Lecture Outline 4: Functions and Calculus of Several Variables

This lecture note is based on Chapter 13 and 14 of *Mathematics for Economists* by Simon and Blume.

1. Functions Between Euclidean Spaces: I leave this to your own reading.
2. Geometric Representation of Functions
   - Two variables: the slice method

   Exercise *Use the slice method to sketch the graphs of the following functions: a)* $z = -x^2 - y^2$; *b)* $z = y - x^2$; *c)* $z = ye^{-x}$.

   - Level curves

   Exercise *If you had the graph of* $z = f(x,y)$, *how would you use it to draw level curves of* f*?*

   Exercise *Describe another example where a map with level curves occurs naturally in real life. What are the implications of curves being close together in this situation?*

   - Planar level sets in economics
     - Isoquants
     - Indifference curves
     - Level sets
3. Special Kinds of Functions
   - Linear functions on R^k: $f(x) = Ax$ for all $x \in R^k$
   - Quadratic forms: $Q(x_1,\cdots,x_k) = \sum_{i,j=1}^{k} a_{ij}x_ix_j$, or in matrix form, $x'Ax$(Q: what is the geometric representation of a general quadratic form on R^2?)
   - Monomials and polynomials

   Exercise *Write the following functions in matrix form:*

   a. $f(x_1,x_2,x_3) = 2x_1 - 3x_2 + 5x_3$
   b. $f(x_1,x_2) = x_1^2 - 2x_1x_2 + x_2^2$
4. Continuous Functions: I leave this to your own reading.
5. Vocabulary of Functions
   - Basic vocalbulary: domain,target space, range, image, preimage
   - Onto functions and one-to-one functions: surjective vs. injective
   - Inverse functions
   - Compositions of functions

   Exercise *For each of the following functions, what is the domain and the range of f? Which ones are one-to-one? For those which are one-to-one, write the expression for the inverse. Which ones are onto?*

   a. $f(x) = 3x - 7$
   b. $f(x) = x^3 - x$
6. Calculus
   - Total derivatives
     - Geometric interpretation
     - Linear approximation

Exercise *Consider the constant elasticity demand function* $Q = 6p_1^{-2}p_2^{3/2}$*, where Q is the demand for good 1 and* p_i *is the price of good i for* $i = 1, 2$*. Suppose current prices are* $p_1 = 6$ *and* $p_2 = 9$*.*

a. *What is the current demand for Q?*

b. *Use differentials to estimate the change in demand as* p_1 *increases by 0.25 and* p_2 *increases by 0.5.*

c. *Estimate the total demands for situation b and compare your estimates with the actual demands.*

Exercise *Use Calculus and no calculator to estimate the output given by the production function* $Q = 3K^{2/3}L^{1/3}$ *when: a)* $K = 1000$ *and* $L = 125$*, b)* $K = 998$ *and* $L = 128$*.*

- The Chain Rule
  - Coordinates functions $x(t)$
  - Velocity vector $x'(t)$ (tangent vector)
  - Differentiating along a curve

Exercise *Let* $f(x,y) = 3xy^2 + 2x$ *where* $x(t) = -3t^2$ *and* $y(t) = 4t^3 + t$*.*

a. *Use the Chain Rule to find a general expression for the rate of change of the composite* $f(x(t), y(t))$ *with respect to t.*

b. *Use substitution and direct differentiation to compute the rate of change of the composite* $f(x(t), y(t))$ *with respect to t. Compare this answer with your answer to part a*