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Problem Set 8

1. Problem 19.2.
2. Problem 19.3.
3. Problem 19.5.
4. Problem 19.7.
5. Let's do a concrete example illustrating the use of the Envelope Theorem. Return to Example 17.3, about a price-discriminating monopolist. Since the demand and cost functions are linear, there are six possible parameters we could vary in this problem: the slope and intercept of the inverse demand curve in market 1, the slope and intercept of the inverse demand curve in market 2, and the slope and intercept of the cost function.
    a. As a benchmark, compute how much profit the firm makes at the optimal solution in this worked-out example.
    b. Now suppose that the intercept of the inverse demand curve in market 2 changes. Replace the 100 with the parameter a, and rewrite the profit function. Use the Envelope Theorem to compute how much the monopolist's profit would change with a marginal increase in a from 100 to something slightly larger than 100. In other words, compute the derivative of the optimal value of F with respect to a.
    c. To see how convenient it was to use the Envelope Theorem, redo the calculation in (b) by doing things more explicitly. Solve for the optimal Q_1 and Q_2 as a function of a, and then plug these values into the profit function to find out the optimal profit as a function of a. Finally, take the derivative of the optimal profit with respect to a. (Notice how many more calculations are required by this method than in (b), but see that you get the same answer both ways.)
    d. Now consider a different parameter. This time, suppose we change the slope of the cost function from 20 to some number c slightly greater than 20. What is the marginal change in the monopolist's profit as the parameter c increases? Use the Envelope Theorem to obtain your result.
    e. Check your answer to (d) by doing things the long way: compute optimal profit as a function of the parameter c, then take the derivative directly, as in part (c).
6. Problem 19.10.
7. Problem 19.11.
8. Problem 19.13.
9. In Example 18.6, Simon and Blume found four candidates that satisfied the first-order conditions. Check the second-order conditions at each of these four points. According to the SOCs, which of the points are local maxima and which are local minima?
10. In Example 18.7, Simon and Blume again found four candidates that satisfied the first-order conditions. Check the second-order conditions at each of these four

points. According to the SOCs, which of the points are local maxima and which are local minima?

11. In Example 18.8, Simon and Blume find the first-order conditions for a utility-maximization problem. Write down the second-order sufficient conditions for a maximum. Show that "diminishing marginal utility" will guarantee that the first-order conditions produce a local maximum.
12. Problem 19.14. **(for this question, you need to refer to your answers to PS #7)**
13. Problem 19.18.
14. Problem 19.21.
15. Problem 19.22.
16. Let's return to the Sales-Maximizing Firm example of Section 18.7, and show that the constraint qualifications are satisfied everywhere on the constraint set. Remember that if the constraint qualifications are satisfied everywhere on the constraint set, then we won't accidentally miss the true maximum when we look at the first-order conditions (that is, the constraint qualifications are required for the FOCs to be necessary conditions). Let's check the NDCQ in part (a) of Theorem 19.12, which is the version of the constraint qualifications we originally learned (as stated in Theorem 18.4). Write out the constraints so that they fit the statement of the theorem, with the inequalities going in the right direction, so that you know what the right *g* functions are.

    Note that at each point, the NDCQ only concerns itself with the constraints that are binding at that point. There are three constraints, but they can't all be binding at once (I assume the profit constraint doesn't go through the origin, because if it did the problem woudln't be very interesting). What are the possible combinations of constraints that could be binding? Just one of them could be binding, or any two of them could be binding, so there are six possible cases. For each of these six cases, show that the Jacobian of the binding constraints has full rank. (Hint: use the assumptions in the model about the signs of the derivatives of the functions R and C, and perhaps add another economically reasonable assumption of your own if the ones in the model aren't enough.)
17. Now let's examine the second-order sufficient conditions for a constrained maximum in the Sales-Maximizing Firm example.
    a. Write down the equation(s) that must be true for the bordered-Hessian test of Theorem 19.8. For simplicity, assume that there is a positive amount of advertising, so that only one constraint is binding.
    b. What can you assume about the various partial derivatives of R and C to guarantee that the model's solution is sufficient for a maximum? (Note that such conditions are often called "regularity conditions" when assumed by economic theorists.) Do you think that these assumptions are economically reasonable?