

# Introduction

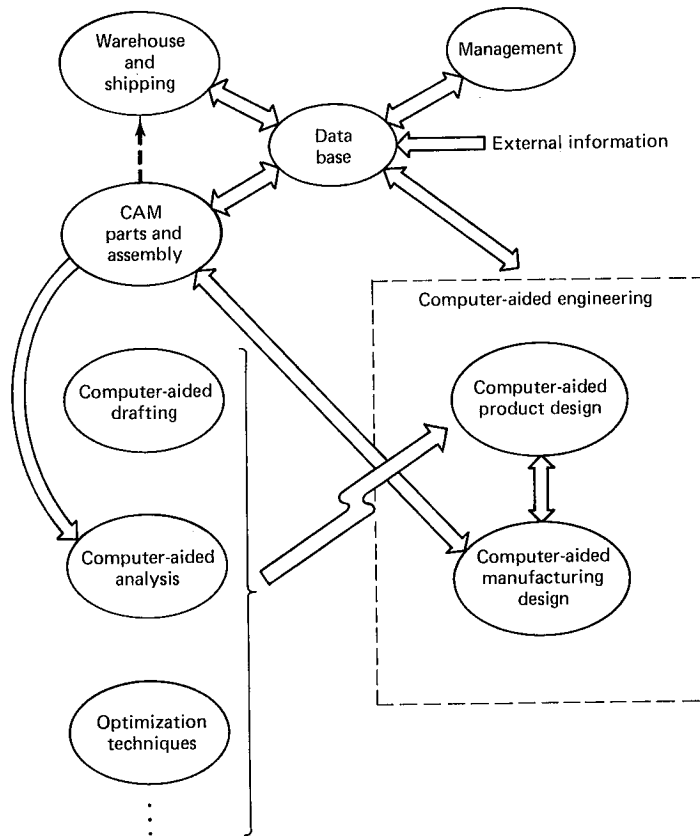
The major goal of the engineering profession is to design and manufacture marketable products of high quality. Today's industries are utilizing computers in every phase of the design, management, manufacture, and storage of their products. The process of design and manufacture, beginning with an idea and ending with a final product, is a closed-loop process. Almost every link in the loop can benefit from the power of digital computers.

## 1.1 COMPUTERS IN DESIGN AND MANUFACTURING

Factory automation is one of the major objectives of modern industry. Although there is no one plan for factory automation, a general configuration is presented in Fig. 1.1. In this configuration, all branches of the factory communicate and exchange information through a central data base. Various parts of the product are designed in the computer-aided engineering (CAE) branch, and then the design is sent to the computer-aided manufacturing (CAM) branch for parts manufacturing and final assembly. Two of the major subbranches of CAE are computer-aided product design and computer-aided manufacturing design.

The computer-aided product design branch, better known as computer-aided design (CAD),<sup>†</sup> may consider the design of single parts or it may concern itself with the final product as an assembly of those parts. Computerized product design requires such capabilities as computer-aided analysis, computer-aided drafting, design sensitivity analysis, or optimization. The computer-aided analysis capability serves as part of the design proc-

<sup>†</sup>The abbreviation CAD is commonly used for both computer-aided drafting and computer-aided design. Most of the CAD systems available today are intelligent computerized drafting systems with limited design capability.



**Figure 1.1** Automated design and manufacturing.

ess and is also used as a model simulator for the finished manufactured product. Analysis may be considered especially appropriate for a product whose initial design has to be modified several times during the manufacturing process. Thus computer-aided analysis can be used as a substitute for laboratory or field tests in order to reduce the cost.

The computer-aided manufacturing design branch is concerned with the design of the manufacturing process. This branch considers the manufacturability of newly designed parts and employs techniques to improve the manufacturing process, in addition to on-line control of the manufacturing process.

### 1.1.1 Computer-Aided Analysis

The computer-aided analysis process (CAA) allows the engineer to simulate the behavior of a product and modify its design prior to actual production. In contrast, prior to the introduction of CAA, the manufacturer had to construct and test a series of prototypes, a process which was not only time-consuming but also costly. Most optimal design techniques require repetitive analysis processes. Although one of the major goals of an automated factory is computer-aided design, computer-aided analysis techniques must be developed first.

Computer-aided analysis techniques may be applied to the study of electrical and electronic circuits, structures, or mechanical systems. The development of algorithms for analyzing electrical circuits began in the early days of electronic computers. Similar techniques were also employed to develop computer programs for structural analysis. Today, these programs, known as finite-element techniques, have become highly advanced and are used widely in various fields of engineering.

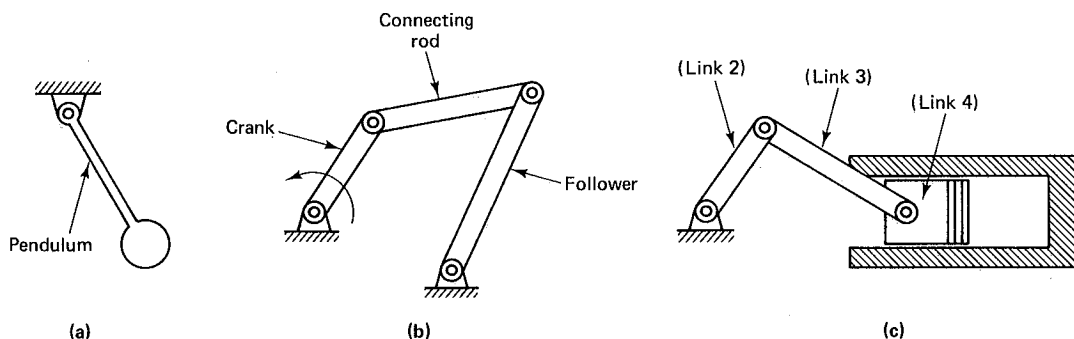
It was not until the early 1970s that computational techniques found their way into the field of mechanical engineering. One of the areas of mechanical engineering where computational techniques can be employed is the analysis of multibody mechanical systems.

## 1.2 MULTIBODY MECHANICAL SYSTEMS

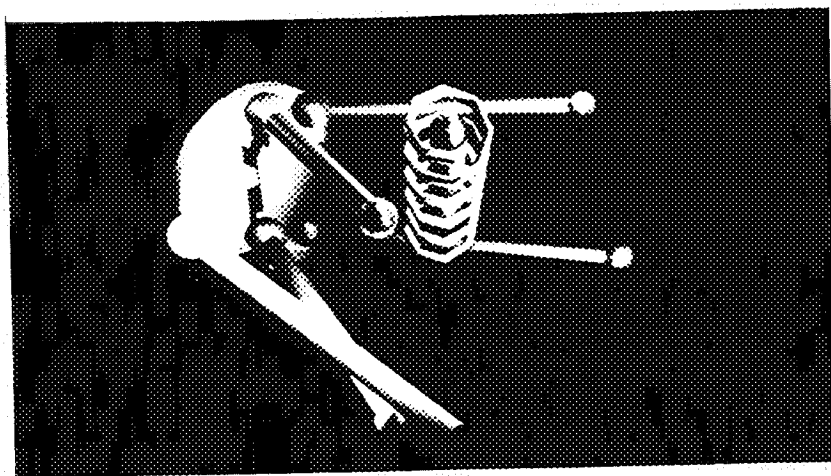
A mechanical system is defined as a collection of bodies (or links) in which some or all of the bodies can move relative to one another. Mechanical systems may range from the very simple to the very complex. An example of a simple mechanical system is the single pendulum, shown in Fig. 1.2(a). This system contains two bodies—the pendulum and the ground. Examples of more complex mechanical systems are the four-bar linkage and the slider-crank mechanism, shown in Fig. 1.2(b) and (c), respectively. The four-bar linkage is the most commonly used mechanism for motion transmission. The slider-crank mechanism finds its greatest application in the internal-combustion engine.

While the motion of the systems in Fig. 1.2 is planar (two-dimensional), other mechanical systems may experience spatial (three-dimensional) motion. For example, the suspension and the steering system of an automobile, shown in Fig. 1.3, contain several spatial mechanisms. This system as a whole has several degrees of freedom. While the kinematics of the individual linkages in this vehicle are more complicated than those of the mechanisms shown in Fig. 1.2, the concept remains the same.

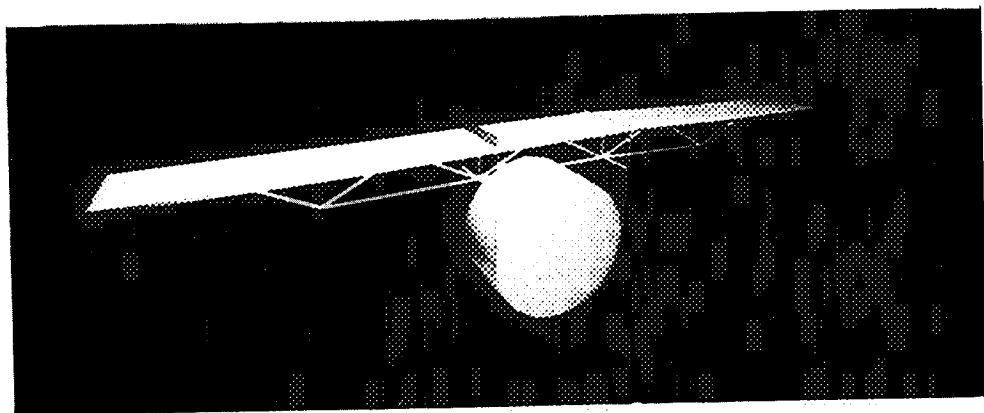
A cascade of simple planar linkage systems can be put together to perform rather complex tasks. The deployable satellite antenna shown in Fig. 1.4 contains such a cascade of six four-bar linkages.<sup>6</sup> Before deployment, the panels of the antenna are folded in order to occupy the minimum space. Once the satellite is in orbit the panels are unfolded in a predefined sequence, as shown in Fig. 1.5. When the unfolding process is completed, the four-bar linkages become a truss structure to support the panels.



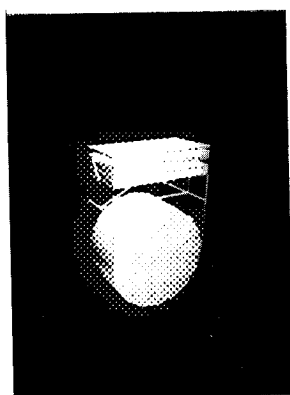
**Figure 1.2** Examples of simple mechanical systems: (a) a single pendulum, (b) a four-bar mechanism, and (c) a slider-crank mechanism.



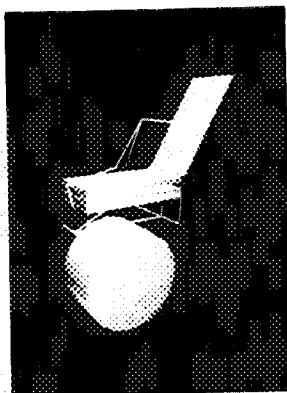
**Figure 1.3** The suspension system and the steering mechanism of an automobile.



**Figure 1.4** A deployed satellite antenna.



(a)

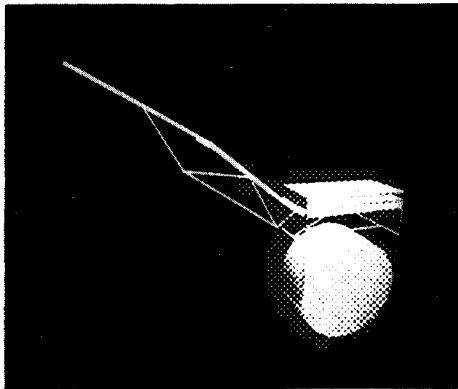


(b)

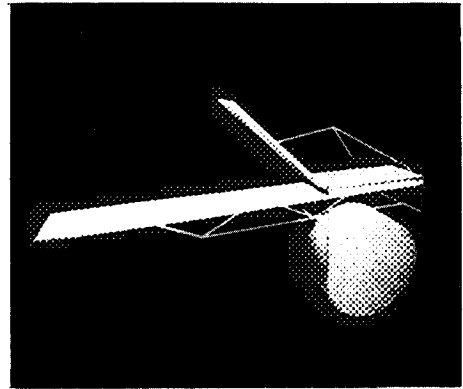


(c)

**Figure 1.5** Unfolding process of the antenna in orbit: (a) folded panels; (b–e) unfolding process.



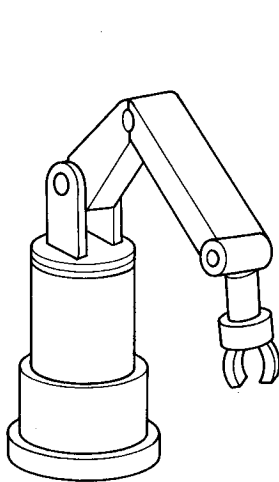
(d)



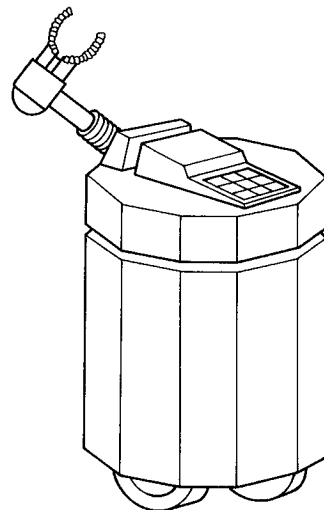
(e)

**Figure 1.5** (continued)

Another example of a mechanical system is a robotic device. A robot can be fixed to a stationary base or to a movable base, as shown in Fig. 1.6. The motion and the position of the end effector of a robot are controlled through force actuators located about each joint connecting the bodies that make up the robot.



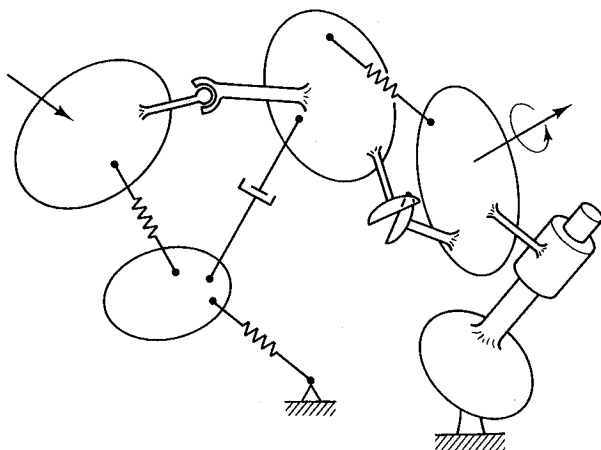
(a)



(b)

**Figure 1.6** Examples of robots with (a) stationary base and (b) movable base.

Any mechanical system can be represented schematically as a multibody system in the manner shown in Fig. 1.7. The actual shape or outline of a body may not be of immediate concern in the process of analysis. Of primary importance is the connectivity of the bodies, the inertial characteristics of the bodies, the type and the location of the joints, and the physical characteristics of the springs, dampers, and other elements in the system.



**Figure 1.7** Schematic representation of a multibody system.

### 1.3 BRANCHES OF MECHANICS

There are two different aspects to the study of a mechanical system: analysis and design. When a mechanical system is acted on by a given excitation, for example, an external force, the system exhibits a certain response. The process which allows an engineer to study the response of an already existing system to a known excitation is called *analysis*. This requires a complete knowledge of the physical characteristics of the mechanical system, such as material composition, shape, and arrangement of parts. The process of determining which physical characteristics are necessary for a mechanical system to perform a prescribed task is called *design* or *synthesis*. The design process requires the application of scientific techniques along with the engineer's judgment. The scientific techniques in the design process are merely tools to be used by the engineer. These are mainly *analysis* techniques and *optimization* methods. Although these techniques can be employed in a systematic manner in the design process, the overall process hinges on the judgment of the design engineer. Since the scientific aspect of the design process requires analysis techniques as a tool, it is important to learn about methods of analysis prior to design.

The branch of analysis which studies motion, time, and forces is called *mechanics*. It consists of two parts—*statics* and *dynamics*. Statics considers the analysis of stationary systems—systems in which time is not a factor. Dynamics, on the other hand, deals with systems that are nonstationary—systems that change their response with respect to time. Dynamics is divided into two disciplines—kinematics and kinetics. *Kinematics* is the study of motion regardless of the forces that produce the motion. More explicitly, kinematics is the study of displacement, velocity, and acceleration. *Kinetics*, on the other hand, is the study of motion and its relationship with the forces that produce that motion.

The focus of this book is on the dynamics of mechanical systems, with an emphasis on computational methods. In addition, one chapter is devoted to computational methods in static equilibrium analysis, since this may be needed prior to dynamic analysis for certain mechanical systems.

#### 1.3.1 Methods of Analysis

Before we analyze the motion of any mechanical system, we must make some simplifying assumptions. For example, if the overall acceleration of a vehicle under the applied

load of the engine is to be determined, then the vibrational motions of certain parts of the vehicle are of no significance. If one decides to consider the vibration and local deformation of every part of the vehicle, then determining the response of the system becomes highly complicated, if not impossible. Therefore, these simplifying assumptions serve two purposes: to make the problem solvable and to eliminate the expenditure of effort on unnecessary or insignificant responses.

Classical methods of analysis in mechanics have relied upon graphical and often quite complex techniques. These techniques are based on geometrical interpretations of the system under consideration. As an example, consider the slider-crank mechanism shown in Fig. 1.8. The crank is rotating with a constant angular velocity. The objective is to find the velocity of the slider. A graphical solution to this problem can be achieved rather easily. The velocity of point A,  $\vec{v}^A$ , has a magnitude of  $v^A = (1.0)(0.1) = 0.1$  m/s and is perpendicular to the crank OA, as shown in Figure 1.9(a). The velocity of point B,  $\vec{v}^B$ , is in the direction of the motion of the slider, and the velocity of point B relative to point A, denoted by vector  $\vec{v}^{BA}$ , is perpendicular to the connecting rod AB. A vector expression relating these velocities is given as

$$\vec{v}^B = \vec{v}^A + \vec{v}^{BA} \quad (1.1)$$

A vector diagram (velocity polygon) corresponding to this expression is shown in Fig. 1.9(b). From this diagram the magnitude and the direction of  $\vec{v}^B$  can be found.

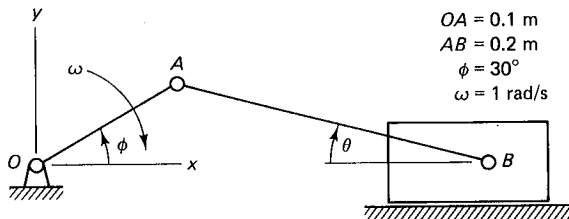


Figure 1.8 A slider-crank mechanism.

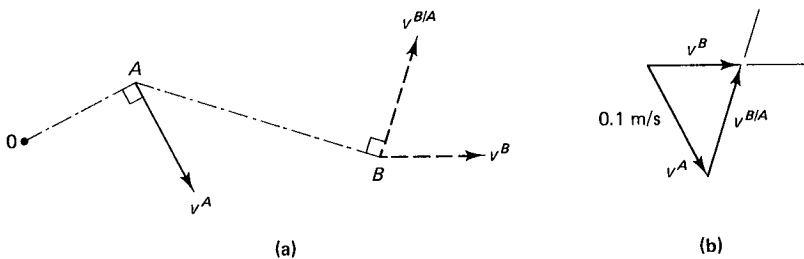
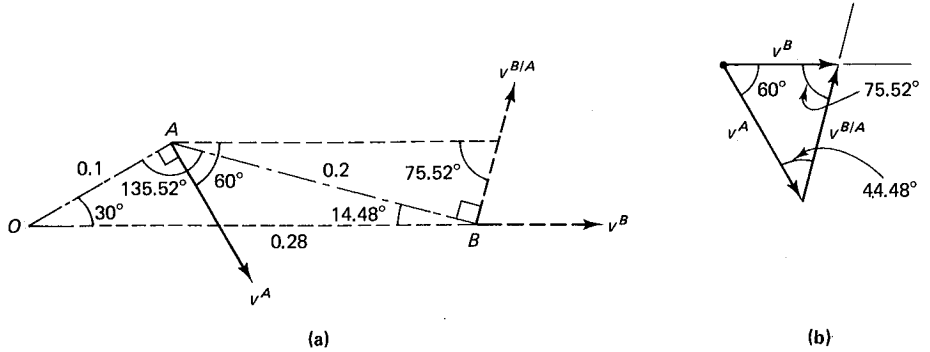


Figure 1.9 Graphical solution.

Although a graphical solution to this problem is rather simple, its accuracy is limited. The graphical approach can yield more accurate results if some trigonometric formulas and geometric relations are introduced into the process. For example, for the slider-crank mechanism, since the angle  $\phi$  and the lengths of the crank and the connecting rod are known, other geometric information for this system can be found easily, as depicted in Fig. 1.10(a). Then a vector diagram can be constructed with complete details as shown in Fig. 1.10(b). From this diagram,  $v^B$  can be calculated from the elementary



**Figure 1.10** Geometric approach with detailed information.

relationship between the sides and the angles of a triangle:

$$v^B = (0.1) \frac{\sin 44.48}{\sin 75.52} = 0.072 \text{ m/s}$$

and of course the direction of  $\vec{v}^B$  is known.

An alternative method to the graphical approach is the solution by vector algebra. In this method, all the vectors are expressed in terms of their components in a common coordinate system. For example, the components of the velocity vectors of the slider-crank mechanism in the  $xy$  coordinate system are

$$\mathbf{v}^A = \begin{bmatrix} 0.1 \cos 60 \\ -0.1 \sin 60 \end{bmatrix} \quad \mathbf{v}^B = \begin{bmatrix} v^B \\ 0 \end{bmatrix} \quad \mathbf{v}^{BA} = \begin{bmatrix} v^{BA} \cos 75.52 \\ v^{BA} \sin 75.52 \end{bmatrix}$$

Substitution of these components in Eq. 1.1 yields

$$\begin{aligned} v^B &= 0.1 \cos 60 + v^{BA} \cos 75.52 \\ 0 &= -0.1 \sin 60 + v^{BA} \sin 75.52 \end{aligned}$$

which results in  $v^{BA} = 0.089 \text{ m/s}$  and  $v^B = 0.072 \text{ m/s}$ .

Kinematic analysis with vector algebra may lead to solving linear or nonlinear simultaneous algebraic equations. For example, the geometric relations between the sides and the angles of the triangle made by the slider-crank mechanism can be expressed as

$$\begin{aligned} a \cos \phi + b \cos \theta - d &= 0 \\ a \sin \phi - b \sin \theta &= 0 \end{aligned} \quad (1.2)$$

where  $a = OA$ ,  $b = AB$ , and  $d = OB$ . Since  $a$  and  $b$  have known values, Eq. 1.2 can be written as

$$\begin{aligned} 0.1 \cos \phi + 0.2 \cos \theta - d &= 0 \\ 0.1 \sin \phi - 0.2 \sin \theta &= 0 \end{aligned} \quad (1.3)$$

For any given value of the crank angle  $\phi$ , the solution to Eq. 1.3 yields values for  $\theta$  and  $d$ . For example for  $\phi = 30^\circ$ , it is found that  $\theta = 14.48^\circ$  and  $d = 0.28 \text{ m}$ . The time derivative of Eq. 1.3 yields the velocity equations,

$$\begin{aligned} -0.1 \dot{\phi} \sin \phi - 0.2 \dot{\theta} \sin \theta - \dot{d} &= 0 \\ 0.1 \dot{\phi} \cos \phi - 0.2 \dot{\theta} \cos \theta &= 0 \end{aligned} \quad (1.4)$$



The angular velocity of the crank is  $\omega = \dot{\phi} = -1$  rad/s. (Since the direction of the angular velocity of the crank is opposite to the defined positive direction of  $\phi$ , a negative sign is given to  $\dot{\phi}$ .) Substituting this in Eq. 1.4 yields  $\theta = -0.45$  rad/s and  $d = 0.072$  m/s, where  $d$  represents the velocity of point  $B$ .

The method of solution with vector algebra is an analytic approach. This approach is more systematic when compared with the graphical method. A problem formulated analytically can be solved repeatedly for different values of input. For example, if the angle  $\phi$  and the angular velocity  $\omega$  of the crank are varied as a function of time, Eqs. 1.3 and 1.4 can be solved repeatedly to obtain the solution. Although this process can be performed with pencil and paper, a computer program can do the job more efficiently.

The usefulness of writing a computer program becomes even more apparent when the mechanical system under consideration is more complex than a planar slider-crank mechanism. For example, the spatial five-bar linkage shown in Fig. 1.11 has two input angles,  $\phi_1$  and  $\phi_2$ . If this system is considered for kinematic analysis, a graphical approach would be very tedious as well as inaccurate. In contrast, if the problem is solved analytically, the solution is accurate and is found efficiently.

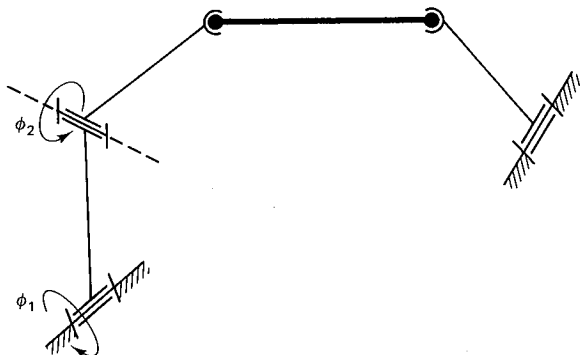


Figure 1.11 A spatial five-bar mechanism.

An analytical approach using a computer program manifests itself when a mechanical system is considered for dynamic (kinetic) analysis. Equations to represent the motion of a system that contains the applied loads and other characteristics of the system are either differential equations or mixed algebraic-differential equations. An exact (closed-form) solution to these equations cannot be found except for highly simplified cases. Regardless of the complexity of the equations of motion, it is always possible to solve them numerically.

## 1.4 COMPUTATIONAL METHODS

The purpose of computer-aided analysis of mechanical systems is to develop basic methods for computer formulation and solution of the equations of motion. This requires systematic techniques for formulating the equations and numerical methods for solving them. A computer program for the analysis of mechanical systems can be either a *special-purpose* program or a *general-purpose* program.

A special-purpose program is a rigidly structured computer code that deals with only one type of application. The equations of motion for that particular application are

derived a priori and then formulated into the program. As input to the program, the user can provide information such as the dimensions and physical characteristics of each part. Such a program can be made computationally efficient and its storage requirement can be minimized, with the result that it will be suitable for implementation on small personal computers. The major drawback of a special-purpose program is its lack of flexibility for handling other types of applications.

A general-purpose program can be employed to analyze a variety of mechanical systems. For example, the planar motion of a four-bar linkage under applied loads and the spatial motion of a vehicle driven over a rough terrain can be simulated with the same general-purpose program. The input data to such a program are provided by the user and must completely describe the mechanical system under consideration. The input must contain such information as number of bodies, connectivity between the bodies, joint types, force elements, and geometric and physical characteristics. The program then generates all of the governing equations of motion and solves them numerically. A general-purpose program, compared with a special-purpose program, is not computationally as efficient and requires more memory space, but it is flexible in use.

The computational efficiency of a general-purpose program depends upon several factors, two of which are the choice of coordinates and the method of numerical solution. The choice of coordinates directly influences both the number of the equations of motion and their order of nonlinearity. Furthermore, depending upon the form of these equations, one method of numerical solution may be preferable to another in terms of efficiency and accuracy.

### 1.4.1 Efficiency versus Simplicity

The governing equations of motion for a mechanical system can be derived and expressed in a variety of forms, dependent mainly upon the type of coordinates being employed. A set of coordinates  $\mathbf{q}$  selected for a system can describe the position of the elements in the system either with respect to each other or with respect to a common reference frame. In order to show how different sets of coordinates can lead to different formulations describing the same system, a simple example is given here. In this example, a four-bar linkage is considered for kinematic analysis. Therefore, all of the governing equations of motion are algebraic equations: i.e., no differential equations are involved.

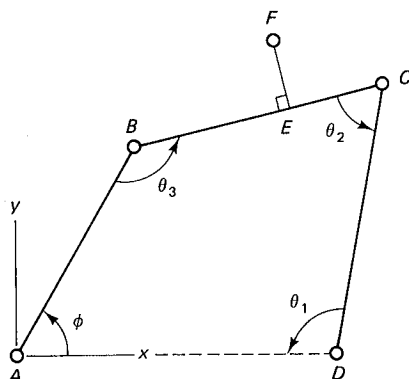
The first formulation shown here considers only one coordinate to describe the configuration of the system, since a four-bar linkage has only one degree of freedom. This is referred to as the *generalized coordinate* of the system. In a system of  $n$  degrees of freedom, there will be  $n$  generalized coordinates. As shown in Fig. 1.12, the angle  $\phi$ , describing the orientation of the crank with respect to the ground, can be selected as the generalized coordinate; i.e.,

$$\mathbf{q} = [\phi] \quad (1.5)$$

For any given configuration, i.e., known  $\phi$ , any other information on the position of any point in the system can be calculated. For example, the angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  can be found from the following formulas:

$$(r^2 + l^2 + s^2 - d^2) - 2rl \cos \phi + 2ls \cos \theta_1 - 2rs \cos(\phi - \theta_1) = 0 \quad (1.6)$$

$$(r^2 + l^2 + s^2 - d^2) - 2rl \cos \phi + 2ds \cos \theta_2 = 0 \quad (1.7)$$



**Figure 1.12** A four-bar mechanism with generalized coordinate  $\phi$ .

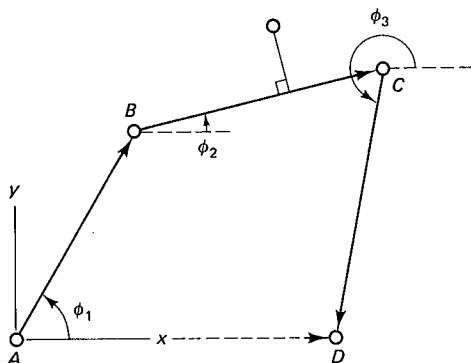
$$\phi + \theta_1 + \theta_2 + \theta_3 - 2\pi = 0 \quad (1.8)$$

In these equations  $r$ ,  $d$ , and  $s$  represent the lengths of the links, and  $l$  represents the distance between points  $A$  and  $D$ . These formulas are derived from simple geometric realizations. It is clear that for a given  $\phi$ , Eq. 1.6 yields  $\theta_1$ , then Eq. 1.7 yields  $\theta_2$ , and finally Eq. 1.8 yields  $\theta_3$ . The solution of these equations requires direct substitution—there is no need to solve a set of simultaneous algebraic equations. Now, it should be clear that the coordinates of a typical point attached to one of the links, in this case the point  $F$  on link  $BC$ , can be found easily.

The second way of formulating the kinematic equations for the four-bar linkage considers three coordinates. For this or any other mechanism, the selected coordinates may define the orientation of each moving body with respect to a nonmoving body or with respect to another moving body. Therefore, in this book these coordinates are referred to as *relative coordinates*. As shown in Fig. 1.13, angles  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are selected as a set of coordinates

$$\mathbf{q} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \quad (1.9)$$

These angles are measured between the positive  $x$  axis and the positive vectors representing the links. Since the four-bar linkage has only one degree of freedom, the three



**Figure 1.13** Relative coordinates describing the configuration of a four-bar system.

coordinates are not independent. Two loop equations relating these coordinates can be written as follows:

$$\begin{aligned} r \cos \phi_1 + d \cos \phi_2 + s \cos \phi_3 - l &= 0 \\ r \sin \phi_1 + d \sin \phi_2 + s \sin \phi_3 &= 0 \end{aligned} \quad (1.10)$$

For any given configuration, i.e., known  $\phi_1$ , the set of two simultaneous algebraic equations must be solved for  $\phi_2$  and  $\phi_3$ . After Eq. 1.10 is solved, other information such as the  $xy$  coordinates of a point  $F$  can be calculated.

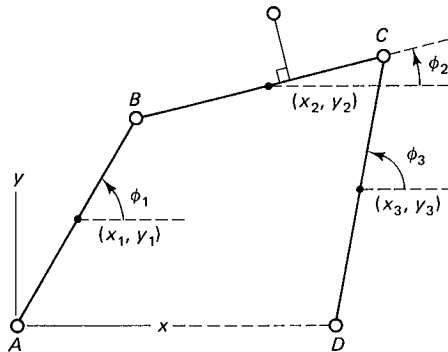
The third formulation uses three *Cartesian coordinates* per link—the  $x$  and  $y$  coordinates of the center point of each link and the angle of the link which is measured with respect to the  $x$  axis, as shown in Fig. 1.14. Thus, the set of coordinates describing the configuration of the four-bar linkage is

$$\mathbf{q} = [x_1 \ y_1 \ \phi_1 \ x_2 \ y_2 \ \phi_2 \ x_3 \ y_3 \ \phi_3]^T \quad (1.11)$$

These nine coordinates are dependent upon each other through eight equations:

$$\begin{aligned} x_1 - \frac{r}{2} \cos \phi_1 &= 0 \\ y_1 - \frac{r}{2} \sin \phi_1 &= 0 \\ x_1 + \frac{r}{2} \cos \phi_1 - x_2 + \frac{d}{2} \cos \phi_2 &= 0 \\ y_1 + \frac{r}{2} \sin \phi_1 - y_2 - \frac{d}{2} \sin \phi_2 &= 0 \\ x_2 + \frac{d}{2} \cos \phi_2 - x_3 - \frac{s}{2} \cos \phi_3 &= 0 \\ y_2 + \frac{d}{2} \sin \phi_2 - y_3 - \frac{s}{2} \sin \phi_3 &= 0 \\ x_3 - \frac{s}{2} \cos \phi_3 - l &= 0 \\ y_3 - \frac{s}{2} \sin \phi_3 &= 0 \end{aligned} \quad (1.12)$$

For any known configuration, any of the nine variables can be specified, and then the remaining eight variables can be found by solving the set of eight nonlinear algebraic equations in eight unknowns.



**Figure 1.14** Cartesian coordinates describing the configuration of a four-bar linkage.

The three preceding forms of formulation with generalized coordinates, relative coordinates, and Cartesian coordinates describe the kinematics of a four-bar mechanism. For dynamic analysis, the differential equations of motion for the four-bar linkage, or for any other mechanical system, can also be derived in terms of any of these sets of coordinates. For the four-bar linkage, formulation with generalized coordinates yields one second-order differential equation in terms of  $\phi$ ,  $\dot{\phi}$ , and  $\ddot{\phi}$ . This equation is highly nonlinear and complex in terms of  $\phi$  and  $\dot{\phi}$ . The equations of motion, for the four-bar linkage in terms of the relative coordinates, consist of three second-order differential equations in terms of  $\phi_i$ ,  $\dot{\phi}_i$  and  $\ddot{\phi}_i$  for  $i = 1, 2$ , and  $3$ . The order of nonlinearity of these equations is not as high or as complex as in the first case. However, with these three differential equations, the two algebraic constraint equations of Eq. 1.10 must be considered. Therefore, the governing equations of motion for this system in terms of relative coordinates are a mixed set of algebraic-differential equations. Similarly, in terms of the Cartesian coordinates, nine second-order differential equations can be derived. Those, in conjunction with the eight algebraic constraint equations of Eq. 1.12, would define the governing equations of motion for the four-bar linkage. These algebraic-differential equations are loosely coupled and have a relatively low order of nonlinearity when compared with the previous sets.

A crude but general comparison between these three sets of coordinates, with regard to several crucial and important aspects, is summarized in Table 1.1. A general conclusion that can be made from this table is that the smaller the number of coordinates and equations, the higher the order of nonlinearity and complexity of the governing equations of motion, and vice versa. Other aspects for comparison, not listed in this table, are the numerical solution of the governing equations of motion and the numerical error encountered in the solution for different formulations.

TABLE 1.1

	Generalized coordinates	Relative coordinates	Cartesian coordinates
Number of coordinates	Minimum <sup>†</sup>	Moderate	Large
Number of second-order differential equations	Minimum <sup>†</sup>	Moderate	Large
Number of algebraic constraint equations	None <sup>†</sup>	Moderate	Large
Order of nonlinearity	High	Moderate	Low <sup>†</sup>
Derivation of the equations of motion	Hard	Moderately hard	Simple <sup>†</sup>
Computational efficiency <sup>‡</sup>	Efficient <sup>‡</sup>	Efficient <sup>‡</sup>	Not as efficient
Development of a general-purpose computer program	Difficult	Relatively difficult	Easy <sup>†</sup>

<sup>†</sup>An advantage over the other two sets of coordinates

<sup>‡</sup>Computational efficiency in solving the governing equations of motion is dependent on the form and the number of equations, and the method of numerical solution. Therefore, this is a very general remark.

Numerical methods for solving ordinary differential equations have been well known for decades. Well-developed algorithms with reliable error control mechanisms have been used extensively in every area of science and engineering. This can be considered another advantage in formulating the equations of motion in terms of a set of generalized coordinates. On the other hand, the governing equations of motion in terms of a set of relative coordinates or Cartesian coordinates are mixed algebraic-differential equations. Methods of numerical solution for such equations, compared with those for the ordinary differential equations, are still in their infancy (this subject is discussed in detail in Chap. 13).

Numerical solutions for differential equations provide only an approximation to the actual (exact) solution. The deviation between the numerical solution and the actual response is the numerical error inherent in the solution. One of the main factors influencing the amount of error in the solution is the number of equations. Generally, the larger the number of equations, the greater the chances for accumulation of numerical error. This can be considered one more advantage in using a minimum number of coordinates.

At this point, it can be concluded that the points in favor of using a set of generalized coordinates for formulating the governing equations of motion outnumber those favoring the other coordinates. However, the disadvantages must not be overlooked. The complexity in deriving the equations of motion and the difficulty in developing a versatile computer program for general usage require an advanced knowledge of dynamics and prior experience in developing large-scale codes. In contrast to the generalized coordinates, the derivation of the equations of motion with Cartesian coordinates is simple. The resulting equations can easily be put into general usage and into a versatile computer program. If computational efficiency is not the decisive factor, then a set of Cartesian coordinates can be an attractive candidate.

It can be concluded that a set of relative coordinates falls in the middle of the "comparison scale." Therefore, selection of a set of relative coordinates might be a good compromise between the generalized and Cartesian coordinates in formulating the governing equations of motion. In Chap. 13, it will be shown how a dynamic analysis algorithm can be developed to take advantage of the simplicity of the Cartesian coordinates for formulating the equations of motion and the efficiency of the generalized or relative coordinates for the numerical solution.

### 1.4.2 A General-Purpose Program

A general-purpose computer program for the dynamic analysis of mechanical systems must perform four basic functions:

1. Accepting data from the user
2. Generating the governing equations of motion
3. Solving the equations
4. Communicating the result to the user

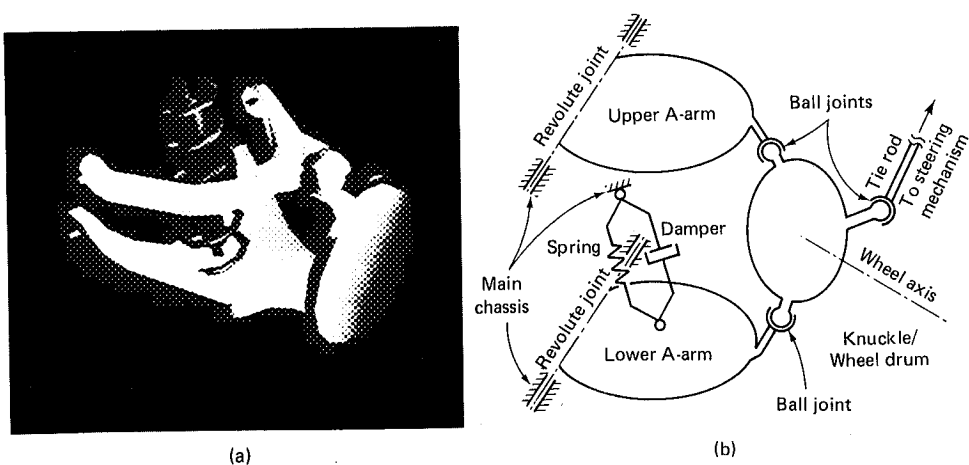
The first step is referred to as the input phase, the second and third steps are the analysis phase, and the fourth step is the output phase.

**Input.** The user must furnish for the program a description of the system under consideration through a set of engineering data. As an example, assume that the double-wishbone suspension system with steering shown in Fig. 1.15(a) is considered for dynamic analysis. A schematic representation of this system is shown in Fig. 1.15(b). The system consists of six moving bodies, a nonmoving body (the ground), a spring and a damper, four spherical joints, and four revolute joints. The input data describing this system must contain such information as:

1. Number of bodies, number and types of joints
2. Mass and moments of inertia of each moving body
3. Connectivity information between the bodies
4. Connectivity information and characteristics of the spring and damper
5. Tire characteristics (if its deformation is to be considered)
6. Direction of gravity
7. Initial conditions on the position and velocity of each body
8. Steering input (from the driver) and applied forces to the wheel (from the road)

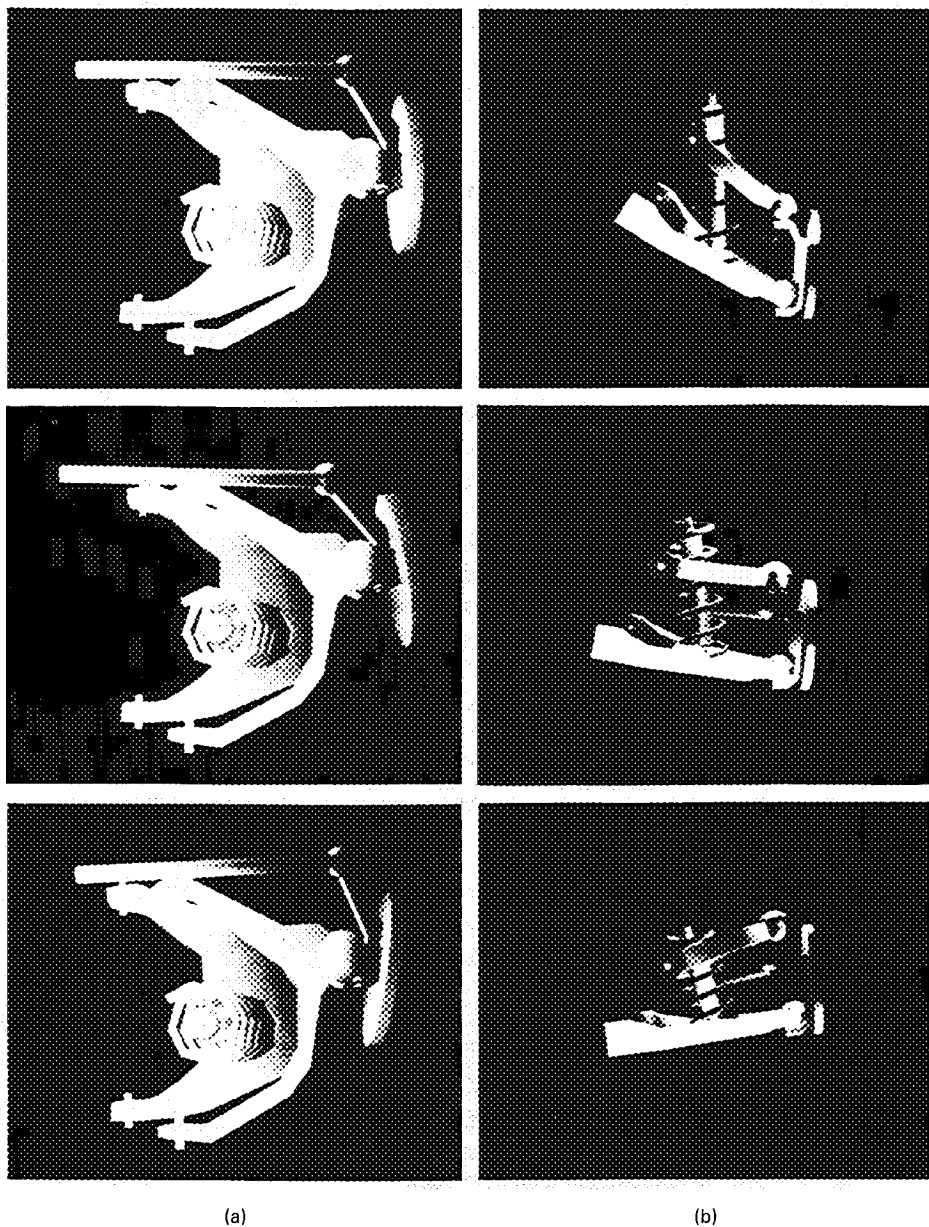
Note that in rigid-body dynamic analysis, the shape of bodies need not be described—this information is needed only if a graphical display of the system is required.

The minimum requirement for generating and communicating a set of input data to the analysis program is an alphanumeric computer terminal. The input can be entered manually via the keyboard and transmitted to the analysis program, which may reside on a mainframe computer, a minicomputer, or a personal microcomputer. The process of generating the input data can be facilitated by developing a preprocessing program and employing a digitizer tablet, a graphics terminal, or a CAD system.



**Figure 1.15** (a) A double-wishbone suspension system, and (b) its corresponding schematic representation.

**Analysis.** On the basis of the input data, the analysis program generates all of the necessary equations describing the system. These equations are then solved numerically in order to obtain the response of the system under the specified loads. The numeri-



**Figure 1.16** Graphic display of the response for the double-wishbone suspension system (a) to a steering command, and (b) to an obstacle on the road.



cal algorithms provide the solution to the equations at discretized points in time. The response at each time point is communicated to the user; it contains such information as the position, velocity, and acceleration of each moving body and the reaction forces at the joints.

**Output.** The minimum output-device requirement is either a terminal screen or a printer. The numerical result of the dynamic simulation for systems undergoing planar motion and having only a few moving bodies may not be too extensive. When it is not, one can interpret and understand the dynamic response for such systems from a printed output. But the task can become extremely time-consuming and tedious when the number of moving bodies is large, particularly when the system undergoes spatial motion. The difficulty of interpreting the dynamic response can be resolved by developing a post-processor program capable of communicating the result to the user through various forms of output device, e.g., a printer, a plotter, or a graphic display unit.

Possibly one of the most expressive forms of communication is computer graphics. For this purpose, the user must provide for the graphics package the exact or an approximate shape of each body. This can be done by defining a set of points on each body and specifying the connectivity between these points. The lines produced from the connectivity information, when displayed, represent surfaces of the outline of each body. The outlines can be positioned in their proper orientation according to the position data provided by the output at any required point in time.

Figure 1.16 is a graphical display of the result of a dynamic simulation of the suspension system of Fig. 1.15. Figure 1.16(a) shows the response of the wheel to a steering command, and Fig. 1.16(b) shows the response of the system when it encounters an



**Figure 1.17** Dynamic response of the double-wishbone suspension system presented as a series of graphic displays to form an animation.

obstacle on the road. Figure 1.17 shows the graphic presentation of the system at several instants of time. When a sequence of graphic displays at small and successive increments of time is generated and displayed at a rate of at least 30 frames per second, an animated picture of the motion is created. This requires a high-speed graphic display device that is capable of displaying several thousand lines or polygons, flicker-free, in one second.