Professor R.L. Oaxaca — Assignment #1
Econ 522a — Spring 2015

Due Wednesday, January 28
(40 points)

This assignment is on estimation with the standard linear regression model. The necessary data are in the Excel file dats105.xls available at http://u.arizona.edu/∼rlo/ under econ 522a. In order to receive full credit be sure to 1) attach the supporting computer print out to the completed assignment, 2) make clear where your answers are shown, and 3) use matrix commands to carry out the empirical calculations.

A model of the aggregate demand for money is given by

$$Y_t = \beta_0 + \sum_{j=1}^{3} \beta_j X_{tj} + u_t, \quad t = 1959, ..., 1989,$$

where $Y_t = \ell n\,(M_{1t})\,, X_{t1} = \ell n(r_t), X_{t2} = \ell n(P_t\,), X_{t3} = \ell n(Q_t\,)$, M_1 is a measure of the money stock (billions of \$'s), r is the 6 month Treasury bill rate, P is the GNP implicit price deflator, Q is real GNP (billions of constant 1982 \$'s), and u is the disturbance term.

1. Estimate the model by OLS and empirically verify the conditions given below.

   a. $\mid M \mid = 0$, trace $(M) = 27$, where $M = \mathrm{I}_{31} - X\,(X'X)^{-1}\,X'$, and X is the 31 x 4 observation matrix for the money demand model.

   b. Let X_j represent the 31 x 1 column vector of the observations for the jth regressor, $j = 1, 2, 3$, let $\hat{u}$ represent the 31 x 1 column vector for the OLS residuals from the estimated model, and let ι be a 31 x 1 column vector of 1's. Empirically verify that $X_j'\hat{u} = 0$ for $j = 1, 2, 3$ and $\iota'\hat{u}' = 0$.

   c. $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}_1 - \hat{\beta}_2\,\bar{X}_2 - \hat{\beta}_3 \bar{X}_3$, where $\bar{Y}$ and $\bar{X}_j$ are sample means.

2. Construct the deviation form of the variables in the model, i.e. $y_t = Y_t - \bar{Y}, x_{tj} = X_{tj} - \bar{X}_j$ and estimate the following equation by OLS:

$$y_t = \sum_{j=1}^{3} \beta_j x_{tj} + u_t.$$

   a. Obtain the variance-covariance matrix for $\underline{\hat{\beta}} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{pmatrix}$ from the model in deviation form.

   b. Empirically verify that the estimated standard errors of $\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$ from the model estimated in deviation form are identical to those estimated from the original model with a constant term.

c. Obtain the reported (centered) $R^2 = 1 - \left(\frac{\hat{u}'\hat{u}}{y'y}\right) = \left(\frac{\underline{\hat{\beta}}' x'x \underline{\hat{\beta}}}{y'y}\right)$, where y represents the 31 x 1 column vector for the observations on y_t, and x is the 31 x 3 observation matrix for the regressors in deviation form.

(1) Compute the uncentered R^2 defined as $R_u^2 = 1 - \left(\frac{\hat{u}'\hat{u}}{Y'Y}\right) = \left(\frac{\hat{\beta}'X'X\hat{\beta}}{Y'Y}\right)$, where Y is the 31 x 1 observation vector on Y_t.

(2) Prove that in general $\underline{\hat{\beta}}' x'x \underline{\hat{\beta}} = \hat{\beta}'X'X\hat{\beta}$ when the sample mean of Y is equal to 0.

3. Let $\hat{v}_{1t}$, $\hat{v}_{2t}$ and $\hat{v}_{3t}$ represent the residuals from the auxiliary regressions of (1) X_{t1} on X_{t2} and X_{t3},(2) X_{t2} on X_{t1} and X_{t3}, and (3) X_{t3} on X_{t1} and X_{t2}. A constant term is included in all of these auxiliary regressions. Verify that the separate regressions of Y_t on $\hat{v}_{1t}$, Y_t on $\hat{v}_{2t}$, and Y_t on $\hat{v}_{3t}$ yield the multiple regression estimates of β_1, β_2 and β_3 (be sure to include a constant term in each of these regressions). Explain the intuition behind these results.

4. Assume the model $Y = Z_1\alpha_1 + Z_2\alpha_2 + u$, where Y and u are Tx1 vectors, Z_1 and Z_2 are Txk_1 and Txk_2 observation matrices, and α_1 and α_2 are k_1x1 and k_2x1 parameter vectors. In the OLS regression of Y on Z_1 and Z_2 , $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the OLS estimates of α_1 and α_2. Let $\tilde{u}_1$ be the Tx1 residual vector from a regression of Y on Z_1 and $\tilde{u}_2$ be the Txk_2 matrix of residuals from separate regressions of each column of Z_2 on Z_1. According to the Frisch-Waugh theorem for the OLS regression $\tilde{u}_1 = \tilde{u}_2\gamma_2 + error$, it is the case that $\tilde{\gamma}_2 = \hat{\alpha}_2$.

a. Explain the intuition behind the Frisch-Waugh theorem.

b. Empirically verify the theorem in the money demand model, where $Z_1 = (\iota, X_1)$, and $Z_2 = (X_2, X_3)$.

5. In the general regression model $Y = X\beta + u$, if a researcher knew the values of Y, X, and u she could back out the value of β.

a. State the formula to calculate the true value of β from Y, X, and u.

b. Since we know the values of Y and X from the sample data, we know the value of $M = \text{I}_T - X\left(X'X\right)^{-1}X'$. We also know that $\hat{u} = Mu$. Could one back out the value of the true error vector u and then back out the value of the true β? Explain.