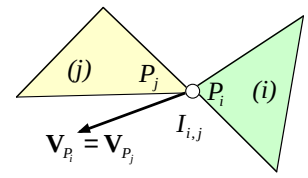
5.3 GRAPHICAL VELOCITY ANALYSIS
Instant Center Method

Instant center of velocities is a simple graphical method for performing velocity analysis on mechanisms. The method provides visual understanding on how velocity vectors are related.
Tools: ruler, right angle, protractor

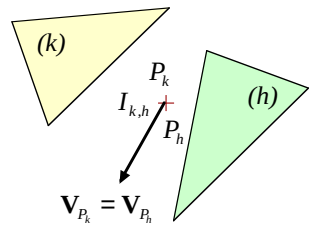
What is An Instant Center?

Instant center of velocities between two links is the location at which two coinciding points, one on each link, have identical velocities.

The most obvious instant center of velocities, or simply the instant center (IC), between two links that are pinned to each other is the point at the center of the pin joint. For example, the center of the pin joint between links *i* and *j* can be viewed as two coinciding points, P_i on link *i* and P_j on link *j*, that have the same velocities. The instant center between these two links is denoted as $I_{i,j}$ or $I_{j,i}$.



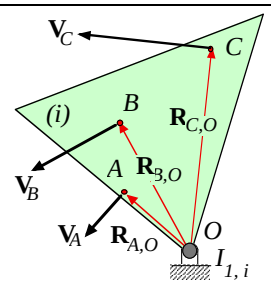
The instant center of velocities may not be located within the physical boundaries of a link. As shown in the second figure, the IC between links *k* and *h*, $I_{k,h}$, is located on imaginary extensions of both links.



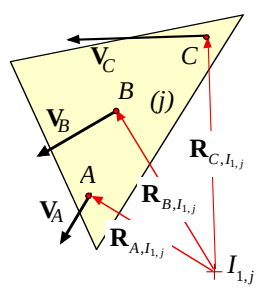
Instant Center Between A Link and The Ground

Consider link *i* that is pinned to the ground at *O*. Point *O* is the instant center between links *i* and the ground, and it is denoted $I_{1,i}$ or $I_{i,1}$ (the ground is always given the index *1*).

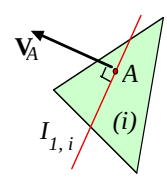
If the link has a non-zero angular velocity ω, every point on the link has a non-zero velocity except for point *O*. The velocity of any point on the link is determined as $\mathbf{V} = \omega\breve{\mathbf{R}}$, where $\mathbf{R}$ is the position vector of that point with respect to *O*. Note that all the velocity vectors are tangent to circles with a common center at $I_{1,i}$.



Now consider link *j* that is not connected to the ground directly. If the link has a non-zero angular velocity, the velocity vectors of all the points on the link *must be* tangent to circles with a common center. This common center is the instant center between link *j* and the ground; i.e., this point has a zero velocity. This center acts as an imaginary pin joint between the link and the ground. It should be obvious that $V_A / R_{A,I_{1,j}} = V_B / R_{B,I_{1,j}} = V_C / R_{C,I_{1,j}} = \omega$.

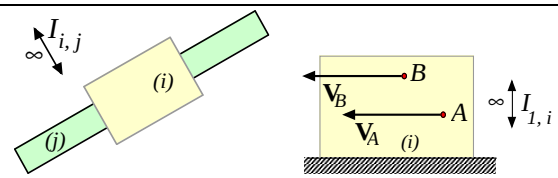


Note: If we know the velocity (absolute) of a point on a link, the instant center between that link and the ground must be located on an axis perpendicular to the velocity vector passing through the point.



Two Links Connected by A Sliding Joint

The instant center between two links that are connected by a sliding joint is located in infinity on any axis perpendicular to the sliding axis. The reason for this instant center being in infinity will be discussed later.



Number of Instant Centers

In a mechanism with *n* links (count the ground as one of the links), the number of instant centers is determined as:

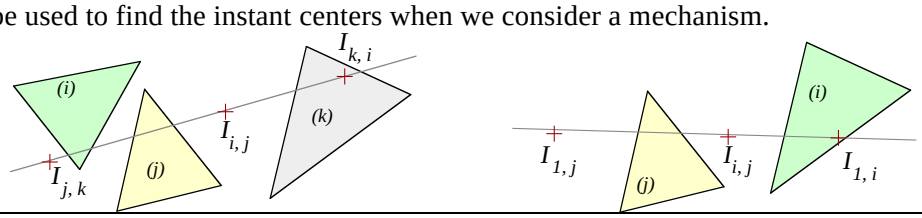
$$C = \frac{n(n-1)}{2}$$

As an example, in a four-bar mechanism or a slider-crank, there are six IC's ($n = 4$). For any six-bar mechanism, $C = 15$.

Kennedy's Rule

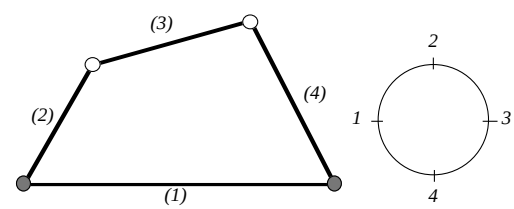
The three instant centers between three planar links must lie on a straight line.

This rule does not tell us where the line is or where the centers are on that line. However, the rule can be used to find the instant centers when we consider a mechanism.

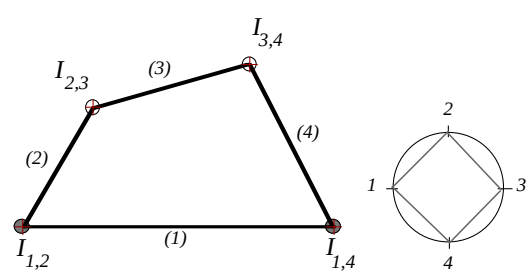


Instant Centers of A Four-bar

A four-bar mechanism has six instant centers regardless of the dimensions or orientation of the links. For bookkeeping purposes in locating the IC's, we draw a circle and place link indices on the circle in any desired order. This bookkeeping procedure may not be necessary for a four-bar, but becomes very useful when mechanisms with greater number of links are considered.

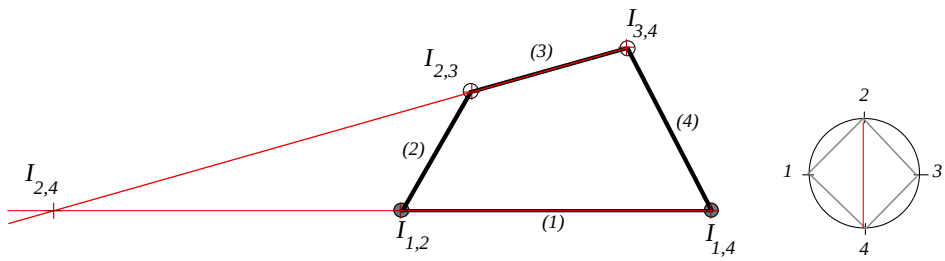


Since pin joints are instant centers, for a four-bar with four pin joints, four IC's are immediately identified. Each found IC is marked on the circle as a line drawn between the two corresponding link indices. These four IC's are actual (not imaginary) pin joints. In order to find the other IC's, we apply Kennedy's rule.

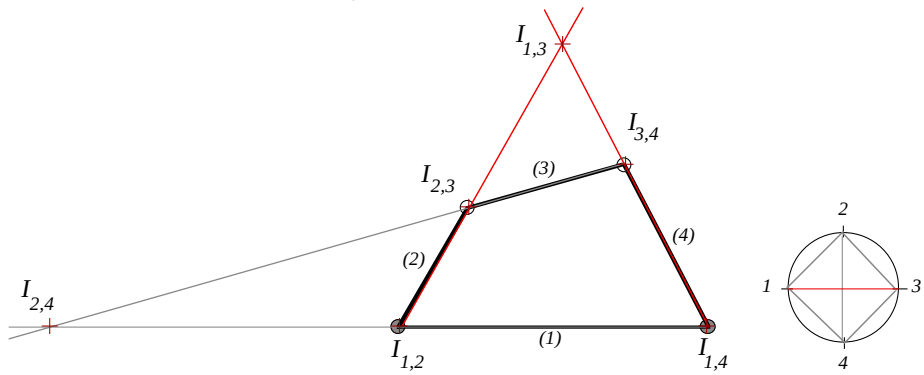


The IC's between links *2*, *3* and *4* must lie on a straight line. These are $I_{2,3}$, $I_{3,4}$, and $I_{2,4}$. Since we already have $I_{2,3}$ and $I_{3,4}$, we draw a line through them; $I_{2,4}$ must also be on this line. The IC's between links *1*, *2* and *4* must lie on a straight line. These are $I_{1,2}$, $I_{1,4}$, and $I_{2,4}$. Since we already have $I_{1,2}$ and $I_{1,4}$, we draw a line through them; $I_{2,4}$ must also be on this line. The intersection of these two lines is $I_{2,4}$.

Note how the circle is used to decide which center to find next. The red line between links *2* and *4* indicates the center we are after. This line is shared between two triangles with known IC's. The triangles tell us to draw a line between $I_{1,2}$ and $I_{1,4}$, then draw another line between $I_{2,3}$ and $I_{3,4}$. The intersection is $I_{2,4}$.



According to the circle, the last center to find is between links *1* and *3*. The two triangles that share this new red line tell us to draw a line between $I_{1,2}$ and $I_{2,3}$, and a second line between $I_{1,4}$ and $I_{3,4}$. The intersection of these two lines is $I_{1,3}$.



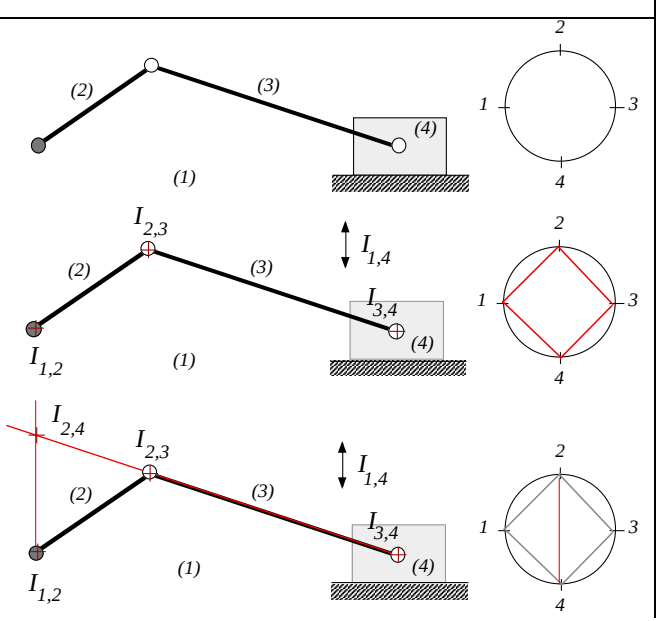
Now we have found all six centers.

Instant Centers of A Slider-crank

A slider-crank mechanism has six instant centers regardless of which inversion it is. Again, for bookkeeping purposes, we draw a circle with link indices.

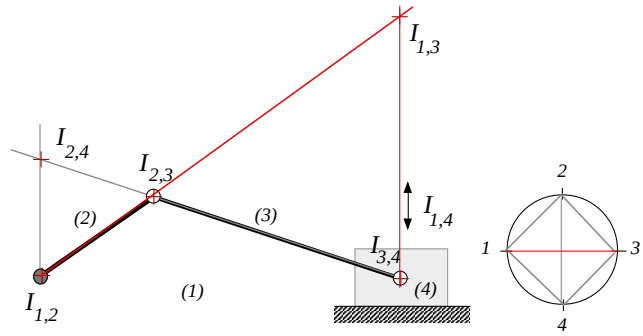
Pin joints provide three of the instant centers, $I_{1,2}$, $I_{2,3}$, and $I_{3,4}$. The center between the slider block and the ground, $I_{1,4}$, is in infinity on an axis perpendicular to the sliding axis.

$I_{2,4}$ must lie on the axis of $I_{2,3}$ and $I_{3,4}$, and on the axis of $I_{1,2}$ and $I_{1,4}$. The intersection of these two axes is $I_{2,4}$.



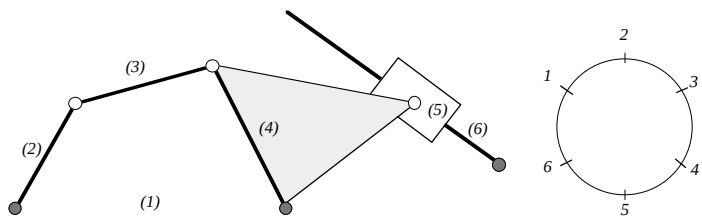
$I_{1,3}$ must lie on the axis of $I_{2,3}$ and $I_{1,2}$, and on the axis of $I_{3,4}$ and $I_{1,4}$. Note that $I_{1,4}$ is in infinity on an axis perpendicular to the slider. The intersection of these two axes is $I_{1,3}$.

Now we have all six centers.

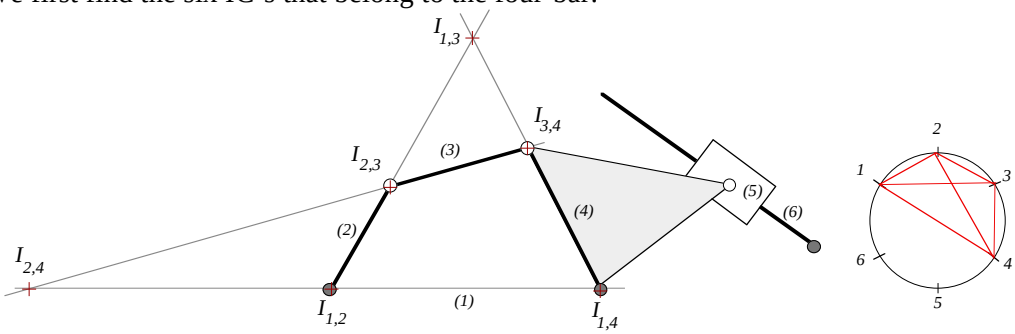


Instant Centers of A Six-bar

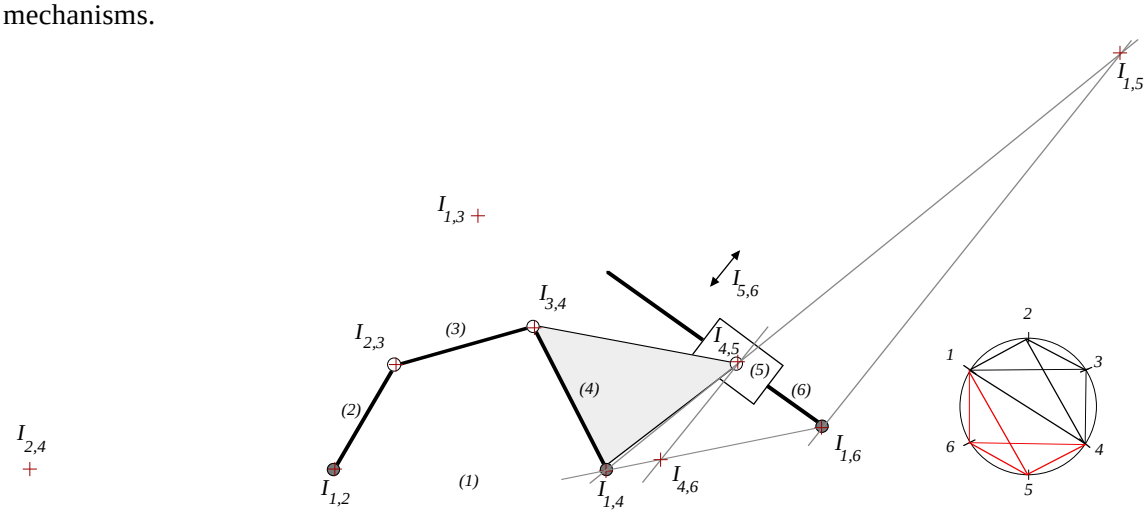
In this example we consider a six-bar mechanism containing a four-bar and an inverted slider-crank that share one link and one pin joint. A circle is constructed with link indices *1* – 6.



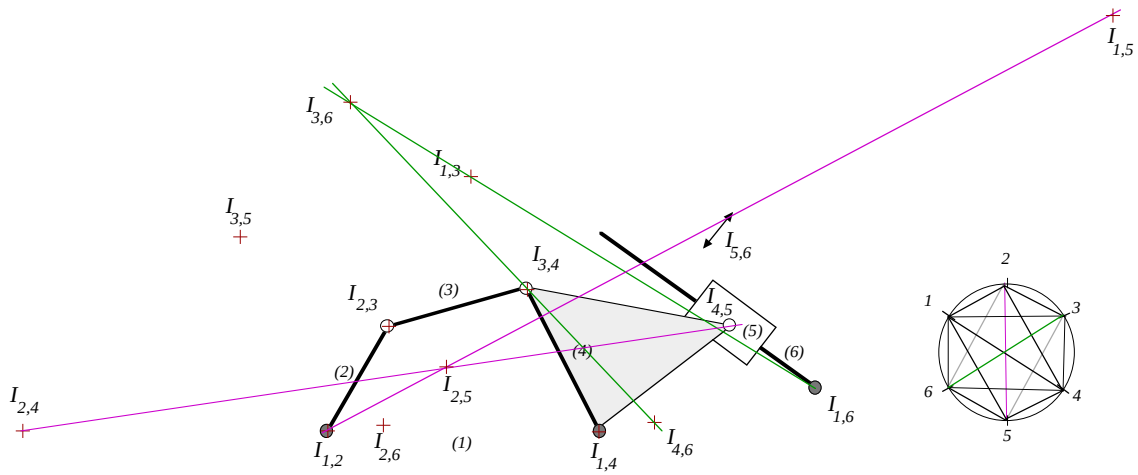
We first find the six IC's that belong to the four-bar.



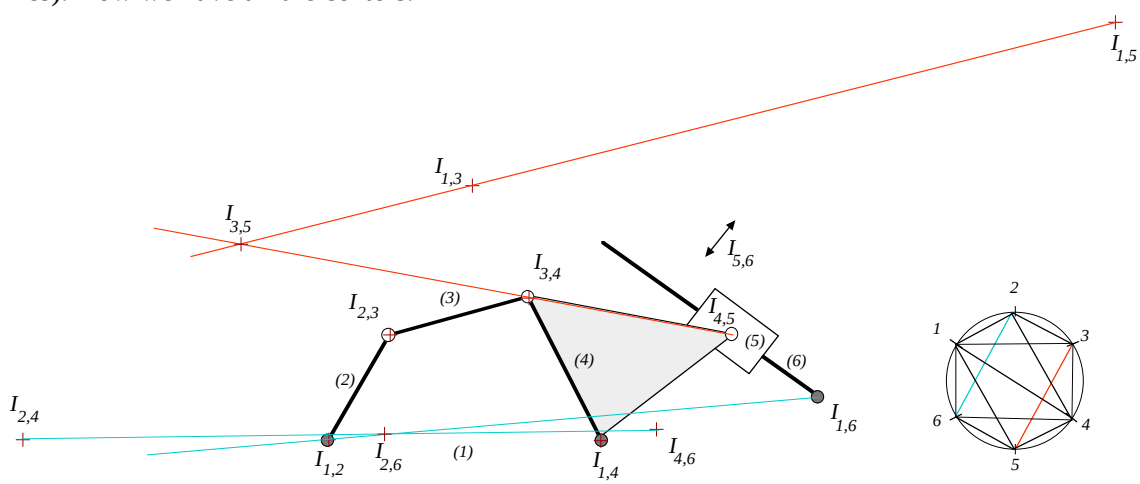
Next we find the IC's for the slider-crank. Note that $I_{1,4}$ is shared between the two sub-mechanisms.



Next, we use the circle to guide us in finding the next IC. $I_{2,6}$ must be on the intersection of lines $I_{2,4}$ - $I_{4,6}$ and $I_{1,2}$ - $I_{1,6}$ (blue lines). $I_{3,5}$ is found at the intersection of lines $I_{1,3}$ – $I_{1,5}$ and $I_{3,4}$ – $I_{4,5}$ (red lines).



The next IC to find is $I_{3,6}$. This center is at the intersection of lines $I_{3,4} - I_{4,6}$ and $I_{1,3} - I_{1,6}$ (green lines). The last center, $I_{2,5}$, is found at the intersection of $I_{2,4} - I_{4,5}$ and $I_{1,2} - I_{1,5}$ (purple lines). Now we have all the centers.

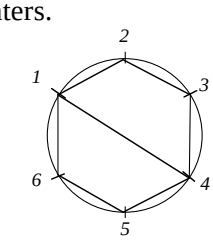


Strategy

The instant center method is a graphical process to perform velocity analysis. A graphical process is a pencil-and-paper approach that requires locating points, drawing lines, finding intersections, and finally taking direct length measurements from the drawing. All of these steps have graphical and measurement errors. Therefore, the accuracy of the analysis depends on the accuracy of our drawings and measurements.

For four-bars and slider-cranks, since four links are involved, there are only six centers to locate. For mechanisms with more links that four, there are many more centers to find. Locating some of the centers requires using some of the other centers that have already been found. The following strategy can reduce the graphical error in locating some of the centers.

Let us use the previous six-bar mechanism as an example. The first seven centers that we locate are at the center of the pin joints. Marking these centers by hand on a diagram contain certain amount or error that we call Order-1 level:



$O-1$: $I_{1,2}, I_{2,3}, I_{3,4}, I_{1,4}, I_{4,5}, I_{5,6}, I_{1,6}$

Next we locate $I_{1,3}$, $I_{2,4}$, $I_{4,6}$ and $I_{1,5}$ using the first seven centers.

These centers add more errors on top of the errors from the original seven. We consider these new centers to contain errors at Order-2 level:

$O-2: \quad I_{1,3}, I_{2,4}, I_{4,6}, I_{1,5}$

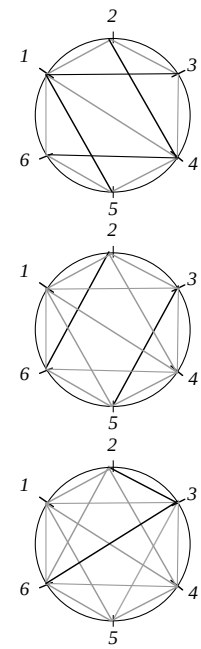
Next we locate $I_{2,6}$ and $I_{3,5}$ using centers with *O*-1 and *O*-2 level errors. Therefore these two centers contain their own graphical error on top of the errors from the other centers:

$O-3: \quad I_{2,6}, I_{3,5}$

Up to this point we did not have any other choices in how to locate the centers, but for the remaining centers we may have more than one choice. For example, to locate $I_{3,6}$ we can use the intersection between any two of these four axes: $I_{3,4}$ - $I_{4,6}$, $I_{1,3}$ - $I_{1,6}$, $I_{2,3}$ - $I_{2,6}$, and $I_{3,5}$ - $I_{5,6}$. Considering the error level in $I_{2,6}$ and $I_{3,5}$, we should not use $I_{2,3}$ - $I_{2,6}$, and $I_{3,5}$ - $I_{5,6}$ axes. Instead, we should use the intersection of $I_{3,4}$ - $I_{4,6}$ and $I_{1,3}$ - $I_{1,6}$ to locate $I_{3,6}$:

$O-3: \quad I_{3,6}$

Note: When locating a new center, use existing centers with the lowest amount of error.

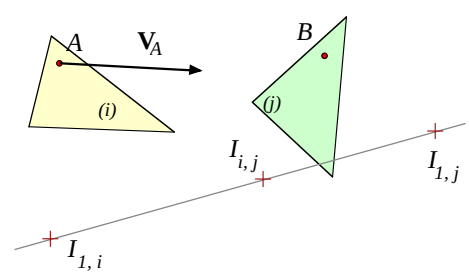


Determining Unknown Velocities

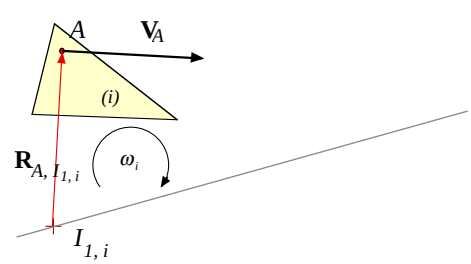
Instant centers are used to determine unknown velocities in a mechanism. Typically the process requires finding the velocity of a point on one link, based on known velocity of a point on another link. The process, in its most efficient form, requires using three instant centers—the centers between the two links and the ground.

Assume that the location of the three instant centers between links *i*, *j*, and the ground (link *1*), and the velocity of point *A* on link *i* are given. The objective is to determine the velocity of point *B* on link *j*.

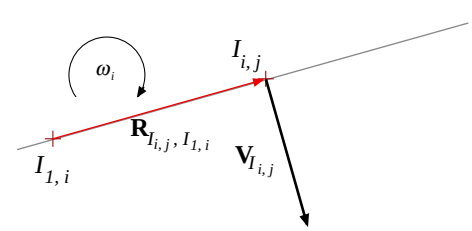
To determine the velocity of *B*, we consider the following four steps:



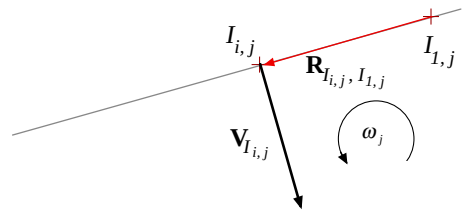
1. We start with the link that has a known velocity. Link *i* rotates about an imaginary pin joint at $I_{1,i}$. We construct vector $\mathbf{R}_{A,I_{1,i}}$ and determine its magnitude. We compute the angular velocity of link *i* as $\omega_i = V_A / R_{A,I_{1,i}}$. The direction of this angular velocity is CW.



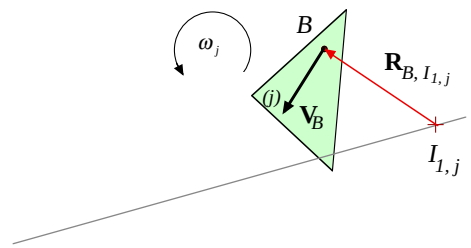
2. The instant center $I_{i,j}$ is an imaginary point on link *i*, and therefore we can determine its velocity. We measure the length of vector $\mathbf{R}_{I_{i,j},I_{1,i}}$ and compute $V_{I_{i,j}} = \omega_i R_{I_{i,j},I_{1,i}}$. The direction of $\mathbf{V}_{I_{i,j}}$ is established based on the direction of ω_i.



3. The instant center $I_{i,j}$ is also an imaginary point on link *j*, and we already know $\mathbf{V}_{I_{i,j}}$. Link *j* rotates about the imaginary pin joint to the ground at $I_{1,j}$. We measure the length of vector $\mathbf{R}_{I_{i,j},I_{1,j}}$, then $\omega_j = V_{I_{i,j}} / R_{I_{i,j},I_{1,j}}$ is computed. The direction of ω_j is established to be CCW.



4. Point *B* is attached to link *j* that rotates about an imaginary pin joint at $I_{1,j}$. We construct vector $\mathbf{R}_{B,I_{1,j}}$ and determine its magnitude. We then compute $V_B = \omega_j R_{B,I_{1,j}}$. The direction of $\mathbf{V}_B$ is established based on the direction of the angular velocity of link *j*.



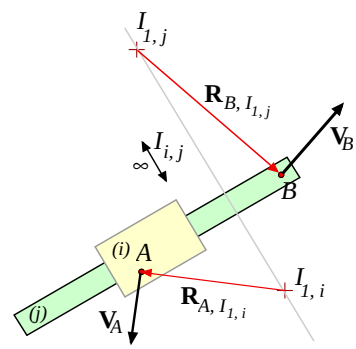
These four steps, either as they are presented or with slight variations, can be applied to find any unknown velocities in mechanisms. Here is another example of applying these four steps when an instant center is in infinity.

Assume that the centers between two links connected by a sliding joint and the ground, and the velocity of *A* on link *i* are given. The objective is to find the velocity of *B* on link *j*. Here are the four steps, slightly revised:

1. Link *i* is pinned (imaginary) to the ground at $I_{1,i}$. The angular velocity of link *i* is computed as $\omega_i = V_A / R_{A,I_{1,i}}$, CCW.

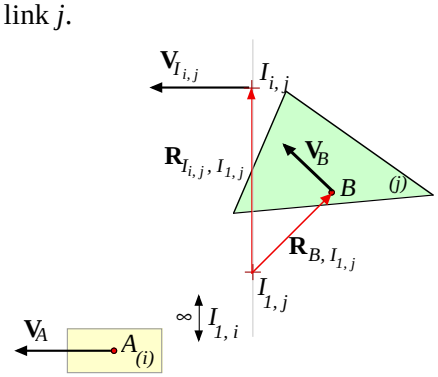
2-3. The two links are connected by a sliding joint, therefore $\omega_j = \omega_i$.

4. Link *j* rotates about $I_{1,j}$. Velocity of *B* is computed as $V_B = \omega_j R_{B,I_{1,j}}$. The direction is established based on the direction of the angular velocity.



In a third example link *i* is connected to the ground by a sliding joint. The velocity of *A* on link *i* is given and the objective is to find the velocity of *B* on link *j*.

1. Since link *i* slides relative to the ground, $\omega_i = 0$.
2. $I_{i,j}$ is a point on link *i*, therefore $\mathbf{V}_{I_{i,j}} = \mathbf{V}_A$.
3. Link *j* rotates with respect to the ground about $I_{1,j}$. The angular velocity of link *j* is computed as $\omega_j = V_{I_{i,j}} / R_{I_{i,j},I_{1,j}}$, CCW.
4. The velocity of *B* is computed as $V_B = \omega_j R_{B,I_{1,j}}$ in the direction shown.

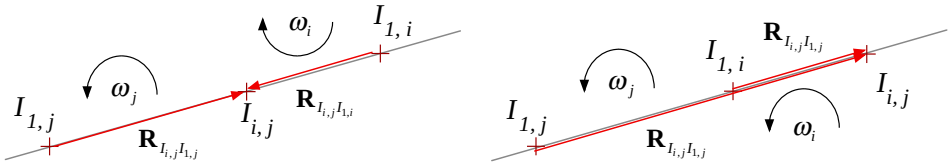


Angular Velocity Ratio

A formula, known as the angular velocity ratio, can be derived between the angular velocities of any two links of a mechanism regardless of the type of joints used in that mechanism. This formula, in general, can be established for links *i* and *j*. Between these two links and the ground link *1*, there are three instant centers $I_{1,i}$, $I_{i,j}$, and $I_{1,j}$. The velocity of the common center $I_{i,j}$

can be determined as $V_{I_{i,j}} = R_{I_{i,j}I_{1,i}}\omega_i = R_{I_{i,j}I_{1,j}}\omega_j$. Therefore, the angular velocity ratio between the two moving links is expressed as

$$\frac{R_{I_{i,j}I_{1,i}}}{R_{I_{i,j}I_{1,j}}} = \frac{\omega_j}{\omega_i}$$



(a) (b)

Note: If $I_{i,j}$ is between $I_{1,i}$ and $I_{1,j}$, as in (a), ω_i and ω_j are in opposite directions ($\mathbf{R}_{I_{i,j}I_{1,i}}$ and $\mathbf{R}_{I_{i,j}I_{1,j}}$ are in opposite directions). But if $I_{i,j}$ is not located between $I_{1,i}$ and $I_{1,j}$, as in (b), ω_i and ω_j are in the same direction ($\mathbf{R}_{I_{i,j}I_{1,i}}$ and $\mathbf{R}_{I_{i,j}I_{1,j}}$ are in the same direction).

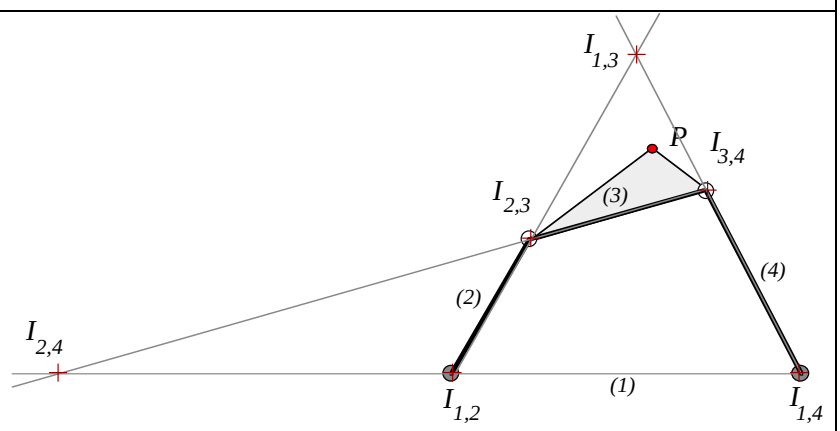
In the following examples we use instant centers to perform velocity analysis for several mechanisms. It is always assumed that either the angular velocity of one link or the linear velocity of one point is given.

Four-bar Mechanism

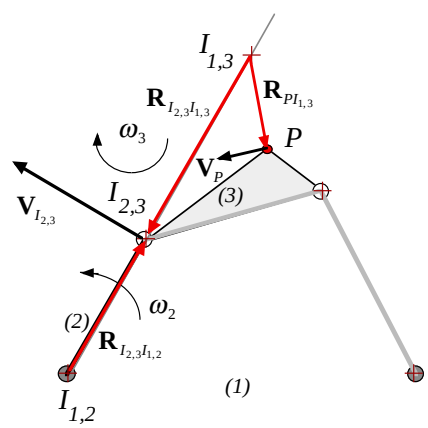
For this four-bar mechanism, we have already found the instant centers. Assume the angular velocity of link *2* is given, CCW. Find:

(a) the velocity of point *P* on link *3*, and

(b) the angular velocity of link *4*.

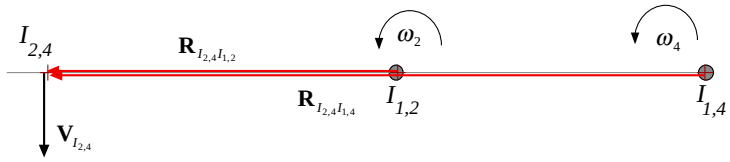


(a) The angular velocity of link *2* is known, and we want to find the velocity of point *P* on link *3*. We need to pick a third link, and that link is always the ground, link *1*. So, we pick the three centers between these three links: $I_{1,2}$, $I_{2,3}$, and $I_{1,3}$. (We can ignore the other centers and links at this point.) Link *2* rotates about $I_{1,2}$. The magnitude of the velocity of $I_{2,3}$ is computed as $V_{I_{2,3}} = \omega_2 R_{I_{2,3}I_{1,2}}$ and its direction is shown. $I_{2,3}$ is also a point on link *3*, and link *3* rotates about $I_{1,3}$. The angular velocity of link *3* is computed as $\omega_3 = V_{I_{2,3}} / R_{I_{2,3}I_{1,3}}$, CW. (We could have also used the angular velocity ratio formula to find ω_3.) Link *3* rotates about $I_{1,3}$, therefore velocity of *P* is computed as $V_P = \omega_3 R_{P,I_{1,3}}$. The direction is shown on the diagram.



b) In order to move from link *2* to link *4*, we only need $I_{1,2}$, $I_{2,4}$, and $I_{1,4}$. Link *2* rotates about $I_{1,2}$. The magnitude of the velocity of $I_{2,4}$ is $V_{I_{2,4}} = \omega_2 R_{I_{2,4}I_{1,2}}$ and its direction is as shown. $I_{2,4}$ is also a point on link *4*, and link *4* rotates about $I_{1,4}$. The angular velocity of link *4* is $\omega_4 = V_{I_{2,4}} / R_{I_{2,4}I_{1,4}}$, CCW.

(We could have also used the angular velocity ratio formula.)

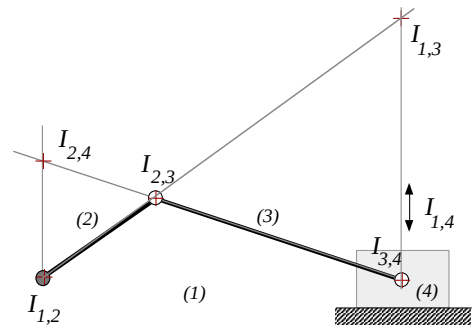


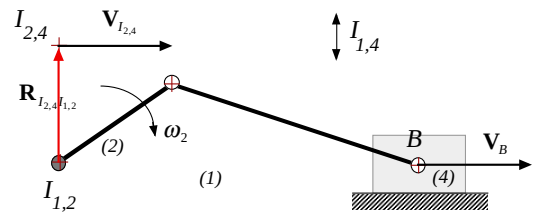
Slider-crank (inversion 1)

The instant centers of this slider-crank have already been located. Assume the angular velocity of link *2* is given, CW. The objective is to find the velocity of link *4*.

Since we have the angular velocity of link *2* and we are interested in the velocity of link *4*, we pick the instant centers $I_{1,2}$, $I_{2,4}$, and $I_{1,4}$.

The center $I_{2,4}$ is a point on link *2*. The magnitude of its velocity is computed as $V_{I_{2,4}} = \omega_2 R_{I_{2,4}I_{1,2}}$, and its direction is as shown. $I_{2,4}$ is also a point on link *4*. Since link *4* does not rotate, all the points on this link have the same velocity. Therefore, $\mathbf{V}_B = \mathbf{V}_{I_{2,4}}$.



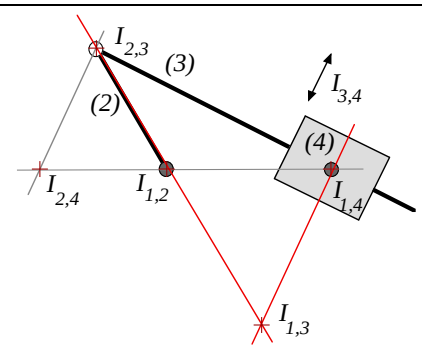


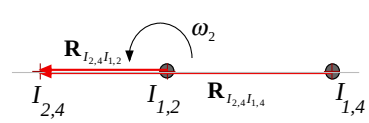
Slider-crank (inversion 3)

For the third inversion of the slider-crank mechanism, the angular velocity of link *2* is given, CCW. We are asked to find the angular velocity of link *4*.

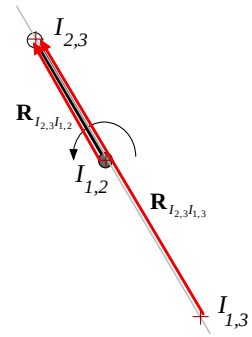
The six instant centers are found as shown. We can determine the angular velocity of link *4* two different sets of instant centers.

(1) We use the instant centers $I_{1,2}$, $I_{2,4}$, and $I_{1,4}$. The angular velocity formula yields $\omega_4 = \omega_2 R_{I_{2,4}I_{1,2}} / R_{I_{2,4}I_{1,4}}$, CCW.



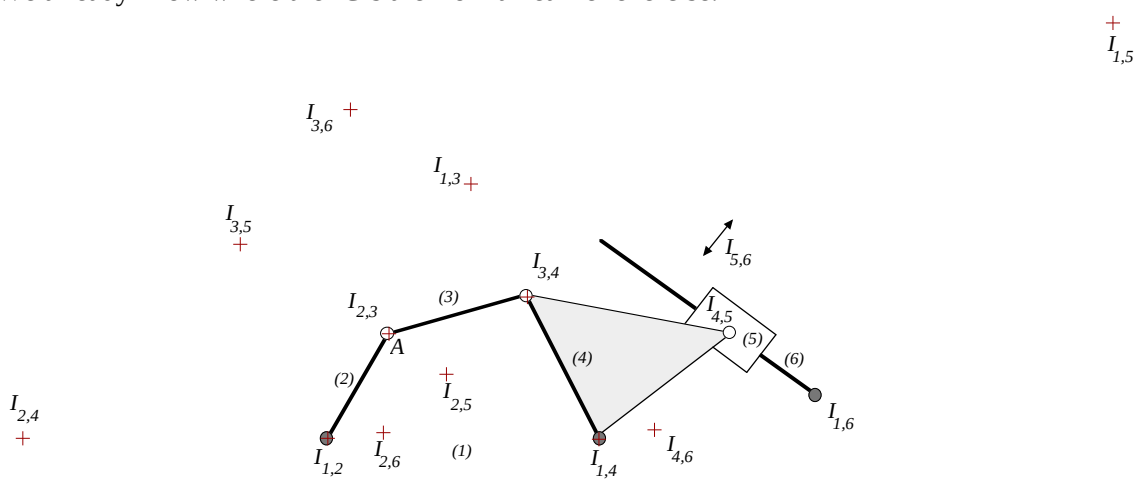


(2) We use the instant centers $I_{1,2}$, $I_{2,3}$, and $I_{1,3}$. The angular velocity formula yields $\omega_3 = \omega_2 R_{I_{2,3}I_{1,2}} / R_{I_{2,3}I_{1,3}}$, CCW. Since links *3* and *4* form a sliding joint, they have the same angular velocities. Therefore, $\omega_4 = \omega_3$, CCW.



Six-bar Mechanism

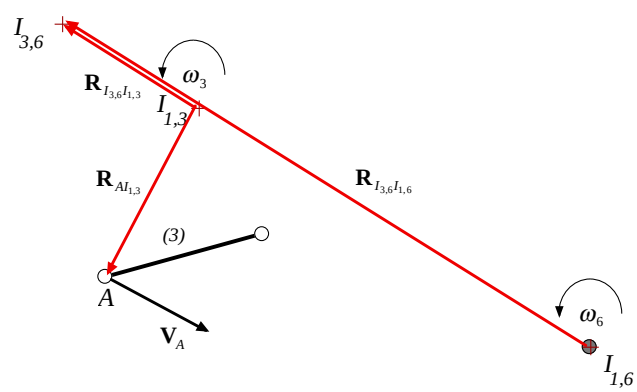
Assume that for this six-bar mechanism the angular velocity of link *6* is given in the CCW direction. We are asked to find (a) the angular velocity of link *3* and (b) the velocity of point *A*. We already know where the IC's are from an earlier exercise.



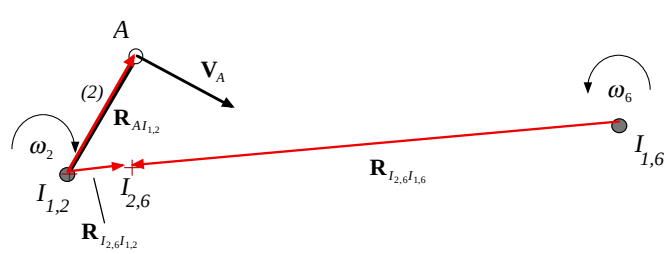
(a) The three IC's between links *6*, 3, and *1* are: $I_{1,6}$, $I_{3,6}$, and $I_{1,3}$. The angular velocity ratio formula yields $\omega_3 = \omega_6 R_{I_{3,6}I_{1,6}} / R_{I_{3,6}I_{1,3}}$, CCW.

(b) We can determine the velocity of point *A* using two different ways:

(1) Point *A* is a point on link *3* which rotates about $I_{1,3}$. The magnitude of the velocity of *A* is $V_A = \omega_3 R_{AI_{1,3}}$ and its direction is perpendicular to $\mathbf{R}_{AI_{1,3}}$ as shown.



(2) Since *A* is also a point on link *2*, we can use $I_{1,6}$, $I_{2,6}$, and $I_{1,2}$. We use the angular velocity ratio formula to determine ω_2, then we determine the velocity of *A*.



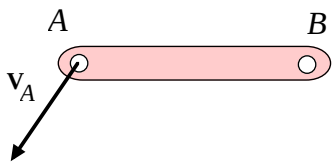
Exercises

In these exercises take direct measurements from the figures for link lengths and the magnitudes of velocity vectors. If it is stated that the angular velocity is known, assume $\omega = 1$ rad/sec CCW, unless it is stated otherwise.

P.1

$\mathbf{V}_A$ and ω are known.

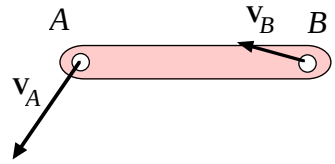
Determine $\mathbf{V}_B$.



P.2

$\mathbf{V}_A$ and $\mathbf{V}_B$ are known.

Determine ω.



P.3

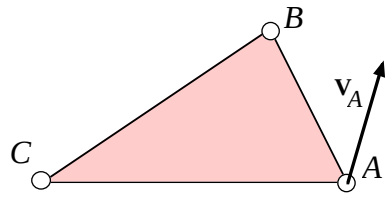
$\mathbf{V}_A$ and $\mathbf{V}_B$ are known.

Determine ω. What do you observe?

P.4

$\mathbf{V}_A$ and ω are known.

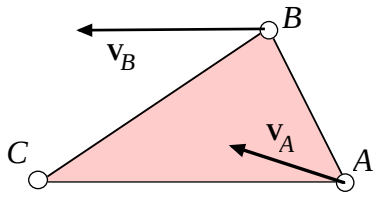
Determine $\mathbf{V}_B$, $\mathbf{V}_C$ and $\mathbf{V}_{BC}$.



P.5

$\mathbf{V}_A$ and $\mathbf{V}_B$ are known.

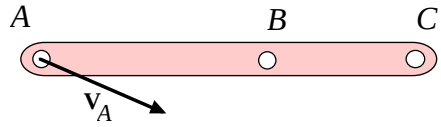
Determine $\mathbf{V}_C$.



P.6

$\mathbf{V}_A$ and ω are known.

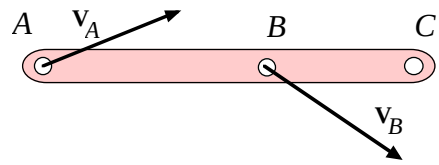
Determine $\mathbf{V}_B$, $\mathbf{V}_C$ and $\mathbf{V}_{BC}$.



P.7

$\mathbf{V}_A$ and $\mathbf{V}_B$ are known.

Determine $\mathbf{V}_C$.

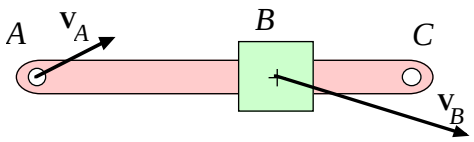


The following exercises (P.8 – P.10) are not typical velocity analysis problems for using the instant center method. They are provided to make you think, to apply the fundamentals of the IC method back and forth, and to better understand the concept and the meaning of the instant centers. The solution to some of the following four exercises can be tricky!

P.8

$\mathbf{V}_A$ and $\mathbf{V}_B$ are known.

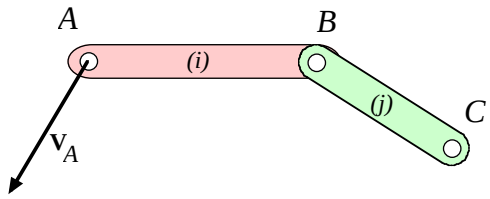
Determine ω and $\mathbf{V}_C$.



P.9

$\mathbf{V}_A$, ω_i and ω_j are known. Assume $\omega_i = 1$ rad/sec CCW and $\omega_j = 1$ rad/sec CW.

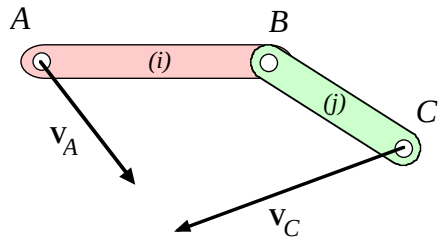
Determine $\mathbf{V}_B$ and $\mathbf{V}_C$.



P.10

$\mathbf{V}_A$ and $\mathbf{V}_C$ are known.

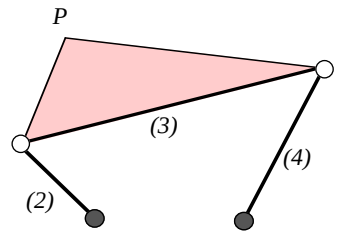
Determine $\mathbf{V}_B$.



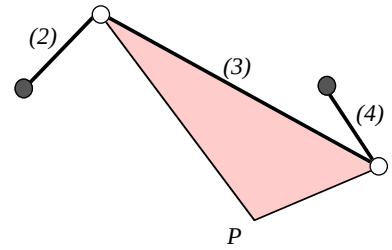
The following exercises are typical problems using the instant center method. Each exercise is a complete mechanism.

Exercises P.11 – P.14 are examples of four-bar mechanism. In each problem, find the instant centers. Assume ω_2 is given, then determine ω_3, ω_4, and $\mathbf{V}_P$.

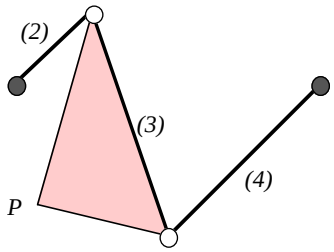
P.11



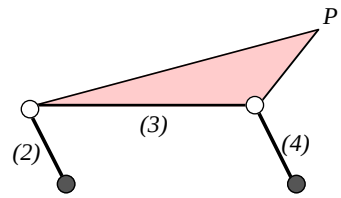
P.12



P.13



P.14

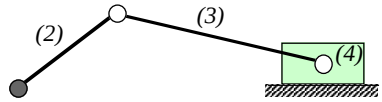


Exercises P.15 – P.18 are examples of slider-crank mechanism. In each problem, find the instant centers. Assume ω_2 is given, then:

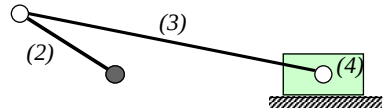
For P.15 and P.16 determine ω_3, ω_4, and the velocity of the slider block;

For P.17 and P.18 determine ω_3, ω_4, and $\mathbf{V}_P$.

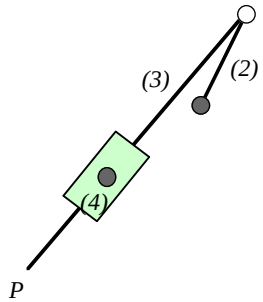
P.15



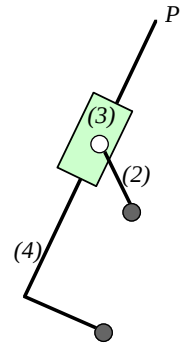
P.16



P.17

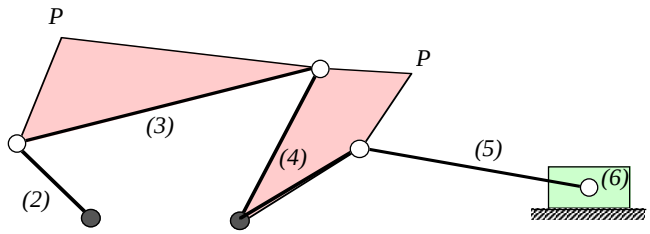


P.18

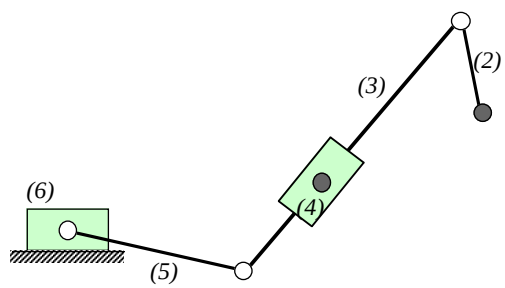


For these six-bar mechanisms ω_2 is given. Find the instant centers. Determine ω_5, and the velocity of the slider block *6*. In P.19, also find the velocity of *P*.

P.19

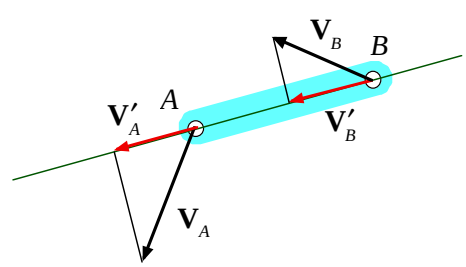


P.20



A Useful Observation

If we consider the absolute velocity of two points on the same link, and project the velocity vectors on the axis that connects the two points, we make the observation that the two projected velocity vectors are equal in magnitude and are in the same direction. Why are the two projected velocity components equal? Because the link is non-deformable—the two points cannot get closer to or move away from one another. In other words, the two points must have identical velocities along the axis that connects them.



This observation can be used to check whether the answer to a computed velocity is incorrect or not. We can also use this observation to find unknown velocities in some problems. For example, in a four-bar mechanism, based on the known velocity of point *A*, we can easily determine the velocity of point *B*. This is performed by projecting $\mathbf{V}_A$ on the axis between *A* and *B* to find $\mathbf{V}'_A$, setting $\mathbf{V}'_B = \mathbf{V}'_A$, and then constructing $\mathbf{V}_B$ on an axis perpendicular to $\mathbf{R}_{BO_4}$.