**Force Analysis**

**Problem 1.** A known force acts on the slider block (6) of this six-bar mechanism. An unknown torque acts on link (2) which keeps the mechanism in static equilibrium. Construct free-body-diagrams for the links while taking advantage of the “two-force member” concept. Use the equilibrium equations to determine the unknown torque. Take direct measurements from the figure for lengths and angles.

![Diagram of six-bar mechanism](image)

**Problem 2.** For the pliers shown, determine the relationship between the magnitude of the applied forces P and the magnitude of the gripping forces on the round object. Use the FBD method. (Hint: Although not necessary, you may consider one of the links to be fixed.)

![Diagram of pliers](image)

**Problem 3.** One link of a mechanism is shown. A known force $F_A = 20$ units in the positive x-axis acts at A. An unknown force $F_B$ acts along the y-axis at B (the shown direction is arbitrary). An unknown force acts at C (it is shown arbitrarily). A torque $T = 30$ units acts on the link in the direction shown. The link is in static equilibrium. Determine the unknown forces. Assume that the units are consistent with the units of measurement that you choose for the lengths.

Take direct measurements from the figure for the lengths. State your unit of measurement.
**Problem 4.** The link lengths of this six-bar mechanism are $L_2 = L_5 = 1.0$, $L_4 = 3.0$ (SI units). The mechanism is in motion according to the shown velocity and acceleration polygons. A spring is attached between the slider $D$ and the ground that has a deformed length of 2.0, an undeformed length of 2.5, and a stiffness $k = 40$. The links have the following masses and moments of inertia: $m_3 = 6.0$, $I_3 = 3.0$, $m_4 = 4.0$, $I_4 = 5.0$, $m_5 = 2.0$, $I_5 = 4.0$, $m_6 = 3.0$, $I_6 = 6.0$. Link 2 has negligible mass and moment of inertia. An unknown torque acts on link 2.

Use the power-formula to determine the unknown torque.
**Problem 5.** This six-bar mechanism is in motion. For the shown configuration the velocity and acceleration polygons are provided. A torque of 10 N.m CW acts on link (2). A force of 50 N acts at B as shown. And a resistive force acts in the horizontal direction on link (6). Use the Power Formula to determine the magnitude and the direction of this resistive force.

Lengths: \( O_2A = 1.0, O_3B = 3.6, BC = 2.1 \), and in the given configuration \( O_3A = 1.85 \) m

Masses and moments of inertias: 
\( m_4 = 10, m_6 = 15, I_4 = 100, I_6 = 150 \) (others are negligible)
**Problem 6.** Consider the inverted slider-crank mechanism shown. The lengths of the links are: \(O_2A = 1.0\), \(AP = 5.0\), \(AG_3 = 4.0\) (\(G_3\) is the mass center of link 3). The links have the following mass and moment of inertia (SI units): \(m_2 = 0\), \(m_3 = 5.0\), \(m_4 = 20.0\), \(I_2 = 0\), \(I_3 = 4.0\), \(I_4 = 6.0\).

A known force acts at point \(P\) having the following components:
\[
F_x = -8.0, \quad F_y = -6.0
\]

An unknown torque acts on link 2. **Do not** consider gravitational forces in this problem.

(a) Assume the system is stationary (nothing is moving). Determine the unknown torque that acts on link 2. Use any method you prefer.

(b) Assume the system is in motion. The corresponding velocity and acceleration polygons are provided. From the polygons the following angular velocities and accelerations have been determined:
\[
\omega_2 = 1.0 \text{ CCW}, \quad \omega_3 = \omega_4 = 0.2 \text{ CCW}
\]
\[
\alpha_2 = 1.0 \text{ CW}, \quad \alpha_3 = \alpha_4 = 0.45 \text{ CW}
\]

Determine the unknown torque that acts on link 2.

**Problem 7.** One link of a mechanism is shown. A known force \(F_A = 20\) units in the negative \(x\)-axis acts at \(A\). An unknown force \(F_B\) acts along the \(y\)-axis at \(B\) (the direction shown is arbitrary). An unknown force acts at \(C\) (it is shown arbitrarily). A torque \(T = 30\) units acts on the link in the direction shown.

The link is in motion. The mass center has an acceleration \(A_g = 10\) units in the direction shown, and the angular acceleration of the link is \(\alpha = 2 \text{ rad/sec}^2\) CCW. The mass and moment of inertia of the link are \(m = 3\) and \(I_g = 20\) units. Determine the unknown forces. Assume that the units are consistent with the units of measurement that you choose for the lengths. Take direct measurements from the figure for the lengths. State your unit of measurement.
Problem 8. The garage-type automobile jack shown is actuated by hydraulic pressure in the cylinder. Links (2), (3) and (4) have negligible weights. The weight of links (1) is 200 N. The jack is supporting a load, \( L \), of 1,000 N. Assume negligible friction.

(a) Construct free-body diagram for each link (next page). Take advantage of 2-force member concept if applicable.

(b) Determine the force exerted by the actuator that keeps the system in static equilibrium (magnitude and direction).

(c) Determine the forces acting on the ground at \( E \) and \( F \) (magnitudes and directions).

Take measurements from the figure using the scale shown.

Problem 9. For this four-bar mechanism, the link lengths are: \( O_2A = 0.40 \text{ m} \), \( AB = 0.55 \text{ m} \), and \( O_4B = 0.20 \text{ m} \). In the given configuration, the velocity polygon is shown. A force of 20 N acts on point \( A \) to the left, and a force of 15 N acts at the center of link 3, point \( C \), perpendicular to the link in the direction shown. What torque (magnitude and direction) should be applied on link 4 in order to keep the four-bar in static equilibrium? (Apply any method you like)
**Problem 10.** For this four-bar mechanism, the following data are given: (SI units)

\[ \ell_1 = 1.2, \quad \ell_2 = 1.0, \quad \ell_3 = 2.0, \quad \ell_4 = 2.5, \quad \ell_{BP} = 1.0 \]

Link 2 has its mass center at \( O_2 \). The mass center of link 3 is at the geometric center of \( AP \). The mass center of link 4 is also at its geometric center. The following mass/inertia data are given:

\[ m_2 = 20, \quad I_2 = 10, \quad m_3 = 4, \quad I_3 = 3.0, \quad m_4 = 2, \quad I_4 = 1.5 \]

A force of 3.0 units acts at point \( P \) in the positive \( x \)-direction. A motor about the axis of \( O_2 \) applies an unknown torque on link 2.

In the configuration shown (crank angle \( \theta_2 = 125 \) degrees) the system is in motion according to the shown velocity and acceleration polygons (next page). The following angular velocities and accelerations are obtained from the polygons:

\[ \omega_2 = 1.0 \text{ CCW}, \quad \omega_3 = 0.24 \text{ CCW}, \quad \omega_4 = 0.50 \text{ CCW} \text{ (all rad/sec )} \]

\[ \alpha_2 = 0.2 \text{ CW}, \quad \alpha_3 = 0.21 \text{ CCW}, \quad \alpha_4 = 0.08 \text{ CW} \text{ (all rad/sec }^2) \]

Apply the Power Formula to determine the unknown torque. Do not consider gravitational forces.

Show the computation of each term in the formula separately before adding them up.

**Problem 11.** (15 points) A force \( F = 100 \text{ N} \) acts on the handle of this device as shown. What force the slider block is applying on the ball?
**Problem 12.** The figure shows an adjustable platform used to load and unload freight trucks. Currently a crate is located as shown having a weight of 5,000 N. The platform weighs 1,000 N, and the weight of all other links is considered insignificant. The measurements are in SI units.

(a) Draw free-body diagram for each link. Consider the platform and the crate as one body. For your convenience the individual links (bodies) are shown separately.

(b) Determine the force that the hydraulic actuator provides to support the system in static equilibrium.

**Problem 13.** The figure shows a small hydraulic jack. At this instant a 20 N force is applied to the handle as shown. This causes link (2) to rotate clockwise at a constant rate of 10 rad/sec. The mass and moment of inertia of links (2) and (3) are negligible. The mass and moment of inertia of the piston, link (4), are \( m_4 = 2 \) and \( I_4 = 1.6 \) (SI units). Determine the force developed on the piston due to the hydraulic fluid, using the Power formula.
**Problem 14.** A front loader is shown. Determine the force required from both hydraulic cylinders to maintain the shovel position. Ignore the mass (weight) of the links.

**Problem 15.** The crank of this slider-crank mechanism is in the form of a disc with a radius of 1 unit (its mass center is at $O_2$). The connecting rod has a length of 3 units with its mass center at its geometric center, $G_3$. The slider block has its mass center at $C$. The mass and moment of inertia for the three moving links are provided as (SI units):

\[ m_2 = 8, I_2 = 2, m_3 = 4, I_3 = 3, m_4 = 6, I_4 = 4 \]

The velocity and acceleration polygons for this mechanism are shown. The angular velocities and accelerations are determined to be:

\[ \omega_2 = -1, \alpha_2 = 1, \omega_3 = -0.24, \alpha_3 = 0.49 \]

The magnitudes of the linear velocities and accelerations are given on the polygons.

A known force $F$ with a magnitude of 1 unit acts on the slider block (to the left). An unknown torque $T_2$ acts on link 2. Determine the magnitude and the direction of this torque in order to maintain the specified motion (do not include gravity in the calculations).
Problem 16. A known force $F$ with a magnitude of 2 N acts at point $P$ of this four-bar mechanism. Known torques of 1.5 N.m and -2.5 N.m act on links 3 and 4 respectively. An unknown torque $T_2$ acts on link 2. The velocity and acceleration vectors for the mass centers and the velocity of point $P$ are shown. The magnitudes of these vectors, the angular velocity and accelerations, and the inertia data are given in the following table. Determine the unknown torque that acts on link 2.

<table>
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<th>m/s</th>
<th>m/s²</th>
<th>rad/s</th>
<th>rad/s²</th>
<th>kg</th>
<th>kg.m²</th>
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<tr>
<td>$V_{G2}$</td>
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<td></td>
<td>1.1</td>
<td>-2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>$V_{G3}$</td>
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<td></td>
<td>1.4</td>
<td>0.6</td>
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</tr>
<tr>
<td>$V_{G4}$</td>
<td>0.4</td>
<td></td>
<td>0.5</td>
<td>-0.5</td>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>$V_P$</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Problem 17. The six-bar mechanism shown is in static equilibrium. A force of 20N acts at point $B$ as shown. A force of 10 acts at point $C$ as shown. An unknown force acts at $A$ along the $x$-axis. A known torque of $T = 40$ Nm acts on link (3) CCW.

Apply the power formula to determine the unknown force (magnitude and direction). For your convenience for $\omega_2 = 2$ rad/sec CCW, it has been determined that $\omega_3 = 1.5$ CCW, $V_A = 5.0$, $V_B = 5.2$, and $V_C = 10.0$ m/sec in the directions shown.
**Problem 18.** Consider the inverted slider-crank mechanism shown below. The lengths of the links are:  
\[ O2_A \text{ (link 2)} = 1.0 \text{ m}, \quad O4_P \text{ (link 4)} = 5.0 \text{ m}, \quad O2_O4 \text{ (ground link)} = 2.0 \text{ m} \]  
In the given configuration the angle of the crank is 130 degrees, the angle of link 4 is 164 degrees, and \( O4_A = 2.75 \text{ m} \).

Links 2 and 4 have negligible mass and moment of inertia. For link 3 (slider block) we have \( m_3 = 3 \text{ kg} \) and \( I_3 = 2 \text{ kg.m}^2 \). A known force acts at point \( P \) that has the following components: \( F_x = 5, \quad F_y = 3 \text{ N} \). An unknown torque acts on link 2.

If we assume that the crank rotates with an angular velocity of 1 rad/sec CCW and an angular acceleration of 1 rad/sec^2 CW, the following velocity and acceleration polygons can be obtained.

Use any of the above information to solve the following problems.

(a) Assume the inverted slider-crank is stationary (nothing is moving). Determine the unknown torque that acts on link 2. Use any method you like. (For your convenience all the figures have been duplicated on this page; use what you need.)

(b) Assume the inverted slider-crank mechanism is in motion according to the velocity and acceleration polygons shown.

(b.1) Show a free body diagram for each link: show all the applied, reaction, and inertial forces/moments.

(b.2) Determine the unknown torque that acts on link 2. Use any method you like.
Problem 19. Consider the inverted slider-crank mechanism shown below. The lengths of the links are:

\[ O2_A \text{ (link 2)} = 1.0 \text{ m}, \quad O4_P \text{ (link 4)} = 3.0 \text{ m}, \quad O2_O4 \text{ (ground link)} = 2.0 \text{ m} \]

In the given configuration the angle of the crank is 30 degrees, the angle of link 4 is 156 degrees, and \( O4_A = 1.24 \text{ m} \).

Links 2 and 4 have negligible mass and moment of inertia. For link 3 (slider block) we have \( m_3 = 3 \text{ kg} \) and \( I_3 = 2 \text{ kg.m}^2 \). A known force acts at point \( P \) that has the following components: \( F_x = 5, \quad F_y = 3 \text{ N} \). An unknown torque acts on link 2.

If we assume that the crank rotates with an angular velocity of 1 rad/sec CCW and an angular acceleration of 1 rad/sec\(^2\) CCW, the following velocity and acceleration polygons can be obtained.

(a) Assume the inverted slider-crank is stationary (nothing is moving). Determine the unknown torque that acts on link 2. Use any method you like. (For your convenience all the figures have been duplicated on this page; use what you need.)

(b) Assume the inverted slider-crank mechanism is in motion according to the velocity and acceleration polygons shown.

(b.1) Show a free body diagram for each link: show all the applied, reaction, and inertial forces/moments.

(b.2) Determine the unknown torque that acts on link 2. Use any method you like.
Problem 20. Consider the inverted slider-crank mechanism shown below. The lengths of the links are:

- $O2_A$ (link 2) = 1.0 m, $A_P$ (link 3) = 5.0 m, $O2_O4$ (ground link) = 2.0 m

In the given configuration the angle of the crank is 300 degrees, the angle of link 3 is 56.6 degrees, and $O4_A$ = 2.24 m. Link3 has negligible mass and links 2 and 3 have negligible moments of inertia. For link 2 we have $m_2 = 2$ kg and for link 4 (slider block) we have $m_4 = 20$ kg and $I_4 = 6$ kg.m$^2$. A known force acts at point $P$ that has the following components: $F_x = -8$, $F_y = -6$ N. An unknown torque acts on link 2.

If we assume that the crank rotates with an angular velocity of 1 rad/sec CCW and an angular acceleration of 1 rad/sec$^2$ CW, the following velocity and acceleration polygons can be obtained.

Use any of the above information to solve the following problems.

(a) Assume the inverted slider-crank is stationary (nothing is moving). Determine the unknown torque that acts on link 2. Use any method you like. (For your convenience all the figures have been duplicated on this page; use what you need.)

(b) Assume the inverted slider-crank mechanism is in motion according to the velocity and acceleration polygons shown. Show a free body diagram for each link: show all the applied, reaction, and inertial forces/moments.

(c) Assume the inverted slider-crank mechanism is in motion according to the velocity and acceleration polygons shown. Determine the unknown torque that acts on link 2. Use any method you like.
Problem 21: The links of this four-bar mechanism have the following masses and moments of inertias (SI units):

\[ m_2 = 5, \ I_2 = 2, \ m_3 = 10, \ I_3 = 4, \ m_4 = 6, \ I_4 = 3 \]

The length of link \( O_2A \) is 1 unit. A force acts on link (3) at \( P \) as shown with a magnitude \( F_P = 100 \). A known torque, \( T_4 = 25 \) CW, acts on link (4). An unknown torque \( T_2 \) acts on link (2).

In the shown configuration, the velocity and acceleration polygons are given. From the polygons, the following angular velocities and accelerations are deduced:

\[ \omega_2 = 2.0, \ \omega_3 = 0.37, \ \omega_4 = 1.35 \ \text{(rad/sec)} \]

\[ \alpha_2 = -4.0, \ \alpha_3 = -0.54, \ \alpha_4 = -3.07 \ \text{(rad/sec}^2) \]

The linear velocities and accelerations can be determined from the polygons based on the velocity of \( A \) (2.0 units) and acceleration of \( A \) (5.7 units).

Determine the unknown torque on link (2), using the Power formula. Computation of each term in the formula must be shown clearly.
Problem 22. One link of a mechanism is shown. The mass and moment of inertia of the link are $m = 5$ and $I_G = 20$ units.
A known force of 20 units acts at $A$ in the negative $x$-axis.
A known force of 10 units acts at $B$ in the positive $y$-axis.
An unknown force $F_C$ acts at $C$.
An unknown torque $T$ acts on the link.
The link is in motion. The mass center has an acceleration of $A_{G(x)} = 6$ and $A_{G(y)} = -14$ units. The angular acceleration of the link is $4 \text{ rad/sec}^2$ CW.
Determine the unknown force and torque. Assume that the units are consistent.

Problem 23. A CCW torque $T_2 = 100 \text{ N.m}$ acts on link (2) of this inverted slider-crank mechanism. Coulomb friction exists between links (3) and (4) where the static coefficient of friction is 0.2. An unknown torque acts on link (4). Determine the range of magnitude and direction of this moment for keeping the system in static equilibrium.