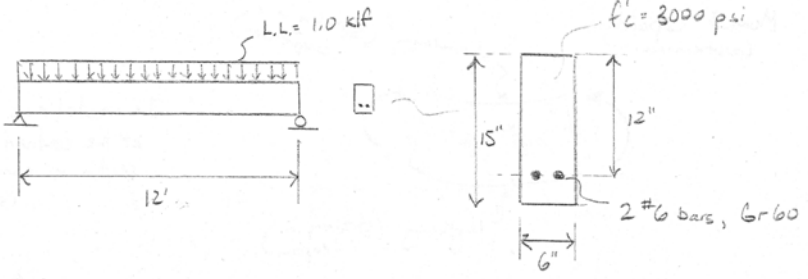
Reinforced Concrete (R.C.)



Notes on loading:

"CAPACITY" "DEMAND"

- ACI 9.1.1 Design Strength ≥ Required Strength

ACI Eqn 9-1

Required Strength U

$$U = 1.4D + 1.7L$$

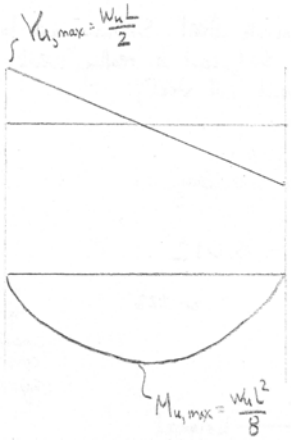
dead load
live load

Dead load: Specified + Self weight!

concrete 145 pcf
steel 450 pcf
$\Rightarrow$ R.C. : 150 pcf

Self-weight:

$$\frac{(6\text{ in})}{12\text{ in/ft}}\frac{(15\text{ in})}{12\text{ in/ft}}\frac{150\text{ lb}}{\text{ft}^3} = 94\text{ plf} \qquad (\text{lb/ft})$$



Factored Distributed Load:

$$W_u = 1.4\,(.094\text{ klf}) + 1.7\,(1.0\text{ klf})$$
$$= 0.1316 + 1.7 = 1.83\text{ klf}$$

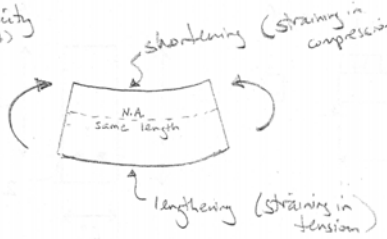
$$V_{u,max} = \frac{\left(1.83\frac{\text{k}}{\text{ft}}\right)(12')}{2} = 11.0\text{ k}$$

$$M_{u,max} = \frac{(1.83\text{ klf})(12')^2}{8} = 33.0\text{ k'}$$

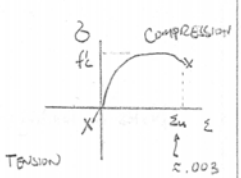
CORRECT ANSWER
3: D

ACI
Ch. 9: Strength Req'm

Moment Capacity (positive moment)



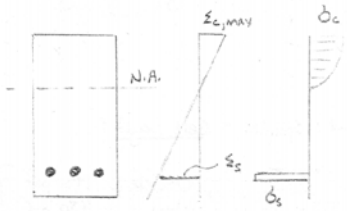
The neutral axis (N.A.) is not at the centroid because the section is composite & cracked (steel & concrete)



ε = .003 or 0.3%
(strain at which concrete crushes)

Because concrete is so weak in tension, we typically ignore its contribution (ie, below N.A concrete is cracked : no help) : Need steel rebar

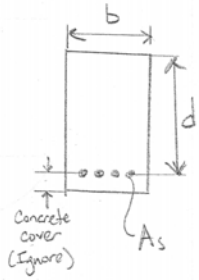
Planar Bending



If we keep on bending, either:

"Balanced" – both happen at same time
- CASE (1): Concrete crushes first (BAD: Brittle no warning, loss of load)
- CASE (2): Steel yields first (GOOD: stretches, carries load Ductile)

CASE (1) happens if we have too much steel. So we want to specify a maximum amount. But also we don't want a really weak beam, so also specify a minimum amount (of steel)



We measure the amount of steel as a percentage:

$$\rho = \frac{A_s}{bd} = \frac{0.88\ in^2}{(12")(6")} = 0.012$$

or 1.2%

A_s: 2 #6 bars*
$= 2\left(\frac{\pi}{4}\left(\frac{6}{8}\right)^2\right)$
$= 2 \times 0.44$ sq in
$= 0.88\ in^2$

* #N = N/8 ths

ACI 10.3.3 $\rho_{max} = 0.75\,\rho_b$ ← "balanced"

ACI 10.5.1 $\rho_{min} = \frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}$

ACI
Ch. 10
Axial
Flexur
loads

ACI Ch. 8: General Req'ms

Notes on max, min steel calculations:

Rather than have you calculated the "balanced" condition (complex), the ACI code gives you an empirical approximation:

ACI Eqn. 8-1: $$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

NOTE: For this calculation & also ρ_{min}, use f'_c & f_y in units of psi !!

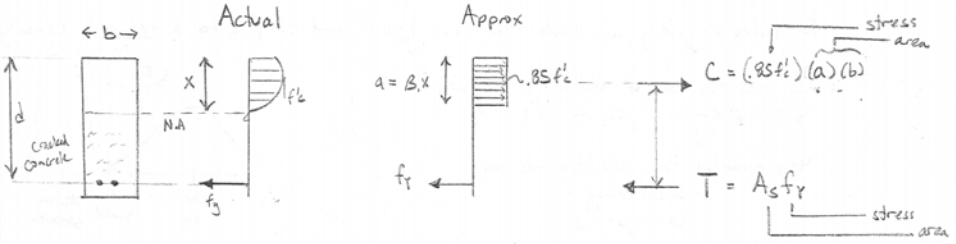
The β_1 constant indicates the depth of the stress block (see below) and is also empirical:

ACI 10.2.7.3

$$\beta_1 = 0.85 \geq 0.85 - .05\left(\frac{f'_c - 4{,}000}{1000}\right) \geq 0.65$$

Now all this only makes sure we get the proper 'limit state' in an overload.
Still need to calculate strength & compare to demand:

At Ultimate: Assume steel has yielded; concrete at crushing



Equilibrium: $\Sigma F_{Horiz} = 0$

$T = C$

∴ solve for a $$a = \frac{A_s f_y}{.85 f'_c\, b}$$ *

*Note: not empirical, so units should be consistent

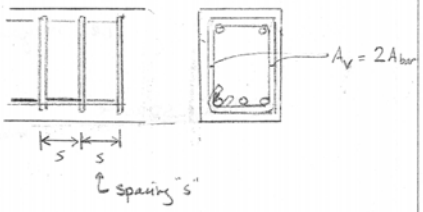
Couple:

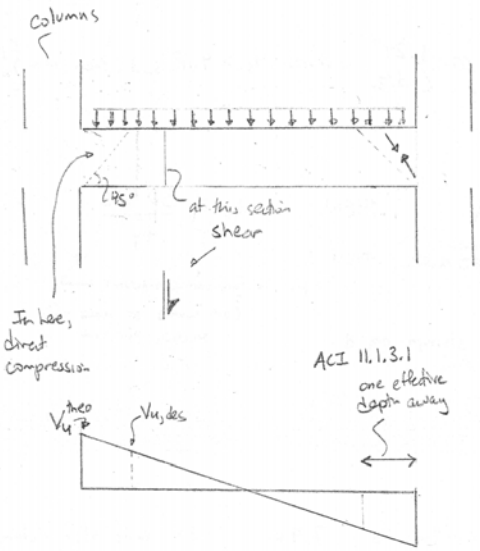
$M = (\text{Force}) \times (\text{Distance between})$

$= (A_s f_y)\left(d - \frac{a}{2}\right)$

Shear Design = Very important in R.C. design because shear causes tension @ 45°

Notes:
- Concrete has inherent shear strength
- This shear capacity is supplemented by vertical bent bars = stirrups





ACI
Ch. 11: shear & torsion

$d = 12" = 1'$ $L = 12'$ $\frac{L}{2} = 6'$

ACI 11.1.3.1

$V_{u,des} = \frac{5}{6} V_u^{theo} = \frac{5}{6}(11k) = 9.2k$ ← from p.1

Concrete Shear strength:

ACI 11.3.1.1 $V_c = 2\sqrt{f'_c}\, b_w d$ ← psi again; ← inches

Note: Beams (no axial) no. P.s. b_w → web

Stirrup shear contribution:

ACI Eqn 11-15 $V_s = \frac{A_v f_y d}{s}$

$\phi V_n > V_u$ ← reduced nominal capacity; ← factored "ultimate" shear (demand, load, in this case 9.2k)

where $V_n = V_c + V_s$

ϕ factor = shear is less behaved, more brittle $\phi = 0.85$

Several limitations exist for shear design:

- At some point, too much steel doesn't provide any additional help. Thus

ACI 11.5.6.8 $\quad V_s \le 8\sqrt{f'_c}\, b_w d$

i.e., if you need more than that, make your cross-section bigger...

- Stirrup spacing:

$$\max s = \min\begin{cases} 24" \\ d/2 \end{cases} \quad \text{unless } V_s > 4\sqrt{f'_c}\, b_w d \text{; then } \min\begin{cases} 12" \\ d/4 \end{cases}$$

- Minimum shear reinforcement

If $V_u > \frac{1}{2}\phi V_c$ THEN $A_v = \dfrac{50\, b\, s}{f_y}$

(DEMAND) > (HALF PROVIDED BY CONCRETE ALONE) THEN

PUT IN THIS SMALL AMOUNT (OF COURSE IF MORE IS NEEDED, PUT THAT)

If $V_u < \frac{1}{2}\phi V_c$ no stirrups req'd

Example

$V_u = 9.2\,k$

$V_c = 2\sqrt{3000}\,(6")(12") = 7900\,lb = 7.9\,k$

Note: • for rectangle $b_w = b$
• $\sqrt{psi} \rightarrow lb \Rightarrow k$

$\phi V_n = \phi(V_c + V_s) = 0.85\,(7.9 + V_s) > 9.2$

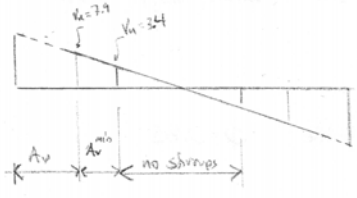
$V_{s,req} = 2.9\,k$

max spacing $= \min\begin{cases} 24" \\ 12/2" \end{cases} = 6"$

$2.9k = \dfrac{A_v f_y d}{s}$ $\quad A_v = \dfrac{(2.9k)(6")}{(60\,ksi)(6")} = .05\,in^2$

However $\frac{1}{2}\phi V_c = \frac{1}{2}(0.85)(7.9k) = 3.4k$ $\therefore$ must check min

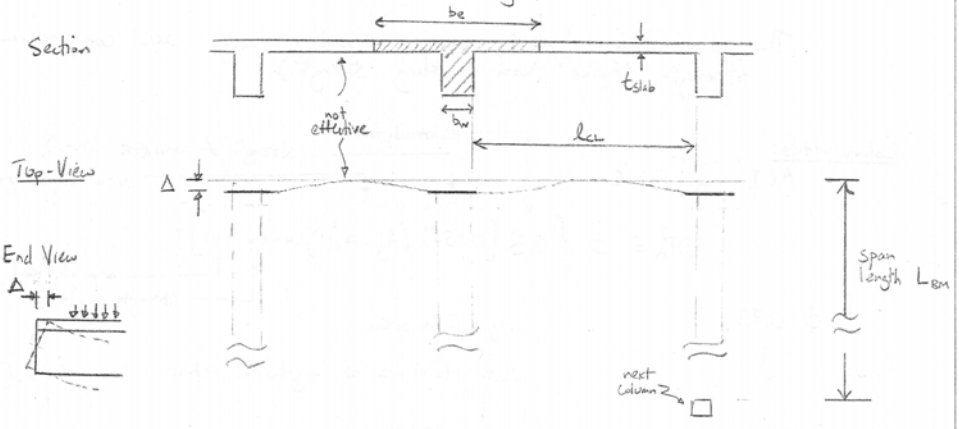
$A_v^{min} = \dfrac{50(6")(6")}{60{,}000} = 0.03\,in^2$



CORRECT ANSWER 4: A

Further R.C. design equations:

T-beams: Concrete beams are rarely poured w/out a slab also.



As you might imagine, much of the slab is partially effective, some not at all

Code designates an effective width, b_e, in which all is fully effective

ACI 8.10.2

$$b_e = \min \begin{cases} L_{BM}/4 & \\ b_w + 16 t_{slab} & \text{(overhang } 8t_{slab}\text{)} \\ b_w + \ell_{CL} & \text{(overhang } \tfrac{1}{2}\ell_{CL}\text{)} \end{cases}$$

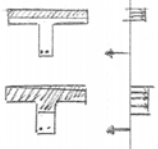
The design of T-beams involves the same principles as rectangular beams except:

Use b_w for ρ calculation:

$$\rho = \frac{A_s}{b_w d}$$

If N.A. is in slab, i.e. $a < t_{slab}$ → same as rectangular

If N.A is in stem, i.e. $a > t_{slab}$



$$\phi M_n = \phi\left[0.85 f'_c h_f (b_e - b_w)(d - h_f/2) + .85 f'_c a b_w (d - a/2)\right]$$

(Code uses h_f for t_{slab})

Concrete Columns: Unlike steel, which because it is strong → less is needed → slender, buckling is not a huge consideration for anything other than really long columns.

Thus, column design typically involves providing enough axial compressive strength (rather than buckling strength)

Column w/ties:

ACI Eq. 10-2

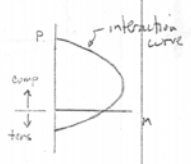
$$\phi P_n = \phi \left\{ 0.8 \left[0.85 f'_c (A_g - A_{st}) + f_y A_{st} \right] \right\}$$

(0.8 = eccentricity factor*; f'_c = strength of concrete (avg.); A_g = area of concrete; f_y = strength of steel; A_{st} = area of steel)

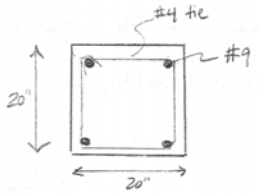
$\phi = 0.75$

A_g = gross area

A_{st} = total area of longitudinal rebar



For instance:



$f'_c = 5000$ psi

Gr 60 steel

$A_g = (20)(20) = 400\ in^2$

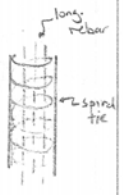
$A_{st} = 4 \times \left(\frac{\#}{8}\right)^2 \frac{\pi}{4} = 4\ in^2$

↑ not exactly #8's, but close enough to look w

$$\phi P_n = 0.75 \left\{ 0.8 \left[.85 (5\ ksi)(400\ in^2 - 4\ in^2) + 60\ ksi\ (4\ in^2) \right] \right\}$$

$$= 1153.5\ K$$

Column w/spirals:



Spirals confine the concrete better

Two modifications: $\phi = 0.8$ (instead of 0.75)

Eccentricity factor = 0.85 (instead of 0.8)

Same column w/spirals

$$\phi P_n = 1153.5\ K \left(\frac{0.8}{0.75}\right)\left(\frac{0.85}{0.8}\right)$$

$$= 1307\ K$$

Limitations: ρ_{st} must be between 1% and 8% $\left(\rho_{st} = \frac{A_{st}}{A_g}\right)$