

Due Thursday, December 8

This assignment covers time series econometric methods. The necessary data are contained in the Excel file data5_11.xlsx available at <http://u.arizona.edu/~rlo>. Be sure to attach the supporting computer print out to the completed assignment and make clear where your answers are shown.

The data for this exercise are quarterly time series for the period 1960q1-2010q4 and are taken from the National Income and Product Accounts and from the Federal Reserve.

The following information pertains to questions 1 & 2. A time-series model of the implicit price deflator for gross domestic product ($pgdp$) is given by

$$\Delta \ln(pgdp)_t = \beta_0 + \sum_{i=1}^3 \beta_i \Delta \ln(pgdp)_{t-i} + \varepsilon_t,$$

where $\Delta \ln(pgdp)_t = \ln(pgdp)_t - \ln(pgdp)_{t-1}$ and ε_t satisfies the standard assumptions.

1. Let $\varepsilon_t = u_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2}$, $u_t \sim N(0, 1)$, $\alpha_0 > 0$, and $0 < \alpha_1 < 1$.
 - a. Estimate the parameters of the above model by MLE.
 - b. Estimate the unconditional variance of ε_t .
 - c. Consider H_0 : ARCH(0); H_1 : ARCH(1)
 - (1) Determine the distribution of ε_t under H_0 .
 - (2) Test the above hypothesis at the 5% level of significance using the following tests:
 - (a) LM test
 - (b) Asymptotic 't' test based on the MLE results.
 - (3) Discuss the feasibility of testing the above hypothesis using the standard LR procedure.
2. Let $\varepsilon_t | \Psi_t \sim N(0, h_t)$, $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \delta_1 h_{t-1}$, $\alpha_0 > 0$, $0 < \alpha_1, \delta_1 < 1$, and $0 < \alpha_1 + \delta_1 < 1$. Ψ_t denotes an information set as of time t .
 - a. Estimate the parameters of the above model by MLE.
 - b. Estimate the unconditional variance of ε_t .

The following information pertains to questions 3 & 4. pce is personal consumption expenditures in \$billions, y_pers is personal income in \$billions, p_pce is the price index for personal consumption expenditures, and $pgdp$ is the implicit price deflator for gross domestic product.

3. Test for unit roots in the log of real consumption expenditures, $\ln(pce/p_pce)$, and in the log of real personal income, $\ln(y_pers/pgdp)$, using the augmented Dickey-Fuller test of the form

$$\Delta Z_t = \beta_0 + \alpha Z_{t-1} + \beta_1 \Delta Z_{t-1} + \beta_2 \Delta Z_{t-2} + u_t.$$

Conduct your test of $H_0: \alpha = 0$, $H_1: \alpha < 0$ at the 5% level of significance using the appropriate Dickey-Fuller critical value.

4. Test for cointegration between $\ln(pce/p_pce)_t$ and $\ln(y_pers/pgdp)_t$ in the model $\ln(pce/p_pce)_t = \alpha + \beta \ln(y_pers/pgdp)_t + u_t$. Conduct your test at the 5% level of significance for a one-tailed test using an Augmented Dickey-Fuller specification with 1 lag . You may use the large sample test statistics for Case 2, Table B.9 (p.766) in Hamilton's *Time Series Analysis* or Table IIb (p.190) in Phillips and Ouliaris, *Econometrica*, vol. 58, 1990.