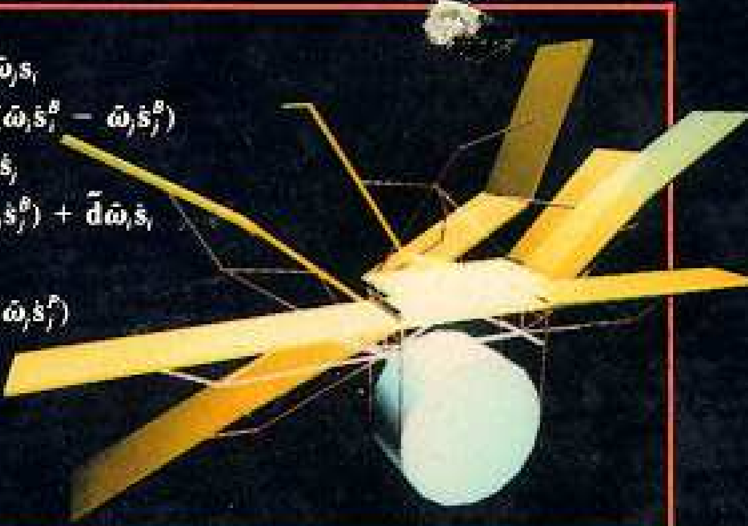
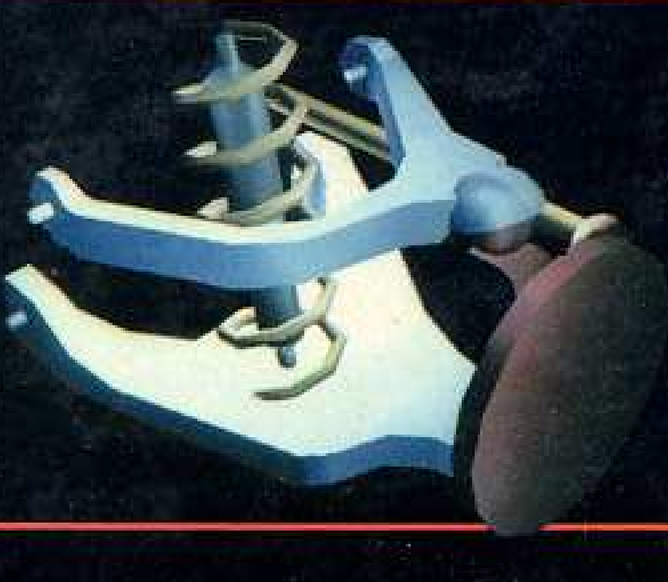
COMPUTER-AIDED ANALYSIS OF MECHANICAL SYSTEMS

Parviz E. Nikravesh

$$\begin{array}{ll}
-\mathbf{s}_i^T\tilde{\mathbf{s}}_j\mathbf{A}_j & -2\dot{\mathbf{s}}_i^T\dot{\mathbf{s}}_j + \dot{\mathbf{s}}_i^T\tilde{\omega}_i\mathbf{s}_j + \dot{\mathbf{s}}_j^T\tilde{\omega}_j\mathbf{s}_i \\
-\mathbf{s}_i^T\tilde{\mathbf{s}}_j^B\mathbf{A}_j & -2\dot{\mathbf{d}}^T\dot{\mathbf{s}}_i - \mathbf{d}^T\tilde{\omega}_i\dot{\mathbf{s}}_i + \mathbf{s}_i^T(\tilde{\omega}_i\dot{\mathbf{s}}_i^B - \tilde{\omega}_j\dot{\mathbf{s}}_j^B) \\
-\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_j\mathbf{A}_j & -2\tilde{\dot{\mathbf{s}}}_i\dot{\mathbf{s}}_j + \tilde{\mathbf{s}}_j\tilde{\omega}_i\dot{\mathbf{s}}_i - \tilde{\mathbf{s}}_i\tilde{\omega}_j\dot{\mathbf{s}}_j \\
-\tilde{\mathbf{s}}_i\tilde{\mathbf{s}}_j^B\mathbf{A}_j & -2\tilde{\dot{\mathbf{s}}}_i\dot{\mathbf{d}} + \tilde{\mathbf{s}}_i(\tilde{\omega}_i\dot{\mathbf{s}}_i^B - \tilde{\omega}_j\dot{\mathbf{s}}_j^B) + \tilde{\mathbf{d}}\tilde{\omega}_i\dot{\mathbf{s}}_i \\
\tilde{\mathbf{s}}_j^P\mathbf{A}_j & -\tilde{\omega}_i\dot{\mathbf{s}}_i^P + \tilde{\omega}_j\dot{\mathbf{s}}_j^P \\
-2\mathbf{d}^T\tilde{\mathbf{s}}_j^P\mathbf{A}_j & -2\dot{\mathbf{d}}^T\dot{\mathbf{d}} + 2\mathbf{d}^T(\tilde{\omega}_i\dot{\mathbf{s}}_i^P - \tilde{\omega}_j\dot{\mathbf{s}}_j^P)
\end{array}$$





$$\mathbf{n}_i^* = 2\mathbf{G}_i^T\mathbf{n}_i$$

$$\mathbf{J}_i'\dot{\omega}_i' + \tilde{\omega}_i'\mathbf{J}_i'\omega_i' - \tfrac{1}{2}\mathbf{L}_i\Phi_{p_i}^T\lambda = \mathbf{n}_i'$$

$$\Phi = \Phi(\mathbf{q}) = 0$$

$$\begin{bmatrix} \mathbf{M} & B^T \\ B & 0 \end{bmatrix}\begin{bmatrix} \dot{\mathbf{h}} \\ -\lambda \end{bmatrix} + \begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{g} \\ \gamma^{\#} \end{bmatrix}$$

NOMENCLATURE

Matrices are in boldface upper-case characters.

Column matrices, algebraic vectors, and arrays are in boldface lower-case characters.

Scalars are in lightface characters.

$\mathbf{a}$	Column vector (array)
$\mathbf{a}^T$	Row vector (array)
$\mathbf{A}$	Matrix
a_{ij}	Element of matrix A in *ith* row and *jth* column
$\boldsymbol{0}$	Zero vector
$\mathbf{0}$	Zero (null) matrix

OVERSCORES

$\vec{}$	Geometric vector
$\tilde{}$	3 × 3 skew-symmetric matrix
$-$	4 × 4 skew-symmetric matrix containing a negative 3 × 3 skew-symmetric matrix
$+$	4 × 4 skew-symmetric matrix containing a positive 3 × 3 skew-symmetric matrix
$\dot{}$	First derivative with respect to time
$\ddot{}$	Second derivative with respect to time

SUPERSCRIPTS

-1	Matrix inverse
i	*i*th time step
T	Matrix or vector transpose
( )	Type of constraint or force
$'$	Components of a vector in a body-fixed coordinate system
$*$	Components of a vector or matrix in Euler-parameter space

SUBSCRIPTS

i	*i*th body in a system
( )	Projection of a vector along a known axis

SYMBOLS

$\boldsymbol{\gamma}$	Vector of right-hand side of acceleration equations
θ	Angle between two vectors
$\boldsymbol{\lambda}$	Vector of Lagrange multipliers
μ_i	Polar moment of inertia for body i
$\xi_i\,\eta_i\,\zeta_i$	Local (body-fixed) Cartesian coordinate system
ρ	Radius of a circle
σ_i	Lagrange multiplier associated with the constraint on $\mathbf{p}_i$
ϕ	Angle of rotation
ϕ_1, ϕ_2, ϕ_3	Bryant angles
ψ, θ, σ	Euler angles
$\vec{\omega}_i$	Angular velocity vector for body i
$\boldsymbol{\omega}_i$	Global components of $\vec{\omega}_i$
$\boldsymbol{\omega}'_i$	Local components of $\vec{\omega}_i$
Φ; $\boldsymbol{\Phi}$	One constraint; vector of constraints
$\boldsymbol{\Phi}_{\mathbf{q}}$	Jacobian matrix of constraints
b	Number of bodies
$\mathbf{b}_i$	Vector containing quadratic velocity terms for body i
$\mathbf{b}$	Vector of quadratic velocity terms
$\vec{d}$	Vector with its ends on two different bodies
$\mathbf{d}$	Global components of $\vec{d}$
e_0, e_1, e_2, e_3	Euler parameters
$\mathbf{e}_i$	Vector of three Euler parameters e_1, e_2, e_3 for body i
$\vec{f}_i$	Force acting on body i
$\mathbf{f}_i$	Global components of $\vec{f}_i$
$\mathbf{g}_i$	Vector of forces for body i containing $\mathbf{f}_i$ and $\mathbf{n}'_i$
$\mathbf{g}$	Vector of forces for a system
$\mathbf{g}^{(c)}$	Vector of constraint reaction forces
$\mathbf{h}_i$	Velocity vector for body i containing $\dot{\mathbf{r}}_i$ and $\boldsymbol{\omega}'_i$
$\mathbf{h}$	Vector of velocities for a system
k	Number of degrees of freedom (DOF)
$\vec{l}$	Vector with its ends on two different bodies
$\mathbf{l}$	Global components of $\vec{l}$
m	Number of constraint equations
$m^{(p)}$	Mass of a particle
m_i	Mass of body i
n	Number of coordinates
$\vec{n}_i$	Moment acting on body i
$\mathbf{n}_i$	Global components of $\vec{n}_i$
$\mathbf{n}'_i$	Local components of $\vec{n}_i$
$\mathbf{n}^*_i$	Components of $\vec{n}_i$ in four-dimensional space
$\mathbf{p}_i$	Vector of four Euler parameters e_0, e_1, e_2, e_3 for body i
$\mathbf{q}_i$	Vector of coordinates for body i
$\mathbf{q}$	Vector of coordinates for a system
$\vec{r}_i$	Translational position vector for body i
$\mathbf{r}_i$	Global coordinates of $\vec{r}_i$
$\vec{s}_i$	Vector with both ends on body i (constant magnitude)
$\mathbf{s}_i$	Global components of $\vec{s}_i$
$\mathbf{s}'_i$	Local components of $\vec{s}_i$
t	Time
t^0	Initial time
t^e	Final (end) time
$\vec{u}$	Unit vector
$\mathbf{u}$	Global components of $\vec{u}$; vector of dependent coordinates
$\mathbf{v}$	Vector of independent coordinates
xyz	Global Cartesian coordinate system
$\mathbf{y}$	Vector of integration variables
$\mathbf{A}_i$	Rotational transformation matrix for body i
$\mathbf{G}_i$	3×4 transformation matrix for body i
$\mathbf{I}$	3×3 or general identity matrix
$\mathbf{I}^*$	4×4 identity matrix
$\mathbf{J}_i$	Global inertia tensor for body i
$\mathbf{J}'_i$	Local (constant) inertia tensor for body i
$\mathbf{J}^*_i$	4×4 inertia tensor
$\mathbf{L}$	Lower triangular matrix
$\mathbf{L}_i$	3×4 transformation matrix for body i
$\mathbf{M}_i$	6×6 mass matrix for body i containing $\mathbf{N}_i$ and $\mathbf{J}'_i$
$\mathbf{M}$	Mass matrix for a system
$\mathbf{N}_i$	3×3 diagonal mass matrix for body i
$\mathbf{U}$	Upper triangular matrix
V	Potential energy

Computer-Aided Analysis of Mechanical Systems

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To the memory of my sister, Henriette.

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Contents

Preface

This book is designed to introduce fundamental theories and numerical methods for use in computational mechanics. These theories and methods can be used to develop computer programs for analyzing the response of simple and complex mechanical systems. In such programs the equations of motion are formulated systematically, and then solved numerically. Because they are relatively easy to use, the book focuses on Cartesian coordinates for formulating the equations of motion. After the reader has become familiar with this method of formulation, it can serve as a stepping stone to formulating the equations of motion in other sets of coordinates. The numerical algorithms that are discussed in this book can be applied to the equations of motion when formulated in any coordinate system.

Organization of the Book

The text is organized in such a way that it can be used for teaching or for self-study. The concepts and numerical methods used in kinematics are systematically treated before the concepts and numerical methods used in dynamics are introduced. Separate chapters on each of these topics allow the text to be used for the study of each topic separately or for some desired combination of topics. Furthermore, the text first treats the less complex problems of planar kinematic and dynamic analysis before it discusses spatial kinematic and dynamic analysis.

With the exception of the first two chapters and the last chapter, the text can be divided into two subjects—kinematics and dynamics. Chapter 1 gives an introduction to the subject of computational methods in kinematics and dynamics. Simple examples illustrate how a problem can be formulated using different coordinate systems. Chapter 1 also explains why Cartesian coordinates provide a simple tool, if not necessarily the most computationally efficient one. Chapter 2 presents a review of vector and matrix algebra, with an emphasis on the kind of formulation that lends itself to implementation in computer programs.

Chapters 3 through 7 deal with kinematics. Chapter 3 introduces the basic concepts in kinematics that are applicable to both planar and spatial systems. Algebraic constraint equations, the various coordinate systems, and the idea of degrees of freedom are presented as a foundation for both the analytical and the numerical aspects of kinematic analysis. Position, velocity, and acceleration analysis techniques are presented and illustrated through the solution of simple mechanisms. Numerical methods for solving the associated kinematic equations are presented and illustrated. These include methods for solving sets of linear and nonlinear algebraic equations. A comprehensive treatment of planar kinematics using Cartesian coordinates is presented in Chapter 4. In that chapter, a library of kinematic constraints is defined and the governing algebraic constraint equations are derived.

Chapter 5 contains a FORTRAN program for planar kinematic analysis. The program is developed and explained as a collection of subroutines that carry out the functions of kinematic analysis. The problems at the end of Chapter 5 provide guidelines for the extensions that allow for the expansion of the program to treat broader classes of planar kinematic systems.

Chapter 6 presents a set of spatial rotational coordinates known as Euler parameters. The physical properties of Euler parameters and the development of their algebraic properties are introduced to allow the reader to become comfortable with and confident in their use. Also, velocity relationships—including the definition of angular velocity—and other identities are developed that are necessary for the formulation of spatial kinematic and dynamic analysis.

Chapter 7 presents a unified formulation of spatial kinematics using Cartesian coordinates and Euler parameters. Vector relationships that are required for the definition of kinematic joints are first presented and then applied to derive the governing equations for a library of spatial kinematic joints. Although this book does not provide a source listing for a spatial kinematic analysis program, the computer program in Chapter 5 and the constraint formulations in Chapter 7 provide all the information that the reader needs to develop a computer program.

Chapters 8 through 13 deal with dynamics. Basic concepts in dynamics are presented in Chapter 8. Discussion begins with familiar concepts of the dynamics of a particle and progresses to the dynamics of systems of particles and, finally, to the dynamics of rigid bodies. By means of a building block formulation, the complete theory of the dynamics of systems of rigid bodies is developed in a systematic and understandable way. The Newton-Euler equations of motion are derived and used as a fundamental tool in the dynamic analysis of systems of rigid bodies that are connected by kinematic joints. The Lagrange multiplier formulation for constrained systems is developed, and the reaction forces between the joints are derived in terms of the Lagrange multipliers.

Chapter 9 discusses the planar dynamics of systems of constrained rigid bodies, drawing upon the kinematics theory discussed in Chapter 4 and the basic dynamics theory discussed in Chapter 8. Even though the numerical methods for solving the differential equations of motion are discussed in detail in Chapters 12 and 13, a FORTRAN program for planar dynamic analysis is presented in Chapter 10. This program, which is a collection of subroutines used to implement a variety of computations required in the formulation and solution of equations of motion, builds upon the kinematic analysis program in Chapter 5. The computer program is demonstrated through the solution of simple examples, and extensions to the program are included as problems at the end of the chapter.

Chapter 11 presents the formulation of spatial system dynamics using Cartesian coordinates and Euler parameters. The equations of motion of kinematically constrained systems of rigid bodies are derived and developed in a form suitable for computational implementation. Chapter 12 presents a brief overview of numerical methods for solving ordinary differential equations. A FORTRAN listing of a fourth-order Runge-Kutta algorithm illustrates the implementation of these numerical methods along with some examples. Chapter 13 presents a number of advanced numerical methods for multibody dynamics. Alternate techniques and algorithms for the solution of mixed systems of differential and algebraic equations that arise in system dynamics are presented.

In the analysis of multibody mechanical systems, it may be necessary to go beyond kinematics and dynamics and find the static equilibrium state of a system. Chapter 14 discusses several computation-based methods for static equilibrium analysis.

Level of Courses

The book can be covered in two successive courses. The student is required to know the fundamentals of kinematics and dynamics, to have a basic knowledge of numerical methods, and to know computer programming, preferably FORTRAN.

The first course — a senior undergraduate or a first-year graduate course — could cover Chapters 1 through 5, 9, and 10, on planar motion; if students do not have the proper background in numerical methods in ordinary differential equations, Chapter 12 should also be covered to the extent necessary. The course could be project-oriented: students could be assigned to find existing medium- to large-scale mechanical systems and analyze them using the computer programs that are provided in the book. The second course would then cover Chapters 6 through 8 and 11 through 14, on spatial motion; this would be quite suitable as a graduate-level course. Students, divided into groups, should be able to develop a spatial-motion dynamic analysis program.

Another possibility would be one course, covering Chapters 1 through 7, on the subject of kinematics, and a second course, covering Chapters 8 through 14, on the subject of dynamics.

Exercises

Problem assignments can be found at the end of most chapters. The problems are designed to clarify certain points and to provide ideas for program development and analysis techniques. However, by no means do these problems represent the ultimate flexibility and power of the formulations and algorithms that are stated in the book. Most realistic multibody problems that arise in engineering practice can be treated by employing similar techniques and ideas.

Computer Programs

Two FORTRAN programs called KAP and DAP, for planar kinematic and dynamic analysis, respectively, are developed and listed in the book. Other programs, for static equilibrium analysis, or for spatial kinematic and dynamic analysis, can be developed by the reader by following the formulations and algorithms that are discussed in various chapters. Source codes for KAP, DAP, and other complementary programs can be obtained on a floppy disk from the publisher.

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NOTE ON UNIT SYSTEM

The system of units adopted in this book is, unless otherwise stated, the international system of units (SI). In most examples and problems, the variables are organized as the elements of arrays suitable for programming purposes. These variables usually represent various different quantities and therefore have different units. If the unit of each element of an array were to be stated, it would cause notational confusion. Therefore, in order to eliminate this problem, the units of the variables are not stated in most parts of the text. The reader must assign the correct unit to each variable. The unit of degree or radian alone is stated for variables representing angular quantities.

SI Units Used in This Book

Quantity	Unit SI	Symbol
(Base Units)		
Length	meter	m
Mass	kilogram	kg
Time	second	s
(Derived Units)		
Acceleration, translational	meter/second^2	m/s^2
Acceleration, angular	radian†/second^2	rad/s^2
Damping coefficient	newton-second/meter	N.s/m
Force	newton	N (=kg.m/s^2)
Moment of force	newton-meter	N.m
Moment of Inertia, mass	kilogram-meter^2	kg.m^2
Pressure	pascal	Pa (=N/m^2)
Spring constant	newton/meter	N/m
Velocity, translational	meter/second	m/s
Velocity, angular	radian†/second	rad/s

† or degree