Lecture Outline 1: Review of Some Basic Logic, Matrix Algebra, and Calculus

In this note I give you an outline of my 1st lecture, based on Chapter 1 to 9 of Mathematics for Economists by Simon and Blume. I will try to give you an outline of every lecture as this course progresses so you know in advance the key concepts we will cover in class.

1. Logic
   • A \Rightarrow B (A implies B), B \iff A (B is implied by A) and A \iff B
   • Sufficient and necessary conditions
   • Rule 1
     If the statement A \Rightarrow B is true, then so too is the statement (not B) \Rightarrow (not A)
     The first statement says that whenever A is true, B is true. Thus if B is false, A must be false—hence the second statement.
   • Rule 2
     The statement not (A and B) is equivalent to the statement (not A) or (not B).

Exercise  A, B, and C are statements. The following theorem is true: if A is true and B is not true then C is true. Which of the following statements follow from this theorem?

a. If A is true then C is true.
b. If A is not true and B is true then C is not true.
c. If either A is not true or B is true (or both) then C is not true.
d. If C is not true then A is not true and B is true.
e. If C is not true then either A is not true or B is true (or both).

2. Matrices
   • Determinant of \begin{pmatrix} a & b \\ c & d \end{pmatrix} and \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}
   • Nonsingular matrix and its inverse
   • Solutions of systems of simultaneous equations \begin{align*} ax + by &= u \\ cx + dy &= v \end{align*}

Exercise  You buy n items in the quantities q_1, \ldots, q_n at the prices p_1, \ldots, p_n. Express your expenditure using (i) \sum notation, (ii) vector notation.

3. Interval and Functions
   • Interval: open, closed, interior
   • Function: domain, range
   • \( f : A \Rightarrow B \)
   • Continuous function and intermediate value theorem

Exercise  Consider the function \( f \) defined by \( f(x) = x^4 - 4x^2 + 2 \). Does the equation \( f(x) = 0 \) have a solution between 0 and 1? The function is a polynomial, and thus is continuous. We have \( f(0) = 2 \) and \( f(1) = -1 \), so the Intermediate Value Theorem implies that the answer to the question is yes: for some value of x
Exercise Use the Intermediate Value Theorem to show that the equation 
\[ x^7 - 5x^5 + x^3 - 1 = 0 \] has a solution between -1 and 1.

4. Calculus: One Variable
   - Differentiation
     - Sum rule \( F(x) = f(x) + g(x) : F'(x) = f'(x) + g'(x) \)
     - Product rule \( F(x) = f(x)g(x) : F'(x) = f'(x)g(x) + f(x)g'(x) \)
     - Quotient rule \( F(x) = f(x)/g(x) : F'(x) = [f'(x)g(x) - f(x)g'(x)]/(g(x))^2 \)
   - Integration: definite vs. indefinite, fundamental theorem of calculus

Exercise \[ \int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + c. \] Note: Integration by parts
\[ \int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx \]

5. Calculus: Many Variables
   - Partial derivative
   - Young’s Theorem

Exercise Let \( f(x_1, x_2) = (x_1)^3 \ln x_2. \) Then \( f'_1(x_1, x_2) = 3(x_1)^2 \ln x_2 \) and \( f'_2(x_1, x_2) = (x_1)^3 / x_2 \)

Exercise For each of the following functions, find the partial derivatives \( f'_1, f'_2, \) and \( f'_{12} \).
   - \( f(x_1, x_2) = 2x_1^3 + x_1 x_2 \)
   - \( f(x_1, x_2) = (x_1 + 2)/(x_2 + 1) \)

6. Graphical Representation of Functions
   - Linear function
   - Quadratic function
   - Reciprocal function
   - Exponential and logarithm function
   - Function of two variables

Exercise Sketch the graph of \( 2x^2 - 8x + 6. \)

Exercise Sketch the function \( 2|x| + x^2. \)

Exercise Sketch the following sets in the plane.
   - \( \{(x,y) : x + 2y \leq 4, x \geq 0, \text{and } y \geq 0\} \)
   - \( \{(x,y) : x^2 + 4y^2 \leq 4\} \)
   - \( \{(x,y) : xy \leq 4, 0 \leq x \leq 4, 0 \leq y \leq 4\} \)