

Due Thursday, October 6  
(40 points)

This assignment is on multiple outcome and limited dependent variable models. The necessary data are contained in the Excel files `dat205a.xls` and `dat201b.xls` available from the website <http://u.arizona.edu/~rlo>. Be sure to attach the supporting computer print out to the completed assignment and make clear where your answers are shown.

The data for question 1 are provided in the file `dat205a.xls`.

1. Individuals traveling between two cities may choose among 4 different modes of travel: Air ( $j=1$ ), Train ( $j=2$ ), Bus ( $j=3$ ), and Car ( $j=4$ ). The utility function of a representative traveler can be expressed as

$$U_{ij} = \beta_1 Time_{ij} + \beta_2 Cost_{ij} + \varepsilon_{ij}$$

where  $Time_{ij}$  represents the number of minutes it would take the  $i$ th individual to reach the destination using transportation mode  $j$ ,  $Cost_{ij}$  represents the monetary cost to the  $i$ th individual to use the  $j$ th transportation mode, and  $\varepsilon_{ij}$  is an i.i.d. error term from the extreme value distribution. Let  $Y_{ij}$  represent a dummy variable defined by  $Y_{ij} = 1[J = j]$  corresponding to the individual choosing transportation mode  $j$ .

- a. Specify the likelihood function for the model and estimate the  $\beta'$ s by MLE.
- b. Consider an individual who faces the travel time and travel cost situation described below.

| Mode  | Time | Cost |
|-------|------|------|
| Air   | 120  | 90   |
| Train | 650  | 150  |
| Bus   | 600  | 50   |
| Car   | 550  | 60   |

- (1) Estimate the probabilities that this individual would travel (a) by Air, (b) by Car.
  - (2) Estimate the elasticity of the probability of traveling by Car with respect to the cost of traveling by Car.
  - (3) Estimate the elasticity of the probability of traveling by Car with respect to the cost of traveling by Air.
- c. Explain what is meant by the Independence of Irrelevant Alternatives assumption. Test this assumption at the 5% level with respect to the availability or non availability of the Car travel option.

The data for questions 2-6 are provided in the file dat201b.xls. The data consist of a national sample of 4,569 adults between the ages of 33 and 41 in 1998 and are drawn from the NLSY79 data set. For the exercises below, restrict the sample to the set of individuals for whom  $grade98 > 0$ ,  $female = 1$ , and  $other = 0$ . In this data set  $grade98$  is the highest grade completed,  $female$  is an indicator variable for gender,  $other$  is an indicator for race other than white or black,  $black$  is a race indicator variable for black, and  $union98$  is a indicator variable for union membership.

2. Consider the following model of job satisfaction

$$Y_i^* = \beta_0 + \beta_1 black_i + \beta_2 union98_i + \sum_{j=1}^4 \beta_{j+2} oc_{ji} + \varepsilon_i,$$

$$i = 1, \dots, n$$

where  $Y_i^*$  is a latent variable,  $\varepsilon_i$  is an i.i.d. logistically distributed random variable, and  $oc_j$  is an occupational indicator variable defined by

$$\begin{aligned} oc_4 &= 1 \text{ if } occup = 4 \text{ (professional/managerial)} \\ &= 0 \text{ otherwise} \\ oc_3 &= 1 \text{ if } occup = 3 \text{ (sales/clerical)} \\ &= 0 \text{ otherwise} \\ oc_2 &= 1 \text{ if } occup = 2 \text{ (craft)} \\ &= 0 \text{ otherwise} \\ oc_1 &= 1 \text{ if } occup = 1 \text{ (service)} \\ &= 0 \text{ otherwise} \end{aligned}$$

Although  $Y_i^*$  is not observed,  $jobsat_i$  is observed:

$$\begin{aligned} jobsat_i &= 0 \text{ if dislikes job} \\ &= 1 \text{ if likes job fairly well} \\ &= 2 \text{ if likes job very much} \end{aligned}$$

According to the model

$$\begin{aligned} jobsat_i &= 0 \text{ if } Y_i^* \leq 0 \\ &= 1 \text{ if } 0 < Y_i^* \leq \mu_1 \\ &= 2 \text{ if } \mu_1 < Y_i^*. \end{aligned}$$

- a. Specify the likelihood function for this model and estimate its parameters by MLE.
- b. Estimate the probability that a union, white female, professional worker would report that she likes her job very much.

3. Consider the following model of occupational employment:

$$\text{Pr ob}(occ_i = 0) = \frac{1}{1 + \sum_{m=1}^2 \exp(X_i' \beta_m)}, \quad i = 1, \dots, n$$

$$\text{Pr ob}(occ_i = j) = \frac{\exp(X_i' \beta_j)}{1 + \sum_{m=1}^2 \exp(X_i' \beta_m)}, \quad j = 1, 2$$

where  $j = 0, 1, 2$  correspond to (1) laborers, operatives, and service workers; (2) craft workers, sales, and clerical workers; and (3) professional and managerial workers. The vector  $X_i' = (1, black_i, union98, grade98_i)$ . The variable  $occ$  is defined as follows:

$$\begin{aligned} occ &= 2 && \text{if } occup = 4 \\ &= 1 && \text{if } occup = 2 \text{ or } 3 \\ &= 0 && \text{otherwise} \end{aligned}$$

- a. Specify the likelihood function for the model and estimate the model's parameters by MLE.
- b. Estimate the marginal effects of the highest grade completed on the probabilities of employment in each occupational group for a union, black female worker with 12 years of schooling.

The following information pertains to questions 4 and 5.

Consider the true model:  $autoval_i = \beta_0 + \beta_1 black_i + \beta_2 \ln(ntinc_i) + \varepsilon_i$ ,  $i = 1, \dots, n$ ,

where  $autoval$  is the market value (\$1,000's) of all vehicles currently owned by the individual,  $ntinc$  is the individual's family net income (\$1,000's),  $\varepsilon_i \sim N(0, \sigma^2)$  and satisfies all of the classical assumptions.

4. Suppose one had access only to the observations for which  $autoval_i > 0$ .

- a. Use MLE to estimate the model

$$(autoval_i | autoval_i > 0) = \beta_0 + \beta_1 black_i + \beta_2 \ln(ntinc_i) + (\varepsilon_i | autoval_i > 0).$$

- b. Estimate the marginal effect on  $E(autoval | X', autoval > 0)$  of net family income ( $ntinc$ ) for a white female automobile owner with the sample average net family income of white female automobile owners.

5. Suppose one observes  $Y_i$  rather than  $autoval$ , where

$$\begin{aligned} Y_i &= 0 && \text{if } autoval_i \leq 0 \\ &= autoval_i && \text{if } autoval_i > 0 \end{aligned}$$

- a. Use a MLE method to estimate the  $\beta$ 's
  - b. Estimate the average marginal effect on  $E(Y | X')$  of net family income ( $ntinc$ ) over the total white female sample. Explain what this marginal effect means.
6. A researcher wishes to estimate a simple model of wage determination among women workers in the professional/managerial occupations :

$$\ln(earnings_i) = \beta_0 + \beta_1 grade98_i + \beta_2 black_i + \beta_3 age_i + \beta_4 union98_i + \varepsilon_i, \quad \{i | oc_{4i} = 1\}$$

where  $earnings_i$  is the individual's annual earnings (\$1000's). The researcher wishes to take account of selection effects in terms of the kinds of women workers who are employed in the professional/managerial occupations. Accordingly, the researcher specifies an occupational selection equation:

$$\text{Prob}(oc_{4i} = 1) = \text{Prob}(u_i > -I_i)$$

where  $I_i = \gamma_0 + \gamma_1 ntinc_i + \gamma_2 grade98_i + \gamma_3 black_i + \gamma_4 union98_i + u_i$ .

Assume that  $\varepsilon_i$  and  $u_i$  satisfy all of the classical assumptions and follow a bivariate normal distribution  $(0, 0, 1, \sigma_u^2, \rho)$ .

- a. Use the Heckit selection method to obtain consistent estimators of the parameters  $\gamma$  and  $\beta$ .
- b. What do the empirical results suggest about the unobserved determinants of earnings of female workers in the professional/managerial occupations? Explain.
- c. Estimate the marginal effects on log earnings of  $ntinc$  and  $grade98$  for a worker with the profile  $ntinc = 66, grade98 = 15, black = 0, union98 = 1$ , and  $age = 37$ .