Abstract:

Diffraction gratings with sub-wavelength spatial frequencies can reduce the reflectivity at an interface. The reflectivity at the interface depends on a number of conditions: filling factor, groove depth, angle of incidence, and polarization. This paper explores the results of research done comparing approximate theory and experimental results. The conditions for zero reflectivity are covered. The benefits of using a diffraction grating are compared to that of thin films, or expensive exotic index matching material.

The search for non-reflecting surfaces is one that is of great importance to the optical community as a whole. Often anti-reflection (hereby referred to as AR) coatings are thin films, or a layer of specific index glass. Each of these AR coatings have their own problems; thin films are easily damaged; and it may be difficult or very expensive to create a batch of an unusual index. Diffraction gratings with the right characteristics are capable of generating the same (or better) effects, and can be applied directly to the surface of the optical surface.

Before understanding the theory behind the phenomenon it is necessary to know some of the basic properties of a diffraction grating.

Groove depth is referred to as $d$ (Fig.1). The fill factor $F$ is referred to as the fraction of the grating period that is filled with the grating material (Fig.1).

For normal incidence where the solutions to the following grating equation:
\[
m \frac{\lambda}{n_1} = \Lambda (\sin \theta' + \sin \theta_{m}') \quad \text{Eq.1}
\]

\(m\) is the order number. \(\lambda\) is the wavelength of the incident light. The indices at the interface are given by \(n_1\) and \(n_2\). The incident angle in material one is given by \(\theta'\), and reflected waves of \(m^{th}\) order are given by \(\theta_{m}''\).

When only the \(m = 0\) term is real, the orders of \(|m| > 1\) are imaginary (Fig. 2). This means that only the zero order terms are used for transmission and reflection.

The use of rigorous coupled wave analysis shows that the optical absorption and index varies periodically between the two planes that make up the boundary of the grating. The \(E\) field inside the grating is expanded in terms of space-harmonic components. After more operations, the expansion is put into the appropriate wave equation and an infinite set of coupled waves are formed. The set of second order coupled waves is then converted to a single set of doubly infinite equations. Eigen values and eigen vectors are then solved for from a differential coefficient matrix. Using the boundary values given because the tangential components of the \(E\) and \(H\) field are continuous, two linear equations are found. These equations are used to calculate the amplitudes of the propagation orders. From these amplitudes, the diffraction efficiency can be calculated. The diffraction efficiency is the quality of the grating and is needed to figure what other parameters must be set to for zero reflectance. For the purpose of obtaining zero reflectance, diffraction efficiency, angle of incidence, wavelength and polarization must be known.

When an electromagnetic wave is incident on the grating it produces both transmitted and
reflected light. The angle of transmitted light is given by the equation

\[ m \lambda = \lambda (n_1 \sin \theta' + n_2 \sin \theta_{m}') \]  
Eq. 2

\( m \) is the order of diffraction. \( \lambda \) is the vacuum wavelength of the incident light. \( n_1 \) is the index of the substrate under the grating and \( n_2 \) is the index of the grating material.

If the vacuum wavelength and angle of incidence satisfy Bragg's condition\(^8\) the equation for reflected waves simplifies to: 
\[ m \lambda / n_1 = 2 \Lambda \sin (\theta') \] . For the case when the \( \vec{E} \perp \vec{K} \), the \( E \) field is parallel to the grating grooves\(^5\). In the case where the \( \vec{E} \parallel \vec{K} \), the magnetic field is parallel to the grating grooves\(^2\). It has been seen that the reflectance is zero for both cases\(^1\). This means that under the right conditions the gratings can be used as an AR coating. The index of the grating can be approximated by considering them as a homogeneous single layer thin film. This approximate is the average value over a grating period, which is dependent on the two other indices and the filling factor. The in air the equation is 
\[ \bar{n} = n_1 + F (n_2 - n_1) \] . Since this equation does not take into account the polarization state of the light a different set of formulas is needed\(^5\). For this the boundary conditions of \( \vec{E} \) and the electric displacement \( \vec{D} \) are needed. Since \( E \) tangential is continuous, the index equation becomes 
\[ n_{(\vec{E} \perp \vec{K})} = [n_1^2 (1-F) + n_2^2 F]^{1/2} \] if \( \vec{E} \perp \vec{K} \) (Fig.3)\(^2\). When \( \vec{E} \parallel \vec{K} \) the index equation becomes 
\[ n_{(\vec{E} \parallel \vec{K})} = [(1-F)/n_1^2 + F/n_2^2]^{1/2} \] (Fig.4)\(^2\).

\[ \text{Effective index vs Fill factor for } \vec{E} \perp \vec{K} \text{ polarized light as a function of fill factor} \]

\[ \text{Effective Index vs Fill Factor for } \vec{E} \parallel \vec{K} \text{ polarized light as a function of fill factor} \]

In the long wavelength area such as the mid to far IR, the approximation as a thin homogeneous layer is a good estimate of \( \vec{E} \perp \vec{K} \) and \( \vec{E} \parallel \vec{K} \) as the wave length become larger than the grove depth and grating width\(^1\). The groove density \( d = \lambda / 4 (n_1 n_2)^{1/2} \) becomes a function of the desired index needed for an AR coating \( (n_1 n_2)^{1/2} \). An example uses an air to
glass interface $n_1=1, n_2=1.5$, where if we were using solid glass it would need an index of 1.227. Using the formula for $d$ the calculated thickness required is $0.20412 \lambda$. The calculated groove density $\frac{d}{\lambda}$ is 0.204, which is a agrees approximate compared to $0.20412 \lambda$. Through boundary conditions the filling factor for $\vec{E} \perp \vec{K}$ becomes $F = n_1/(n_1 + n_2)$, and for $\vec{E} \parallel \vec{K}$ the filling factor becomes $F = n_2/(n_1 + n_2)$. If the indicies nearly equal, the filling factor is ~50%. As the difference in indicies increases the filling factor required for each polarizations diverge. If the polarization is random the lowest reflectivity is reached at $F \sim 50\%$.

**Conclusion:**

Under specific conditions, diffraction gratings with sub-wavelength spatial frequencies have zero reflectance. Reflectance is dependent on groove depth, fill factor, and the incident polarization of the light (as well as incident angle). The index of the gratings can be approximated by treating them as a single homogeneous layer, where the index is an average over the grating period. Having long wavelength allows for the calculation of fill factor required for the grating to act as an AR coating, depending on the polarization of the incident light. In the mid to far range infrared (IR), specific polarizations have better lasing properties for applications in metal working\(^6\). In the 10.6μm range, an AR diffraction grating on an optical surface can replace a costly quarter wave retarder\(^6\). The gratings have application ranging from AR coating, to transmission increasing polarizing coating.

**Biography:**

M. G. Moharam currently does research at CREOL in the area of holograms and diffraction elements. He has published many papers on the different aspects of diffraction gratings, including several on the subject of zero reflectivity gratings. His research in this area goes back to the early 80's. In January 2001 he won the Teaching Incentive Program (TIP) Award\(^4\).
Citations:


