

EQUILIBRIUM AND EFFICIENCY IN THE PRESENCE OF UNCERTAINTY

(BEGIN WITH FIRST PAGE OF THE EXTENDED EXAMPLE:
PARETO EFFICIENCY AND WALRASIAN EQUIL'N)

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MODELING MARKETS WHEN THERE IS UNCERTAINTY

MODELING UNCERTAINTY

IT IS NOW STANDARD TO MODEL UNCERTAINTY IN TERMS OF A SET S OF STATES OF THE WORLD, WHERE IT IS ASSUMED THAT EXACTLY ONE STATE $s \in S$ WILL OCCUR, BUT IT IS NOT KNOWN WHICH ONE WILL OCCUR. WE ALSO OFTEN ASSUME THERE IS SOME PROBABILITY MEASURE ON S , REPRESENTING A DECISION-MAKER'S SUBJECTIVE PROBABILITY ASSESSMENT OF THE LIKELIHOOD OF EACH STATE OCCURRING. (THESE ASSESSMENTS COULD DIFFER ACROSS DECISION-MAKERS.) THIS IS THEREFORE JUST A STANDARD PROBABILITY MODEL, WITH S AS THE SET OF ELEMENTARY EVENTS.

WE WILL NOT (FOR THE MOST PART) NEED TO USE A PROBABILITY ON S . AND WE WILL USE ONLY A FINITE SET S , ALTHOUGH ONE COULD USE AN INFINITE S . ($|S|$ WILL ALSO DENOTE THE NUMBER OF ELEMENTS OF S .)

WITH THIS STANDARD PROBABILITY STRUCTURE, ANYTHING THAT WILL DEPEND ON THE STATE (i.e., ON HOW THE UNCERTAINTY RESOLVES ITSELF) CAN BE REPRESENTED AS A RANDOM VARIABLE. FOR EXAMPLE, A CONTINGENT PLAN, OR STRATEGY, COULD BE WRITTEN AS $(x_s)_{s \in S} \in \mathbb{R}_+^{|S|}$, OR $(x(s))_{s \in S} \in \mathbb{R}_+^{|S|}$.

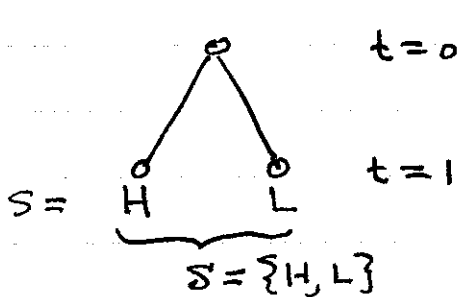
THE TEMPORAL RESOLUTION OF UNCERTAINTY

WE MODEL THE TEMPORAL CHARACTER OF THE UNCERTAINTY BY USING

(1) A SET $T = \{0, 1, \dots, T\}$ OF DATES (THIS SET MAY BE INFINITE — e.g., \mathbb{R}_+ , OR \mathbb{N} — BUT WE WILL USE ONLY FINITE SETS T HERE), AND

(2) A TREE STRUCTURE (GEOMETRICAL) } ALTERNATIVE (EQUIVALENT) WAYS OF DOING THE SAME THING
 OR
 PARTITIONS (ANALYTICAL)

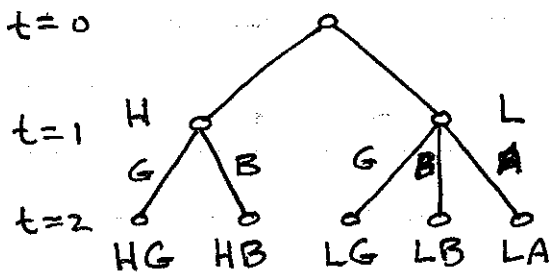
EXAMPLE 1:



$$S = \{H, L\}$$

$$T = \{0, 1\}$$

EXAMPLE 2:



$$S = \{HG, HB, LG, LB, LA\}$$

$$T = \{0, 1, 2\}$$

$$\mathcal{A}_0 = \{S\}, \text{ TRIVIAL PARTITION OF } S.$$

$$\mathcal{A}_1 = \{H, L\} = \{\{HG, HB\}, \{LG, LB, LA\}\},$$

A PARTITION OF S .

$$\mathcal{A}_2 = \{\{HG\}, \{HB\}, \{LG\}, \{LB\}, \{LA\}\},$$

A PARTITION OF S AND A

REFINEMENT OF \mathcal{A}_1 .

\mathcal{A}_0 IS THE COARSEST PARTITION OF S .

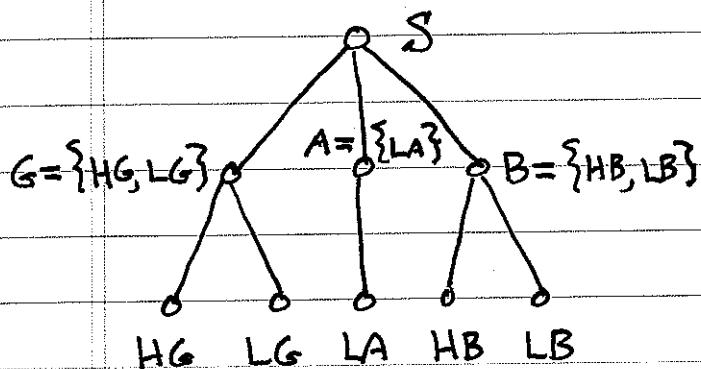
\mathcal{A}_2 IS THE FINEST PARTITION OF S .

$$\forall E \in \mathcal{A}_2: \exists E' \in \mathcal{A}_1: E \subseteq E'$$

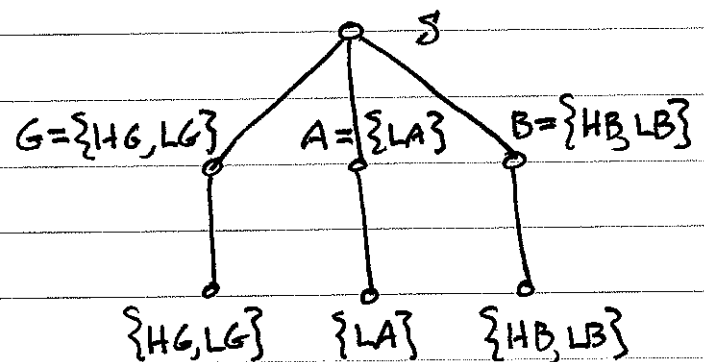
DIFFERENTIAL ("ASYMMETRIC") INFORMATION

IN OUR EXAMPLE 2, INFORMATION WAS DISCOVERED OR "REVEALED" IN A PARTICULAR ORDER: FIRST IT WAS DISCOVERED WHETHER H OR L IS TRUE (i.e., WHETHER $S \in H = \{HG, HB\}$ OR $S \in L = \{LG, LA, LB\}$); AND THEN WHETHER G, A, OR B IS TRUE (THUS, AT $t=2$ IT WAS KNOWN EXACTLY WHICH MEMBER OF $S = \{HG, HB, LG, LA, LB\}$ IS THE TRUE STATE S). BUT DIFFERENT INDIVIDUALS MIGHT, AT ANY STAGE, HAVE DIFFERENT INFORMATION WHICH IS NEVERTHELESS CONSISTENT WITH OTHERS' INFORMATION. HERE ARE SOME EXAMPLES OF OTHER RESOLUTIONS OF THE UNCERTAINTY IN EXAMPLE 2. NOTICE THAT WHICHEVER MEMBER OF S IS THE TRUE STATE S , WE CAN IDENTIFY "WHAT EACH INDIVIDUAL WILL KNOW" AT EACH STAGE.

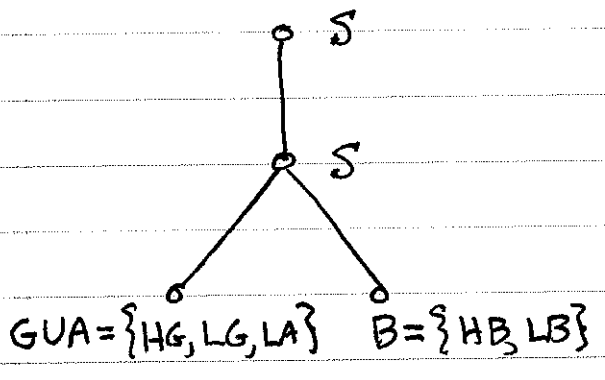
(a) "OBSERVE WHETHER G, A, OR B; THEN WHETHER H OR L."



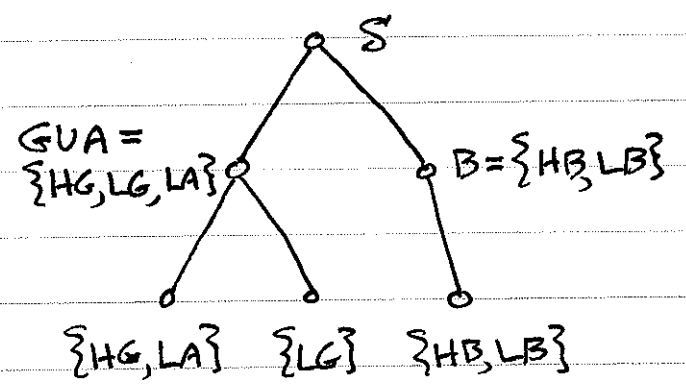
(b) "OBSERVE ONLY WHETHER G, A, OR B — AT $t=1$."



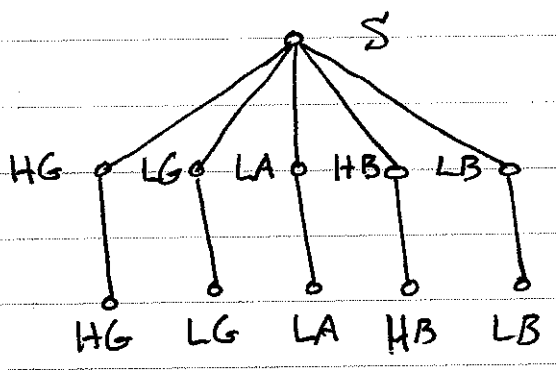
(c) "OBSERVE ONLY WHETHER B OR 'G OR A' - AT t=2."



(d) "OBSERVE WHETHER B OR 'G OR A'; THEN (IF 'G OR A') WHETHER 'L AND G' OR NOT."



(e) "OBSERVE THE WHOLE STATE AT t=1."



IN TERMS OF PARTITIONS, (d) FOR EXAMPLE WOULD BE

$$\begin{aligned}
 \mathcal{I}_0 &= S \\
 \mathcal{I}_1 &= \{ \{HG, LG, LA\}, \{HB, LB\} \} \\
 \mathcal{I}_2 &= \{ \{HG, LA\}, \{LG\}, \{HB, LB\} \}
 \end{aligned}$$