$$\begin{aligned}\ln\left(\frac{w_{jt}^{f}}{w_{ht}^{m}}\right)+\ln\left(1+d_{jt}\right) &= \ln\left(\frac{MP_{jt}^{f}}{MP_{ht}^{m}}\right)\\ &= \beta_{jh0}^{fm}+\frac{\beta_{jh1}^{fm}}{t}+(\rho-1)\ln\left(\frac{L_{jt}^{f}}{L_{ht}^{m}}\right)+\epsilon_{jht}^{fm},\end{aligned}$$

where $\beta_{jh0}^{fm}=\alpha_{j0}^{f}-\alpha_{h0}^{m}$, $\beta_{jh1}^{fm}=\alpha_{j1}^{f}-\alpha_{h1}^{m}$, and $\epsilon_{jht}^{fm}=\epsilon_{jt}^{f}-\epsilon_{ht}^{m}$ for j, h $= 1, ..., 4$.

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- It follows that

$$\ln(\frac{w^f_{jt}}{w^m_{ht}}) + \ln(1 + d_{jt}) = \ln\left(\frac{MP^f_{jt}}{MP^m_{ht}}\right)$$
$$= \beta^{fm}_{jh0} + \frac{\beta^{fm}_{jh1}}{t} + (\rho - 1)\ln(\frac{L^f_{jt}}{L^m_{ht}}) + \epsilon^{fm}_{jht},$$

  where $\beta^{fm}_{jh0} = \alpha^f_{j0} - \alpha^m_{h0}$, $\beta^{fm}_{jh1} = \alpha^f_{j1} - \alpha^m_{h1}$, and $\epsilon^{fm}_{jht} = \epsilon^f_{jt} - \epsilon^m_{ht}$ for j, h $= 1, ..., 4$.

- The wage equations for female workers in occupation j, relative to the wage of female workers in occupation h, with $j \neq h$ can be written as

$$\ln(\frac{w^f_{jt}}{w^f_{ht}}) = \ln\left(\frac{MP^f_{jt}}{MP^f_{ht}}\right) + \ln(1 + d_{ht}) - \ln(1 + d_{jt})$$

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$$\ln(\frac{w^f_{jt}}{w^f_{ht}}) - \ln(1 + d_{ht}) + \ln(1 + d_{jt}) = \ln\left(\frac{MP^f_{jt}}{MP^f_{ht}}\right)$$

$$= \beta^{ff}_{jh0} + \frac{\beta^{ff}_{jh1}}{t} + (\rho - 1)\ln\left(\frac{L^f_{jt}}{L^f_{ht}}\right) + \epsilon^{ff}_{jht},$$

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- The wage equations for female workers in occupation j, relative to the non-labor input can be expressed as

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where $\beta_{jk0}^{f}=\alpha_{j0}^{f}$, $\beta_{jk1}^{f}=\alpha_{j1}^{f}$, and $\epsilon_{jkt}^{f}=\epsilon_{jt}^{f}$ for $j=1,...,4$.

- The wage equations for male workers in occupation j, relative to the non-labor input can be expressed as

$$\begin{aligned}\ln\left(\frac{w_{jt}^{m}}{r_t}\right) &= \ln\left(\frac{MP_{jt}^{m}}{MP_{Kt}}\right)\\ &= \beta_{jk0}^{m}+\frac{\beta_{jk1}^{m}}{t}+(\rho-1)\ln\left(\frac{L_{jt}^{m}}{K_t}\right)+\epsilon_{jkt}^{m},\end{aligned}$$

where $\beta_{jk0}^{m}=\alpha_{j0}^{m}$, $\beta_{jk1}^{m}=\alpha_{j1}^{m}$, and $\epsilon_{jkt}^{m}=\epsilon_{jt}^{m}$ for $j=1,...,4$.

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- Separate wage equations for each occupation and gender can be estimated using monthly micro data on individual characteristics (schooling, potential experience, potential experience squared).

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- Consider first the wage equation for a male worker l, in occupation j, quarter t,

$$\ln\left(w_{jtl}^{m}\right) = X_{jtl}^{m}\hat{\beta}_{jt}^{m} + \upsilon_{jtl}^{m},$$

where X_{jtl}^{m} is the vector of individual characteristics of male workers.

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where X_{jtl}^{m} is the vector of individual characteristics of male workers.
- Similarly, consider the wage equation for a female worker l, in occupation j, quarter t,

$$\ln\left(w_{jtl}^{f}\right) = X_{jtl}^{f}\hat{\beta}_{jt}^{f} + \upsilon_{jtl}^{f}, \tag{11}$$

where X_{jtl}^{f} is the vector of individual characteristics of female workers.

Unexplained wage gaps

- The wage decomposition for workers in occupation j, quarter t, is given by

$$\ln(w_{jt}^{m}) - \ln(w_{jt}^{f}) = (\overline{X}_{jt}^{m} - \overline{X}_{jt}^{f})\hat{\beta}_{jt}^{m} + \overline{X}_{jt}^{f}(\hat{\beta}_{jt}^{m} - \hat{\beta}_{jt}^{f}),$$

  where the first right hand side term represents the wage gap due to differences in skills and the second term represents the unexplained wage gap.

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  where the first right hand side term represents the wage gap due to differences in skills and the second term represents the unexplained wage gap.

- A measure of unexplained differences can be obtained as

$$\begin{aligned}\ln(1 + d_{jt}) &= \overline{X}_{jt}^{f}(\hat{\beta}_{jt}^{m} - \hat{\beta}_{jt}^{f}) \\ &= \ln\left(\frac{w_{jt}^{m}}{w_{jt}^{f}}\right) - (\overline{X}_{jt}^{m} - \overline{X}_{jt}^{f})\hat{\beta}_{jt}^{m},\end{aligned}$$

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- $\overline{X}_{jt}^{f(m)}$ is the sample weighted average of female (male) workers' characteristics, $\overline{X}_{jt}^{f(m)} = \sum_{l_f}(X_{jtl}^{f(m)}) * weight_{jtl}^{f(m)}$, and $weight_{jtl}^{f(m)}$ is the sampling weight.

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- Because the unexplained gap is a residual after netting out the effects of gender differences in characteristics, wage equations were only estimated for males.

User cost of capital and measurement of nonlabor inputs

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  - the K_t series is obtained residually from (12):

$$K_t = \frac{(P_t Q_t - w_t L_t)}{r_t}$$

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  - $\frac{1}{t}$ or $1/RD_t$

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- These constraints insure invariance of the estimated coefficients to the choice of any 8 equations.

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- Avoidance of the "overidentification" problem necessitates estimating $\binom{9}{2} = 36$ equations for all possible wage differential pairings with cross-equation restrictions that uniquely identify the estimated parameters.
- Because any 8 equations can span the remaining 28 equations, internal consistency requires that additional cross-equation restrictions be imposed on the constant term and the coefficient of the time variable.
- These constraints insure invariance of the estimated coefficients to the choice of any 8 equations.
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  - Therefore, Three Stage Least Squares estimation cannot be performed for the system of 36 demand equations.
  - Two Stage Least Squares (2SLS) is used to estimate the system jointly with appropriate cross-equation restrictions.

Empirical Results

Table 1: Definition of Occupation Variables

Occupational Categories	
Occ 1	Managerial and Professional Specialty
Occ 2	Technical, Sales and Administrative Support
Occ 3	Service Occupations and Precision Production, Craft and Repair
Occ 4	Operators, Fabricators and Laborers, Farming, Forestry and Fishing

Table 2: Definition of Variables

Variable	Description
w_{jt}^f	Hourly wage of full time female employees, occupation j, quarter t [dollars]
w_{jt}^m	Hourly wage of full time male employees, occupation j, quarter t [dollars]
L_{jt}^f	Full time female employees, occupation j, quarter t [thousands]
L_{jt}^m	Full time male employee, occupation j, quarter t [thousands]
r_t	Non-labor Input factor price in quarter t
K_t	Non-labor Input, in quarter t [thousand 2000 dollars]
i_t	3-month T-bill rate
RD_t	Total R&D expenditure for quarter t [million 2000 dollars]

Table 3: Summary Statistics of Main Variables

Variable	Mean	Std. Dev.	No.Obs.
$\frac{w^f_{1t}}{w^m_{1t}}$	0.791	0.164	92
$\frac{w^f_{2t}}{w^m_{2t}}$	0.783	0.189	92
$\frac{w^f_{3t}}{w^m_{3t}}$	0.786	0.431	87
$\frac{w^f_{4t}}{w^m_{4t}}$	0.895	0.070	92
L^f_{1t}	1,694,543	374,661.6	92
L^f_{2t}	1,195,887	157,752.6	92
L^f_{3t}	908,408.7	96,894.8	92
L^f_{4t}	85,589.4	13,714.7	92
L^m_{1t}	1,602,240	240,879.3	92
L^m_{2t}	343,033.3	77,278.38	92
L^m_{3t}	794,278.2	83,561.8	92
L^m_{4t}	231,720.2	39,422.2	92
r_t	0.055	0.006	92
K_t	58,448.3	28,594.7	92
i_t	0.016	0.007	92
RD_t	33,426	8,417.57	92

Data source: 1979-2001 Quarterly CPS data and NSF R&D data.

Table 5: The Relation between Technological Change and the Gender Wage Gaps in the U.S. Service Industry: Two Stage Least Squares Estimates of Inverse Relative Input Demand Functions taking into Account the Unexplained Gender Wage Gaps

Independent Variables	Dependent Variables: Relative Marginal Products of Female and Male Workers, by Occupation							
	$ln(MP_1^f/MP_1^m)$		$ln(MP_2^f/MP_2^m)$		$ln(MP_3^f/MP_3^m)$		$ln(MP_4^f/MP_4^m)$	
$Constant$	-.398***	(.017)	.392***	(.018)	-.909***	(.033)	-.428***	(.022)
$\frac{1}{t}$	-.415***	(.086)	-.038	(.090)	-.039	(.117)	.052	(.050)
$ln\left(\frac{L_j^f}{L_j^m}\right)$	-.323***	(.018)	-.323***	(.018)	-.323***	(.018)	-.323***	(.018)
$\sigma = \frac{1}{(1-\rho)}$	3.10							
$No.\ of\ Observations = 89$								

Note: Standard errors are in parentheses. *Statistically significant at .10, level;** at the .05 level; *** at the .01 level (two tailed t-tests).

Table 7: The Relation between Technological Change and the Gender Wage Gaps in the U.S. Service Industry: Two Stage Least Squares Estimates of Inverse Relative Input Demand Functions taking into Account the Unexplained Gender Wage Gaps and using R&D as Direct Measure of Technological Change

Independent Variables	Dependent Variables: Relative Marginal Products of Female and Male Workers, by Occupation							
	$ln(MP_1^f/MP_1^m)$		$ln(MP_2^f/MP_2^m)$		$ln(MP_3^f/MP_3^m)$		$ln(MP_4^f/MP_4^m)$	
$Constant$	-.026	(.038)	.731***	(.066)	-1.018***	(.094)	-.714***	(.059)
$\frac{1}{RD}$	-1.510***	(.152)	-.797***	(.140)	-.271	(.197)	.460***	(.067)
$ln\left(\frac{L_j^f}{L_j^m}\right)$	-.398***	(.031)	-.398***	(.031)	-.398***	(.031)	-.398***	(.031)
$\sigma = \frac{1}{(1-\rho)}$	2.51							
$No.\ of\ Observations = 89$								

Note: Standard errors are in parentheses. *Statistically significant at .10, level;** at the .05 level; *** at the .01 level (two tailed t-tests).