Lecture Outline 1: **Review of Some Basic Logic**, **Matrix Algebra**, **and Calculus**

In this note I give you an outline of my 1st lecture, based on Chapter 1 to 9 of *Mathematics for Economists* by Simon and Blume. I will try to give you an outline of every lecture as this course progresses so you know in advance the key concepts we will cover in class.

1. Logic

- $A \Rightarrow B$ (A implies B), $B \Leftarrow A$ (B is implied by A) and $A \Leftrightarrow B$
- Sufficient and necessary conditions
- Rule 1

  If the statement $A \Rightarrow B$ is true, then so too is the statement $(not\ B) \Rightarrow (not\ A)$

  The first statement says that whenever A is true, B is true. Thus if B is false, A must be false—hence the second statement.
- Rule 2

  The statement *not* (*A and B*) is equivalent to the statement (*not A*) *or* (*not B*).

Exercise *A, B, and C are statements. The following theorem is true: if A is true and B is not true then C is true. Which of the following statements follow from this theorem?*

a. *If A is true then C is true.*
b. *If A is not true and B is true then C is not true.*
c. *If either A is not true or B is true (or both) then C is not true.*
d. *If C is not true then A is not true and B is true.*
e. *If C is not true then either A is not true or B is true (or both).*

2. Matrices

- Determinant of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$
- Nonsingular matrix and its inverse
- Solutions of systems of simultaneous equations $\begin{aligned} ax + by &= u \\ cx + dy &= v \end{aligned}$

Exercise *You buy n items in the quantities $q_1, \ldots, q_n$ at the prices $p_1, \ldots, p_n$. Express your expenditure using (i) $\sum$ notation, (ii) vector notation.*

3. Interval and Functions

- Interval: open, closed, interior
- Function: domain, range
- $f : A \Rightarrow B$
- Continuous function and intermediate value theorem

Exercise *Consider the function f defined by $f(x) = x^4 - 4x^2 + 2$. Does the equation $f(x) = 0$ have a solution between 0 and 1? The function is a polynomial, and thus is continuous. We have $f(0) = 2$ and $f(1) = -1$, so the Intermediate Value Theorem implies that the answer to the question is yes: for some value of x*

between 0 and 1 we have $f(x) = 0$.

Exercise *Use the Intermediate Value Theorem to show that the equation* $x^7 - 5x^5 + x^3 - 1 = 0$ *has a solution between -1 and 1.*

4. Calculus: One Variable

- Differentiation
  - **a**. Sum rule $F(x) = f(x) + g(x) : F'(x) = f'(x) + g'(x)$
  - **b**. Product rule $F(x) = f(x)g(x) : F'(x) = f'(x)g(x) + f(x)g'(x)$
  - **c**. Quotient rule $F(x) = f(x)/g(x) : F'(x) = [f'(x)g(x) - f(x)g'(x)]/(g(x))^2$
- Integration: definite vs. indefinite, fundamental theorem of calculus

Exercise $\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + c$. *Note:Integration by parts* $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

5. Calculus: Many Variables

- Partial derivative
- Young's Theorem

Exercise *Let* $f(x_1, x_2) = (x_1)^3 \ln x_2$. *Then* $f'_1(x1, x2) = 3(x_1)^2 lnx_2$ *and* $f'_2(x_1, x_2) = (x_1)^3/x_2$

Exercise *For each of the following functions, find the partial derivatives* f'_1, f'_2, *and* f''_{12}.

- **a**. $f(x_1, x_2) = 2x_1^3 + x_1x_2$
- **b**. $f(x_1, x_2) = (x_1 + 2)/(x_2 + 1)$

6. Graphical Representation of Functions

- Linear function
- Quadratic function
- Reciprocal function
- Exponential and logarithm function
- Function of two variables

Exercise *Sketch the graph of* $2x^2 - 8x + 6$.

Exercise *Sketch the function* $2/x + x^2$.

Exercise *Sketch the following sets in the plane.*

- **a**. $\{(x, y) : x + 2y \leq 4, x \geq 0, and\ y \geq 0\}$
- **b**. $\{(x, y) : x^2 + 4y^2 \leq 4\}$
- **c**. $\{(x, y) : xy \leq 4, 0 \leq x \leq 4, 0 \leq y \leq 4\}$