# LECTURE NOTES

ON

# DEMAND THEORY

FOR

# INTERMEDIATE MICROECONOMICS

# HOUSEHOLD DECISION-MAKING

(INDIVIDUAL CHOICE)

IN EACH EXAMPLE, NOTE DESCRIPTIVE AND PRESCRIPTIVE POINT OF VIEW.

EXAMPLES:

(1) FOR LUNCH (THIS WEEK), HOW MANY CHIMIS WILL I HAVE, AND HOW MANY TACOS?

(2) HOW MUCH NATURAL GAS WILL I USE (PER MONTH) TO HEAT MY HOME?

(3) WILL I CHOOSE AN OIL FURNACE OR A GAS FURNACE. WHEN SHOULD I (OR WILL I) SWITCH?

(4) HOW MANY CHILDREN WILL A COUPLE HAVE? HOW MANY DO *WE* *WANT* TO HAVE?

WE WANT A METHOD OF ANALYSIS — A *THEORY* — THAT WE CAN USE TO ANALYZE MANY DIFFERENT KINDS OF DECISION PROBLEMS.

WE BEGIN BY DEVELOPING THE THEORY OF *HH* CHOICE, OR DECISION-MAKING, BUT THE SAME THEORY APPLIES TO ANY DECISION PROBLEM (PERHAPS SUITABLY AUGMENTED OR ALTERED).

IT'S EXTREMELY USEFUL TO FIRST BREAK A DECISION PROBLEM DOWN INTO THESE THREE PIECES:

(1) ALTERNATIVES
(2) CONSTRAINTS
(3) CRITERION

EXAMPLE: CHIMIS AND TACOS FOR LUNCH.

FIRST WE NEED TO DECIDE THE TIME FRAME AND UNITS: TODAY? PER WEEK? PER YEAR? DOZENS OF CHIMIS?

→ LET'S SAY CHIMIS/WK. AND TACOS/WK.

DIFFERENT WEEKS, OR DIFFERENT PERSONS

| | A | B | C | D | |
|---|---|---|---|---|---|
| CHIMIS | 5 | 2 | 3 | 6 | |
| TACOS | 5 | 7 | 9 | 0 | .... |

DIFFERENT POTENTIAL CHOICES

EACH POSSIBLE CHOICE IS A PAIR OF NUMBERS:

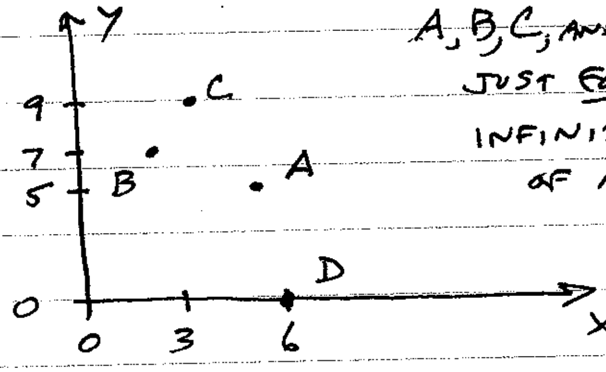
$$A=(5,5) \quad B=(2,7) \quad C=(3,9) \quad D=(6,0).$$

IN OTHER WORDS, A CHOICE IS A PAIR $(x,y)$, WHERE $x$: #CHIMIS/WK AND $y$: #TACOS/WK.

THE ALTERNATIVES ARE ALL THE PAIRS $(x,y)$.
[BUT NOT NEGATIVE, AND PERHAPS ONLY INTEGERS]

THIS SUGGESTS A <u>GEOMETRIC</u> REPRESENTATION OF ALL THE ALTERNATIVES:

WE CALL THE ALTERNATIVES <u>BUNDLES</u> OR COMBINATIONS, OR "PLANS", OR "PROGRAMS"




A, B, C, AND D ARE JUST <u>FOUR</u> OF THE INFINITE NUMBER OF ALTERNATIVES.

WE HAVE SOMETHING <u>REAL</u> THAT WE'RE DESCRIBING AND ANALYZING, AND WE HAVE TWO DIFFERENT WAYS OF <u>REPRESENTING</u> THE REALITY IN ANALYTICAL TERMS:

(1) REALITY
(2) ALGEBRA (ALGEBRAIC REPRESENTATION)
(3) GEOMETRY (GEOMETRIC REPRESENTATION)

THEY ARE, FOR OUR PURPOSES, JUST THREE DIFFERENT ~~[illegible]~~ VERSIONS OF THE SAME THING.

# THE BUDGET CONSTRAINT

LET'S ASSUME I'VE ALLOCATED \$12 TO SPEND ON LUNCHES THIS WEEK, AND LET'S SAY I'M GOING TO SPEND IT ALL ON LUNCHES, AND THAT I'LL ONLY BUY CHIMIS AND TACOS.

WE WANT TO KNOW HOW MANY OF EACH WILL I BUY — i.e., WHICH BUNDLE $(x,y)$ WILL I BUY?

FIRST WE HAVE TO DESCRIBE WHICH BUNDLES I CAN AFFORD — i.e., WHAT CONSTRAINT MY CHOICE IS GOING TO HAVE TO SATISFY. THE BUNDLE I CHOOSE IS OBVIOUSLY GOING TO BE ONE OF THE AFFORDABLE BUNDLES. WHICH ONES ARE THEY?

WE CAN'T SAY, WITHOUT KNOWING THE PRICES I HAVE TO PAY FOR THE TWO GOODS.

SUPPOSE $P_x = \$2$ AND $P_y = \$1$.

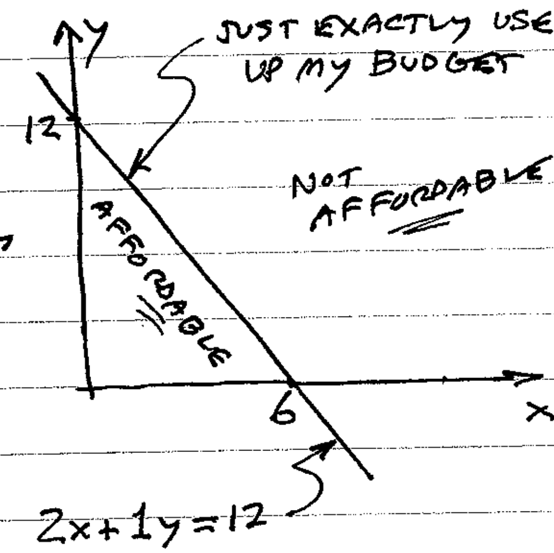
THEN THE BUNDLES THAT JUST EXACTLY USE UP MY BUDGET OF \$12 ARE THE ONES THAT SATISFY THE EQUATION

$$2x + 1y = 12$$

ALGEBRAIC

GEOMETRIC

(AND $x \geq 0$ AND $y \geq 0$).

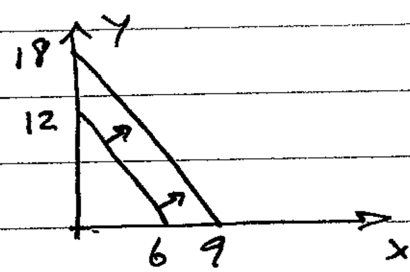



HOW DO CHANGES IN MY "SITUATION" CHANGE THE SET OF AFFORDABLE BUNDLES?

ENVIRONMENT PARAMETERS

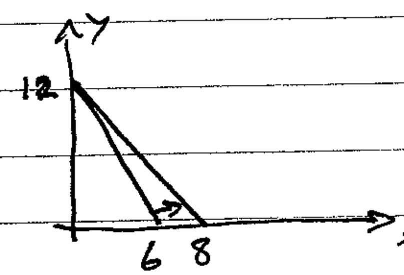
(1) WHAT IF $I = \$18$?

$$2x + 1y = 18$$




(2) WHAT IF $P_x = \$1.50$?

$$1\tfrac{1}{2}x + 1y = 12$$




THE <u>BUDGET CONSTRAINT</u> IS $P_x x + P_y y = I$

OR $P_x x + P_y y \leq I$.

(★) → WE ARE GOING TO WANT TO KNOW
HOW
MY CHOICE (i.e., MY CHOSEN BUNDLE)
IS CHANGED BY
CHANGE IN MY ENVIRONMENT
(i.e., THE "PARAMETERS," $P_x, P_y, I$).

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WILL MY CHOICE CHANGE IN THE SAME WAY IN BOTH (1) AND (2)?

WILL BOTH $x\uparrow$ AND $y\uparrow$?

OR WILL ONE INCREASE AND THE OTHER DECREASE?

... ETC.

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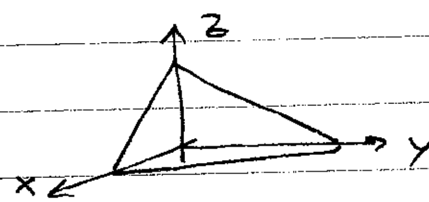
WHAT IF THERE WERE MORE THAN TWO GOODS?

THREE GOODS IS EASY:

$$P_x x + P_y y + P_z z \leqq I$$




MORE THAN THREE GOODS:

$$P_1 x_1 + P_2 x_2 + \dots + P_n x_n \leqq I$$

... FOR $n$ GOODS.

GEOMETRY NOT POSSIBLE.
WE USE LINEAR ALGEBRA
AND DO THIS FOR ECONOMETRICS AND FORECASTING

# THE HH's CRITERION

(UTILITY FUNCTIONS AND INDIFFERENCE CURVES)

WE WANT A CRITERION, OR MEASURE, BY WHICH THE HH CAN EVALUATE ITS ALTERNATIVES (BUNDLES).

THE TEXT TAKES TWO SEEMINGLY DISTINCT APPROACHES TO THIS PROBLEM:

ALGEBRAIC (1) THE HH's "UTILITY" IS "MEASURABLE," VIA SOME UTILITY FUNCTION $u(x,y)$;

GEOMETRIC (2) THE HH's "PREFERENCES" CAN BE REPRESENTED BY AN INDIFFERENCE MAP, i.e., BY A FAMILY OF INDIFFERENCE CURVES.

IT SEEMS IN THE TEXT AS IF THESE TWO APPROACHES ARE SOMEHOW RELATED, BUT IT'S NOT MADE VERY CLEAR, AND IN FACT NOT REALLY DEALT WITH EXPLICITLY.

NOTICE THAT (1) IS AN <u>ALGEBRAIC</u> WAY OF DESCRIBING A HH's PREFERENCES, AND THAT (2) IS A <u>GEOMETRIC</u> WAY OF DOING IT. WE'RE GOING TO DEVELOP THE RELATION BETWEEN THESE TWO WAYS OF REPRESENTING A HH's PREFERENCES.

LET'S START OUT BY JUST <u>ASSUMING</u> THAT THE HH WE'RE INTERESTED IN ALWAYS CHOOSES AMONG ALTERNATIVE BUNDLES BY CHOOSING WHICHEVER ONE (AMONG THOSE AVAILABLE) "MAXIMIZES" THE VALUE OF THE "UTILITY FUNCTION" $u(x,y) = xy$.

FOR EXAMPLE, IF THE HH IS ASKED TO CHOOSE BETWEEN THE TWO BUNDLES

$$A = (10,6) \quad \text{AND} \quad B = (7,9),$$

IT CHOOSES B, BECAUSE

$$u(B) = u(7,9) = (7)(9) = 63$$
$$u(A) = u(10,6) = (10)(6) = 60.$$

IF THE HH IS ASKED TO CHOOSE FROM AMONG THE THREE BUNDLES $A = (10,6)$, $B = (7,9)$, AND $C = (16,4)$, IT CHOOSES $C$, BECAUSE

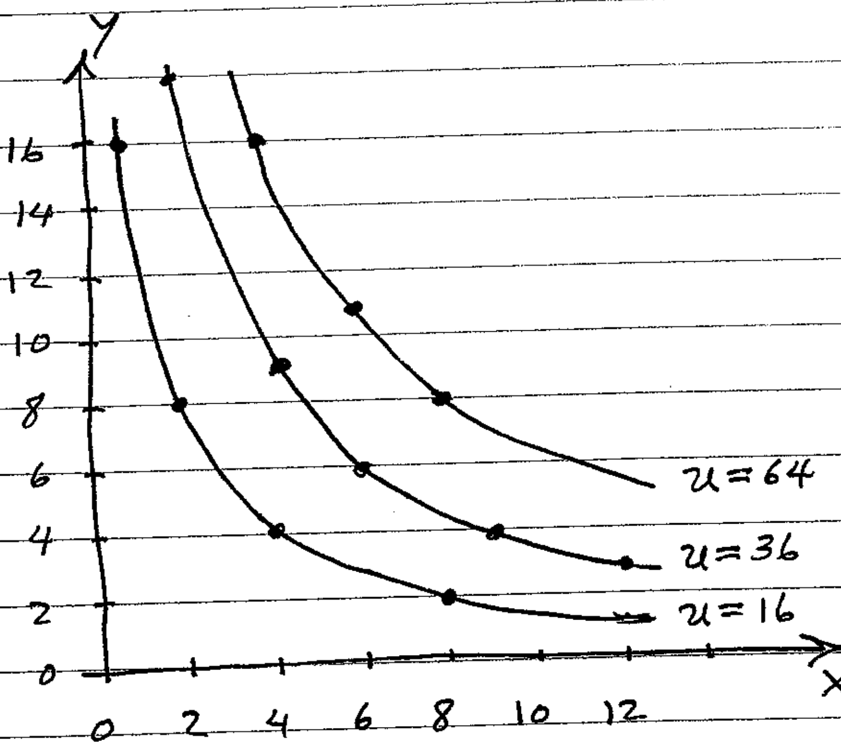
$$u(C) = u(16,4) = (16)(4) = 64; \; u(A) = 60, \; u(B) = 63.$$

LET'S DRAW A FEW OF THE CONTOURS, OR LEVEL CURVES, OF THE FUNCTION $u(x,y) = xy$.
[SAME AS CONTOURS ON A TOPOGRAPHIC MAP, WHERE A CONTOUR IS ALL TWO-DIM'L COORDINATES AT WHICH THE ALTITUDE (HEIGHT) IS A GIVEN VALUE. HERE, "HEIGHT" IS VALUE OF FUNCTION $u(x,y)$ AT $(x,y)$.]

→ ALSO MENTION ISOBARS, WHERE THE VALUES AREN'T ACTUALLY "HEIGHT."

SOME BUNDLES:

| x | y | $u(x,y)$ |
|---|---|---|
| 8 | 8 | 64 |
| 4 | 16 | 64 |
| 16 | 4 | 64 |
| 2 | 32 | 64 |
| 32 | 2 | 64 |
| 6 | 6 | 36 |
| 4 | 9 | 36 |
| 9 | 4 | 36 |
| 2 | 18 | 36 |
| 18 | 2 | 36 |
| 4 | 4 | 16 |
| 2 | 8 | 16 |
| 8 | 2 | 16 |
| 1 | 16 | 16 |
| 16 | 1 | 16 |



XEROX THIS PAGE TO HAND OUT

WHAT ARE SOME OF THE IMPLICATIONS OF OUR ASSUMPTION THAT THE HH CHOOSES BUNDLES ON THE BASIS OF THEIR $u$-VALUES, WHERE $u(x,y) = xy$?

(1) WE CAN DRAW <u>INDIFFERENCE CURVES</u>, WHICH ARE JUST THE LEVEL CURVES, OR CONTOURS, OF $u$.
(2) THE I-CURVES SLOPE <u>DOWNWARD</u>.
(3) THE I-CURVES HAVE <u>DIMINISHING</u> SLOPE (IN ABSOLUTE-VALUE; i.e., IN MAGNITUDE).
(4) WE CAN <u>CALCULATE</u> THE SLOPE OF THE I-CURVE AT ANY BUNDLE $(x,y)$. THIS IS THE MRS AT THE BUNDLE $(x,y)$, WHICH IS <u>VERY IMPORTANT</u>, AND WHICH WE WILL DEVELOP MOMENTARILY. (ACTUALLY, THE MRS IS THE <u>NEGATIVE</u> OF THE SLOPE.)
(5) THE HH'S PREFERENCES ARE "<u>COMPLETE</u>" — i.e., FOR ANY TWO BUNDLES A AND B, WE CAN SAY WHICH THE HH PREFERS, OR THAT THE HH IS INDIFFERENT BETWEEN THEM.
(6) THE HH'S PREFERENCES ARE "<u>TRANSITIVE</u>" — SEE THE TEXT FOR THIS.
(7) THE HH'S PREFERENCES ARE "<u>CONTINUOUS</u>" — i.e., WE CAN'T "CONTINUOUSLY" CHANGE A BUNDLE WORSE THAN A INTO A BUNDLE BETTER THAN A WITHOUT ENCOUNTERING A BUNDLE "INDIFFERENT TO A."

# THE MRS

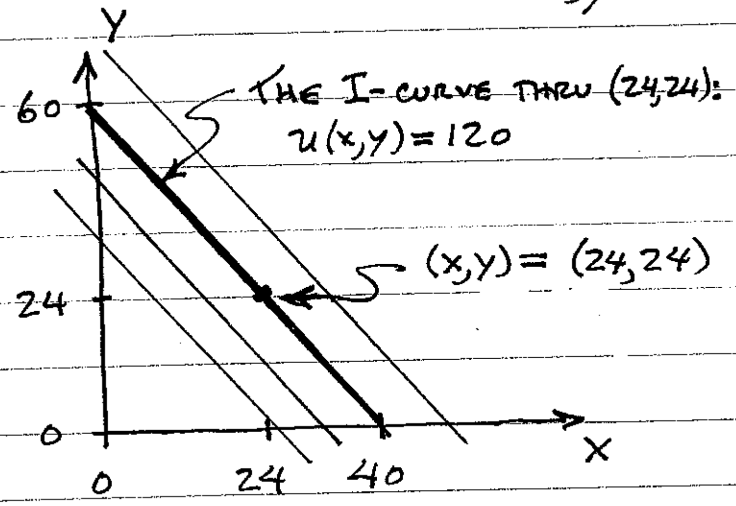
THE MRS (MARGINAL RATE OF SUBTITUTION) IS IMPORTANT BECAUSE IT TELLS US HOW MUCH THE HH VALUES A GOOD IN REAL, MEASURABLE TERMS — HOW MUCH (OF ANOTHER GOOD) THE HH WOULD GIVE UP TO GET MORE OF THE GOOD IN QUESTION.

WHENEVER WE ARE ANALYZING CHOICE INVOLVING JUST <u>TWO</u> GOODS (THAT'LL BE MOST OF THE TIME), WE WILL ALWAYS USE THE MRS TO MEAN THE AMOUNT OF THE <u>Y-GOOD</u> (VERTICAL AXIS) THE HH WOULD GIVE UP TO GET AN ADDITIONAL UNIT OF THE <u>X-GOOD</u> (HORIZONTAL AXIS) — IN OTHER WORDS, THE MRS IS THE VALUE TO THE HH (MEASURED IN Y-GOOD UNITS) OF THE <u>MARGINAL</u> UNIT OF THE X-GOOD.

IN TRYING TO UNDERSTAND THE MRS IDEA IT WILL BE A GOOD IDEA TO BEGIN WITH AN EVEN EASIER EXAMPLE THAN THE $u(x,y)=xy$ UTILITY FUNCTION. WE'LL LOOK AT THE EXAMPLE $u(x,y)=3x+2y$ — IN OTHER WORDS, WE'RE <u>ASSUMING</u> (FOR NOW) THAT THE HH CHOOSES BUNDLES ACCORDING TO THEIR $u$-VALUES, WHERE $u(x,y)=3x+2y$.

# MRS EXAMPLE

$$u(x,y) = 3x + 2y$$




THE I-CURVE THRU (24,24) IS

$$u(x,y) = 120.$$

TO CALCULATE THE SLOPE OF THE I-CURVE:

(1) WRITE EQUATION OF I-CURVE.

(2) WRITE IT AS "Y IN TERMS OF X."

(3) TAKE DERIVATIVE $\frac{dy}{dx}$ AT $(x,y)$.

(4) $MRS = -\frac{dy}{dx}$

SLOPE OF I-CURVE IS $-\frac{3}{2}$.

$\therefore MRS = \frac{3}{2}$

THIS PERSON WOULD GIVE UP 3 TACOS TO GET 2 MORE CHIMIS, OR $\frac{3}{2}$ TACOS TO GET 1 MORE CHIMI.

THIS IS A MEASURE OF THE VALUE TO HIM OF A CHIMI — OF THE MARGINAL CHIMI — MEASURED IN TERMS OF TACOS.

THUS,

(1) $3x + 2y = 120$

(2) $\begin{cases} \text{i.e., } 2y = 120 - 3x \\ \text{i.e., } y = 60 - \frac{3}{2}x. \end{cases}$

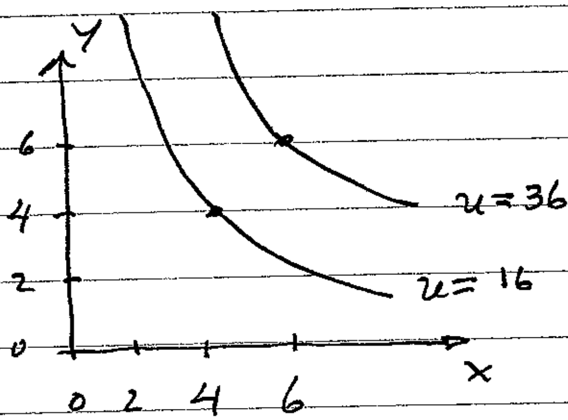
(3) $\therefore \frac{dy}{dx} = -\frac{3}{2}$

(4) $\therefore MRS = \frac{3}{2}$

SUMMARY:

$$\boxed{\begin{aligned} MRS &= -\text{SLOPE} \\ &= -\frac{dy}{dx} \end{aligned}}$$

# MRS EXAMPLE




$u(x,y) = xy$

(1) I-CURVE THRU $(x,y)$

IS $u(x,y) = c$ [FOR SOME $c$]

i.e., $xy = c$.

e.g.,

IF $(x,y) = (6,6)$, THEN $c = 36$;

IF $(x,y) = (2,8)$, THEN $c = 16$.

DO IT FIRST FOR A SPECIFIC $c$.

(2) $y = \dfrac{c}{x}$

$= cx^{-1}$

(3) $\dfrac{dy}{dx} = -cx^{-2}$

$= -\dfrac{c}{x^2}$

NOTE THIS LITTLE TRICK HERE

$= -\dfrac{xy}{x^2}$

$= -\dfrac{y}{x}$

(4) $\therefore$ MRS $= \dfrac{y}{x}$

EVALUATING THE MRS AT SOME BUNDLES:

1 AT $(6,6)$: MRS $= \frac{6}{6} = 1$.

2 AT $(4,4)$: MRS $= \frac{4}{4} = 1$.

3 AT $(4,9)$: MRS $= \frac{9}{4} = 2\frac{1}{4}$.

4 AT $(2,18)$: MRS $= \frac{18}{2} = 9$.

NOTICE THAT FOR THIS HH THE MRS DEPENDS ON WHICH BUNDLE WE'RE TALKING ABOUT. THIS WOULD BE TRUE <u>WHENEVER</u> THE I-CURVES ARE NOT STRAIGHT LINES

## MRS FOR SOME OTHER UTILITY FUNCTIONS

① $v(x,y) = \sqrt{xy}$

(1) $\sqrt{xy} = c$

$xy = c^2$

(2) $y = c^2 x^{-1}$

(3) $\frac{dy}{dx} = -c^2 x^{-2}$

$= -\frac{c^2}{x^2}$

$= -\frac{xy}{x^2}$

$= -\frac{y}{x}$

(4) $\therefore$ MRS $= \frac{y}{x}$

② $w(x,y) = \frac{1}{10} x^2 y^2$

(1) $\frac{1}{10} x^2 y^2 = c$

$x^2 y^2 = 10c$

$xy = \sqrt{10c}$

(2) $y = \sqrt{10c}\; x^{-1}$

(3) $\frac{dy}{dx} = -\sqrt{10c}\; x^{-2}$

$= -\frac{\sqrt{10c}}{x^2}$

$= -\frac{xy}{x^2}$

$= -\frac{y}{x}.$

(4) $\therefore$ MRS $= \frac{y}{x}.$

ANALOGY w/ TOPOGRAPHY AND TOPOGRAPHIC MAP.

FOR BOTH $v(x,y)$ AND $w(x,y)$ THE MRS AT EVERY BUNDLE $(x,y)$ IS EXACTLY THE SAME AS THE MRS AT THAT BUNDLE FOR $u(x,y)$. ALTHOUGH THE UTILITY FUNCTIONS ARE DIFFERENT, THE I-MAPS (i.e., PREFERENCES) AND CHOICES THE THREE HH's WOULD MAKE WOULD ALWAYS BE IDENTICAL. SEE THE I-MAPS (ALL THE SAME) ON THE FOLLOWING PAGE.

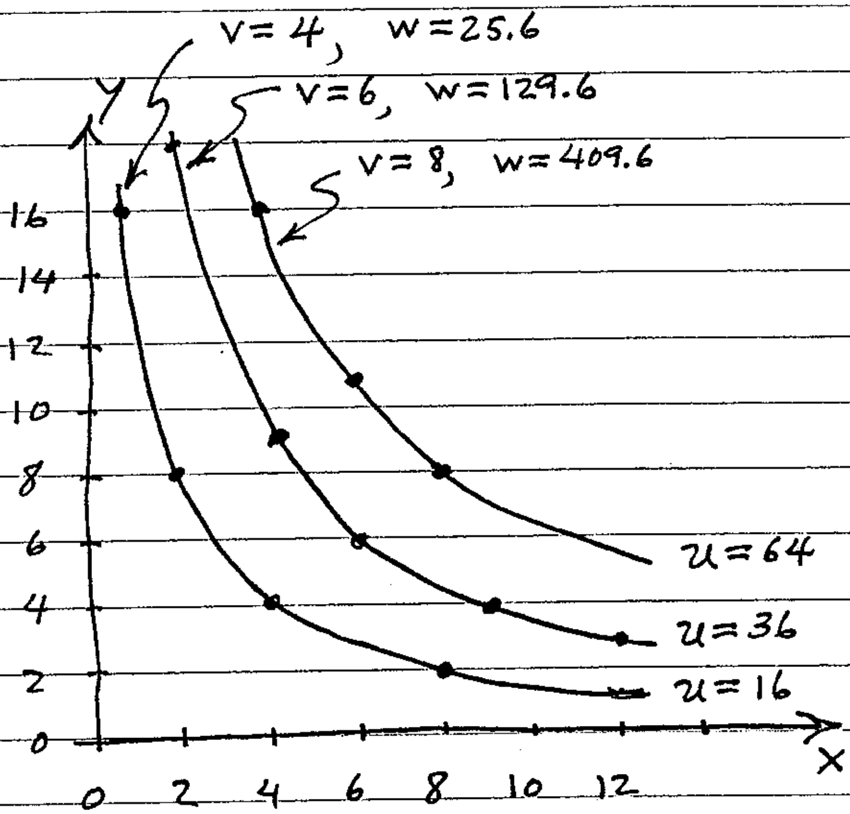
THE UTILITY FUNCTIONS

$$u(x,y) = xy \qquad v(x,y) = \sqrt{xy} \qquad w(x,y) = \frac{1}{10}x^2y^2$$

ALL HAVE EXACTLY THE SAME I-MAPS, EXCEPT THAT THE NUMBERS ASSOCIATED WITH AN I-CURVE ARE DIFFERENT FOR $u$, $v$, AND $w$:

SOME BUNDLES:

| x | y | u(x,y) | |
|---|---|---|---|
| 8 | 8 | 64 | v = 8, w = 409.6 |
| 4 | 16 | 64 | |
| 16 | 4 | 64 | |
| 2 | 32 | 64 | |
| 32 | 2 | 64 | |
| 6 | 6 | 36 | v = 6, w = 129.6 |
| 4 | 9 | 36 | |
| 9 | 4 | 36 | |
| 2 | 18 | 36 | |
| 18 | 2 | 36 | |
| 4 | 4 | 16 | v = 4, w = 25.6 |
| 2 | 8 | 16 | |
| 8 | 2 | 16 | |
| 1 | 16 | 16 | |
| 16 | 1 | 16 | |



③ $u(x,y) = x^2 y$

$x^2 y = c$

$y = c x^{-2}$

$\frac{dy}{dx} = -2cx^{-3}$

$= -2 \frac{c}{x^3}$

$= -2 \frac{x^2 y}{x^3}$

$= -2 \frac{y}{x}$.

$\therefore$ MRS $= 2 \frac{y}{x}$;

i.e, THE MRS AT ANY BUNDLE IS EXACTLY TWICE WHAT IT IS FOR THE FIRST HH WE CONSIDERED, THE ONE THAT HAD $u(x,y) = xy$.

④ $u(x,y) = y + 5 \log x$

$y + 5 \log x = c$

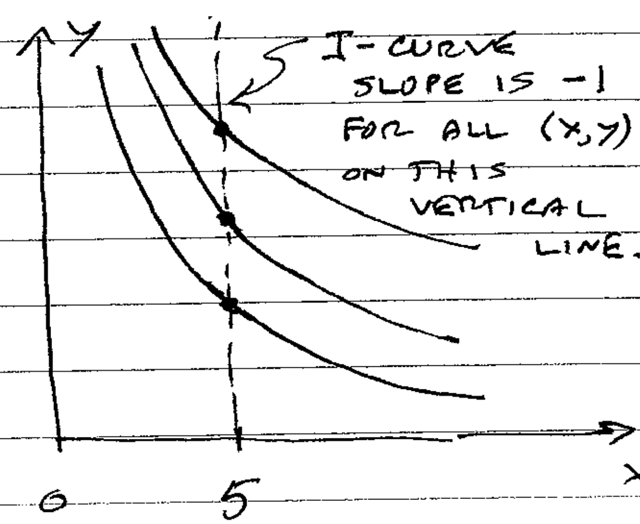
$y = c - 5 \log x$

$\frac{dy}{dx} = -5 \cdot \frac{1}{x} = -\frac{5}{x}$.

$\therefore$ MRS $= \frac{5}{x}$.

THIS HH'S MRS DOES NOT DEPEND ON $y$, ONLY ON $x$. THUS, FOR EXAMPLE, ALL BUNDLES $(x,y)$ IN WHICH $x=5$ WILL HAVE THE SAME MRS: MRS $= 1$.

I-CURVES THRU THESE BUNDLES WILL ALL HAVE SLOPE $-1$.



(5) $u(x,y) = 2\log x + \log y$.

OUR METHOD FOR CALCULATING THE MRS IS VERY MESSY FOR THIS UTILITY FUNCTION — i.e., IT'S WAY TOO COMPLICATED. IN FACT, FOR MOST UTILITY FUNCTIONS OUR METHOD IS TOO COMPLICATED.

ON THE FOLLOWING PAGE IS A METHOD THAT'S MUCH SIMPLER. BUT JUST TO CONVINCE YOU THAT THE METHOD WE'VE BEEN USING IS INDEED MESSY FOR THIS UTILITY FUNCTION, LET'S ACTUALLY DO IT:

$$2\log x + \log y = c$$

$$\log y = c - 2\log x$$

$$y = e^{c - 2\log x}$$

$$\frac{dy}{dx} = -\frac{2}{x} e^{c-2\log x} = -\frac{2}{x} y = -2\frac{y}{x}.$$

$\therefore$ MRS $= 2\frac{y}{x}$. PRETTY BAD, HUH?

AND THIS UTILITY FUNCTION IS ACTUALLY A PRETTY SIMPLE ONE. FOR A MORE COMPLICATED FUNCTION THIS METHOD IS REALLY MESSY!

# THE SIMPLE METHOD TO CALCULATE MRS

DON'T DO ANY OF THE FOUR STEPS IN OUR ORIGINAL METHOD.

INSTEAD, JUST ...

(a) TAKE THE "DERIVATIVE WITH RESPECT TO X" (i.e., AS IF Y WERE CONSTANT), WRITTEN $u_x$ ;

(b) TAKE THE "DERIVATIVE WITH RESPECT TO Y" (i.e., AS IF X WERE CONSTANT), WRITTEN $u_y$ ;

THEN $\boxed{MRS = \frac{u_x}{u_y}}$.

EXAMPLES:

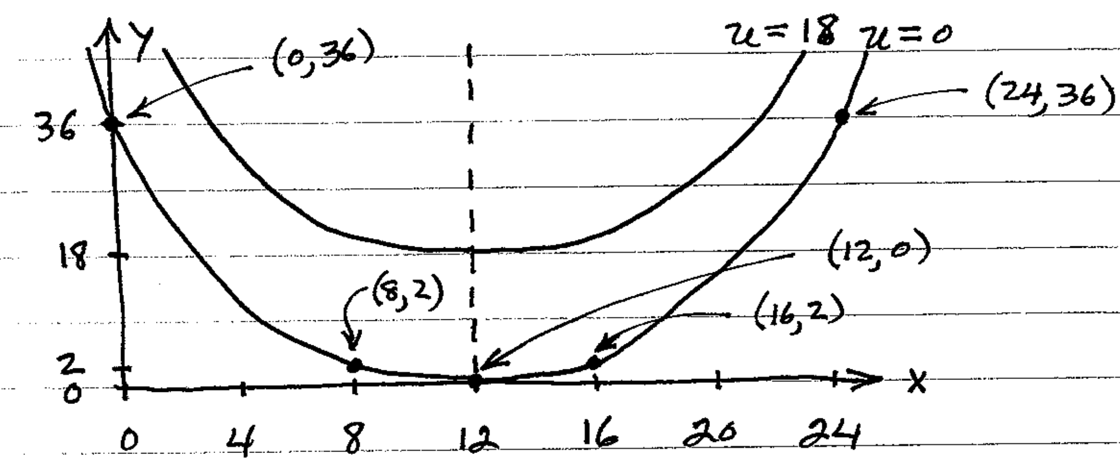
1. $u(x,y) = 2\log x + \log y$. $u_x = \frac{2}{x}$, $u_y = \frac{1}{y}$; $MRS = 2\frac{y}{x}$.
2. $u(x,y) = xy$. $u_x = y$, $u_y = x$; $MRS = \frac{y}{x}$.
3. $u(x,y) = x^2 y$. $u_x = 2xy$, $u_y = x^2$; $MRS = \frac{2xy}{x^2} = 2\frac{y}{x}$.
4. $u(x,y) = y + 5\log x$. $u_x = \frac{5}{x}$, $u_y = 1$; $MRS = \frac{5}{x}$.
5. $u(x,y) = 3x + 2y$. $u_x = 3$, $u_y = 2$; $MRS = \frac{3}{2}$.

## ANOTHER EXAMPLE:

$$u(x,y) = y - \frac{1}{4}(12-x)^2$$
$$= y - \frac{1}{4}(144 - 24x + x^2)$$
$$= y - \frac{1}{4}x^2 + 6x - 36$$

$$u_y = 1 \qquad u_x = -\frac{2}{4}(12-x)(-1) = \frac{1}{2}(12-x)$$
$$= 6 - \frac{1}{2}x.$$

$$\boxed{\therefore \text{MRS} = \frac{1}{2}(12-x) = 6 - \frac{1}{2}x.}$$




AT $x = 12$: MRS $= 0$

AT $x > 12$: MRS $< 0$ (GETTING MORE OF THE x-GOOD HAS NEGATIVE VALUE; HH WOULD "GIVE UP A NEGATIVE AMOUNT" OF y (IT MUST GET MORE y) IN RETURN FOR ACCEPTING MORE x.)

# Do People Really Have Utility Functions?

What we've shown so far is only that...

**IF** the HH has the utility function $u(x,y) = xy$

(i.e., if it chooses bundles according to this $u(x,y)$),

**THEN** its preferences satisfy Conditions (1)-(7)

on page 4.

But here's a cool fact (or "theorem"):

The reverse is also true!

In fact,

**IF** the HH's preferences are simply

(5) complete

(6) transitive

(7) continuous,

**THEN** its choices can be represented by a utility function, and therefore by an indifference map.

So when we assume that HHs or people "have utility functions," what we're really assuming is that they behave as if they have a utility function.

## Do People Really Have Utility Functions?

Do people (or households) really have measurable utility functions? One hundred years ago many people thought so, but today virtually no one believes that. However, it turns out that *it doesn't make any difference*: People may in fact *behave* as if they have measurable utility, even if they don't! If this sounds a little bit crazy, just suspend your skepticism for a few minutes.

Recall that we assumed that a HH's choices among alternative bundles were made according to the utility function $u(x,y) = xy$, even though we didn't really believe that people have "measurable" utility. We just wanted to see where the assumption would take us. Then, on page 4, we listed seven implications of our assumption. When we say that these are implications of the assumption, what we mean is that

> ***if*** the HH chooses bundles according to $u(x,y) = xy$,
>
> ***then*** that HH's preferences satisfy all seven of these conditions.

But here's a really cool mathematical fact, or "theorem" (in fact, most economic theorists would say it's even beyond *cool* ... it's *beautiful*): namely, *the reverse is true too!* That is,

> ***if*** a HH's preferences satisfy these seven conditions,
>
> ***then*** that HH is choosing its bundles according to *some* utility function $u(x,y)$ -- even if the HH isn't *consciously* doing so!

Actually, the thing that's beautiful here is that *even if the HH's preferences satisfy just the conditions (5), (6), and (7) -- i.e., if they're just complete, transitive, and continuous -- then the HH's behavior can be described by some utility function u(x,y).* We're not going to prove this fact; it takes some fairly advanced mathematics. It's only the *fact* that's cool, anyway, not the proof. And by the way, you might notice that Nicholson (like just about every other Intermediate Microeconomics author) includes a discussion of completeness and transitivity. But *why* do they do this, since they never make any use of these ideas? It's because these guys who are writing these textbooks all know this cool fact, so they can't keep themselves from throwing in some of its features, even though they can't find a way to actually explain the reason for including this stuff. (These guys are friends of mine, so I don't think they'll sue me for slander.)

In fact, empirical analysis, including experiments with actual people, has consistently shown that people's behavior generally, if not always, satisfies conditions (5), (6), and (7), and is therefore representable by some utility function -- and therefore by an indifference map. In fact, experiments that have been performed with "animal consumers" indicate that in many situations *animals'* choices are also representable by utility functions and therefore by indifference maps.

## Some Additonal Things to Note

1. Different HHs will typically have different preferences -- i.e., different indifference maps -- and therefore different utility functions.

2. We've already seen that a given indifference map will be representable by many different utility functions, all of which nevertheless yield the same MRS function. Therefore, it is really the MRS function that we always use, not the utility function. ***The MRS function is the critical tool for analyzing choice among alternative consumption bundles.***

3. No matter what utility function describes a HH's preferences, that utility function will satisfy the conditions (1), (5), (6), and (7) in our list of implications. But there are plenty of utility functions (and therefore plenty of perfectly good preferences) that *don't* satisfy conditions (2), (3), and/or (4) -- those implictations only came from the *specific* utility function $u(x,y) = xy$. We'll see some examples of these kinds of utility functions soon, and in fact, we'll quite often encounter preferences that don't satisfy (2) -- i.e., preferences whose I-curves don't slope downward everywhere. You even have one of these on your current Exercise Set. (You should be able to figure out which one it is.)

# THE HH's DECISION

(COMBINING THE HH's PREFERENCES WITH ITS BUDGET CONSTRAINT)

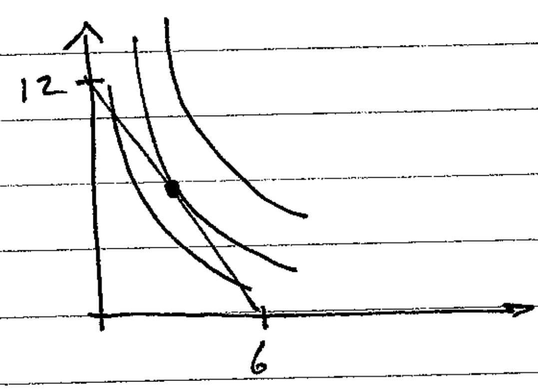
## IN OUR EXAMPLE:

DESCRIPTIVE PRESCRIPTIVE

THE HH WILL CHOOSE (OR SHOULD CHOOSE) THE BUNDLE IT LIKES BEST (i.e., THE ONE THAT YIELDS THE LARGEST VALUE OF $u(x,y) = xy$) AMONG ALL THOSE BUNDLES IT CAN AFFORD.

**GEOMETRICALLY:**

THE BUNDLE CHOSEN IS, AMONG ALL THOSE ON THE B.C., THE ONE ON THE HIGHEST I-CURVE:




**ALGEBRAICALLY:**

THE BUNDLE CHOSEN IS THE ONE THAT IS THE SOLUTION OF THE HH's DECISION PROBLEM,

$$\max\ u(x,y) = xy$$
$$\text{s.t. } 2x + 1y = 12$$
$$(\text{AND } x \geqq 0,\ y \geqq 0).$$

(*) WE WANT TO HAVE A WAY OF CHARACTERIZING, OR DESCRIBING, THE SOLUTION (i.e., THE CHOSEN BUNDLE) IN ANALYTICAL TERMS THAT HAVE ECONOMIC MEANING.

HERE'S ONE WAY WE CAN CHARACTERIZE THE CHOSEN BUNDLE:

(T) $\left\{\begin{array}{l}\text{(x,y) IS THE BUNDLE AT WHICH THE B.C.} \\ \text{IS } \underline{\text{TANGENT}} \text{ TO AN I-CURVE (AND THAT'S} \\ \text{THE HIGHEST I-CURVE THE HH CAN ATTAIN).}\end{array}\right.$

WE CAN USE THIS CONDITION TO "TEST" ANY PROPOSED BUNDLE $(x,y)$ TO DETERMINE WHETHER IT'S THE SOLUTION — THE ONE TO BE CHOSEN.

HERE'S ANOTHER WAY TO SAY THE SAME THING AS (T) — i.e., ANOTHER "TEST":

(C) $(x,y)$ IS <u>ON</u> THE BUDGET CONSTRAINT,

<u>AND</u>

(M) THE SLOPE OF THE I-CURVE AT $(x,y)$ IS THE <u>SAME</u> AS THE SLOPE OF THE B.C.

THESE "TEST" CONDITIONS ARE VERY GEOMETRIC. WE CAN RESTATE THEM ALGEBRAICALLY. THE CONDITION (C) IS EASY:

(C) $2x + 1y = 12$ [IN GENERAL, $p_x x + p_y y = I$.]

HOW CAN WE WRITE THE CONDITION (M) ALGEBRAICALLY?

SLOPE OF I-CURVE AT (x,y) = SLOPE OF B.C.

i.e., $-MRS = -\dfrac{P_x}{P_y}$

i.e., $MRS = \dfrac{P_x}{P_y}$

IN THE EXAMPLE:

$-\dfrac{y}{x} = -2$

$\dfrac{y}{x} = 2$

i.e., PERSONAL VALUE OF MARGINAL X-UNIT, IN TERMS OF Y-UNITS ONE WOULD GIVE UP TO GET THE X-UNIT. = MARKET VALUE OF MARGINAL X-UNIT, IN TERMS OF Y-UNITS ONE HAS TO GIVE UP TO GET THE X-UNIT.

OUR TEST CONDITIONS, THEN, ARE THE FOLLOWING:

$$\boxed{\begin{array}{l} (C)\ P_x x + P_y y = I, \\ (M)\ MRS = \dfrac{P_x}{P_y}, \end{array}}$$

← CONSTRAINT CONDITION

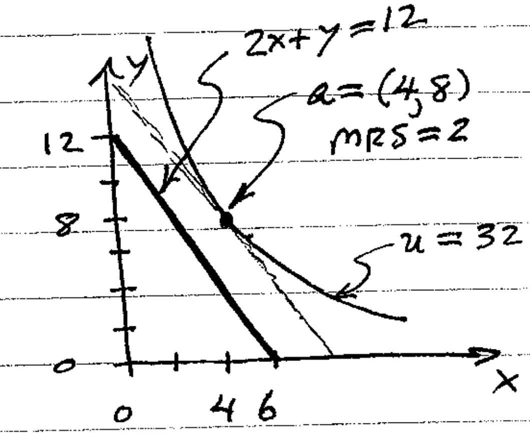
← MARGINAL CONDITION

.... WHICH, IN OUR EXAMPLE, ARE

(C) $2x + 1y = 12$

(M) $\dfrac{y}{x} = 2$.

LET'S "TEST" SOME BUNDLES (IN OUR EXAMPLE) TO SEE WHETHER THEY SATISFY BOTH OF OUR CONDITIONS: [NOTE THAT $\frac{P_x}{P_y} = 2$]

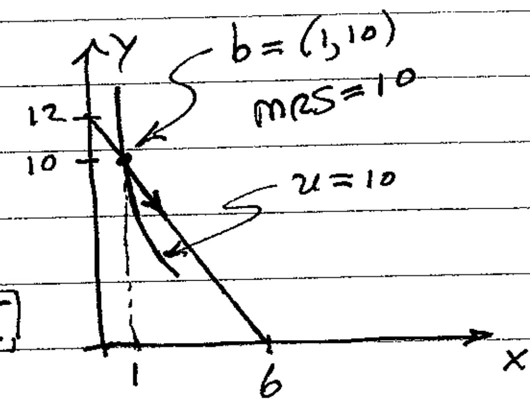



**$a = (4,8)$:**

(m) $MRS = \frac{y}{x} = \frac{8}{4} = 2$ $\boxed{OK}$

(c) $P_x X + P_y Y = (2)(4) + (1)(8)$
$= 16 \neq I$ $\boxed{\text{NOT OK}}$

THIS BUNDLE HAS THE RIGHT MRS, BUT IT COSTS TOO MUCH.




**$b = (1,10)$:**

(c) $P_x X + P_y Y = (2)(1) + (1)(10)$
$= 12 = I$ $\boxed{OK}$

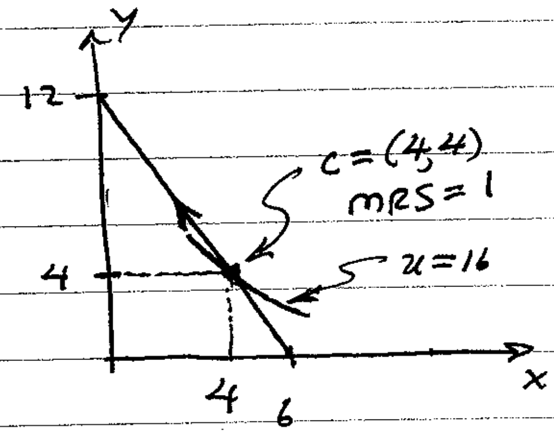
(m) $MRS = \frac{y}{x} = \frac{10}{1} = 10 \neq \frac{P_x}{P_y}$. $\boxed{\text{NOT OK}}$

AT THIS BUNDLE AN ADDITIONAL CHIMI IS WORTH 10 TACOS TO THIS PERSON (i.e., HE WOULD GIVE UP ABOUT 10 TACOS TO GET AN ADDITIONAL CHIMI). (BUT AT THE MARKET PRICES HE ONLY HAS TO GIVE UP 2 TACOS TO GET ONE MORE CHIMI. SO HE WOULD MOVE S.E. ALONG HIS B.C.

$e = (4,4)$:

(c) $p_x x + p_y y = (2)(4) + (1)(4)$
$= 12 = I$ $\boxed{\text{OK}}$

(m) $MRS = \frac{y}{x} = 1 < \frac{P_x}{P_y}$ $\boxed{\text{NOT OK}}$

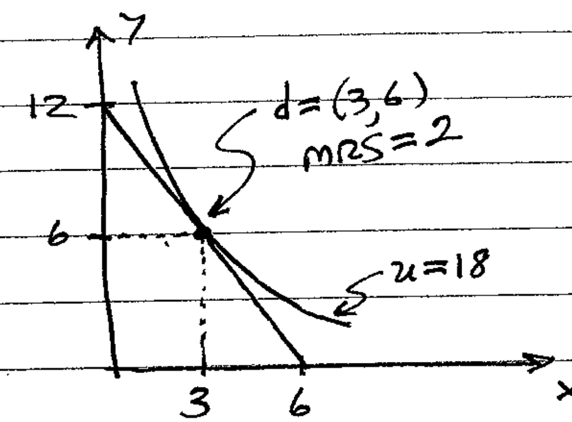



AT THIS BUNDLE AN ADDITIONAL CHIMI IS WORTH 1 TACO TO THIS PERSON, BUT THE MARKET EXCHANGE VALUE IS 2 TACOS PER CHIMI. SO HE WOULD MOVE N.W. ALONG HIS B.C.

$d = (3,6)$:

(c) $p_x x + p_y y = (2)(3) + (1)(6)$
$= 12 = I$ $\boxed{\text{OK}}$

(m) $MRS = \frac{y}{x} = 2 = \frac{P_x}{P_y}$ $\boxed{\text{OK}}$




AT THIS BUNDLE HIS PERSONAL VALUE (MRS) AND THE MARKET VALUE OF CHIMIS IS THE SAME — 2 TACOS PER CHIMI.

AT ALL BUNDLES TO THE SE ON THE B.C., $MRS < \frac{Px}{Py}$, SO "BUY FEWER CHIMIS (& MORE TACOS)."
AT ALL BUNDLES TO THE NW ON THE B.C., $MRS > \frac{Px}{Py}$, SO "BUY MORE CHIMIS (& FEWER TACOS)."

## SOLVING THE TEST CONDITIONS:

FOR NICE, EASY-TO-WORK-WITH UTILITY FUNCTIONS LIKE THE ONE IN OUR EXAMPLE, WE CAN SOLVE THE TEST CONDITIONS TO FIND THE CHOSEN BUNDLE DIRECTLY, INSTEAD OF THE CUMBERSOME TRIAL-AND-ERROR METHOD.

THE CONDITIONS ARE TWO EQUATIONS IN TWO VARIABLES, SO WE CAN OFTEN SOLVE THEM. IN OUR EXAMPLE: $\left[u(x,y) = xy;\ MRS = \frac{y}{x}\right]$

(C) $2x + 1y = 12$

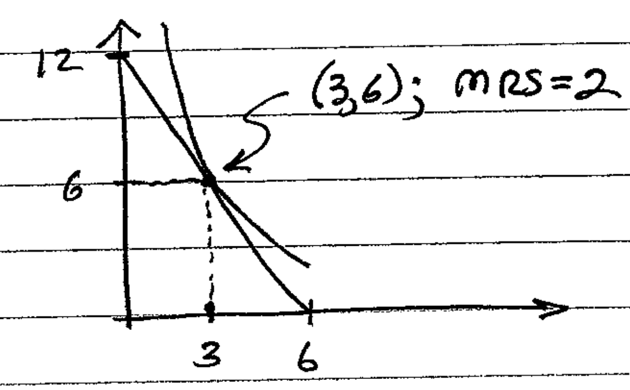
(M) $\frac{y}{x} = 2$; i.e., $y = 2x$,

WHICH WE CAN COMBINE WITH (C):

$$2x + \underbrace{2x}_{=y} = 12$$

i.e., $4x = 12$

$$\therefore \boxed{x = 3;\ \therefore y = 6.}$$



[THE TWO QUESTIONS WE ASKED EARLIER, ABOUT RESPONSES TO PARAMETER CHANGES.]

## WHAT IF $I \uparrow$ TO $I = \$18$?

(c) $2x + 1y = 18$

(m) $\frac{y}{x} = 2$; i.e., $y = 2x$, WHICH YIELDS

(c) $2x + 2x = 18$

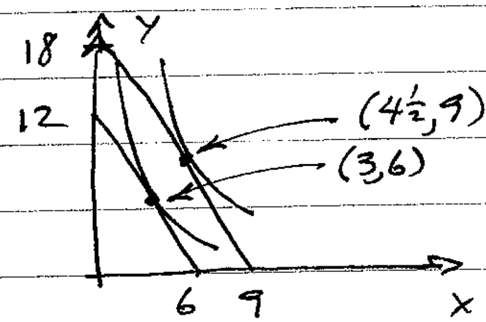
i.e., $4x = 18$

$\therefore \boxed{x = 4\tfrac{1}{2}; \therefore y = 9}$

$\Delta I = +\$6$;

$\Delta x = +1\tfrac{1}{2}$

$\Delta y = +3$




## WHAT IF $p_x \downarrow$ TO $p_x = \$1.50$?

(c) $1\tfrac{1}{2}x + 1y = 12$

(m) $\frac{y}{x} = 1\tfrac{1}{2}$; i.e., $y = 1\tfrac{1}{2}x$, WHICH YIELDS

(c) $1\tfrac{1}{2}x + 1\tfrac{1}{2}x = 12$

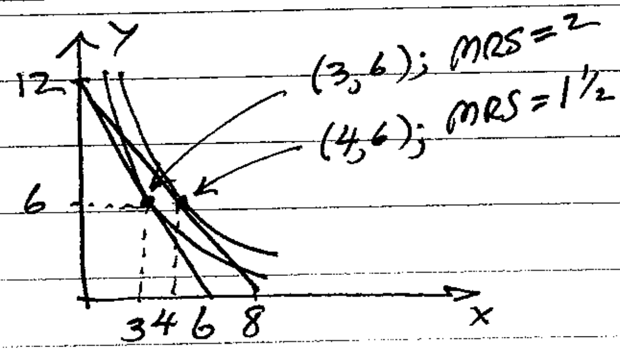
i.e., $3x = 12$.

$\therefore \boxed{x = 4; \therefore y = 6}$

$\Delta p_x = -\tfrac{1}{2}$;

$\Delta x = +1$

$\Delta y = 0$



# A DIFFERENT HH:

A HH WITH $u(x,y) = y + 6\log x$,

IN THE SAME THREE SITUATIONS.

FIRST WE GET THE MRS EXPRESSION, OR FUNCTION:

$$u_y = 1, \quad u_x = \frac{6}{x}; \quad \therefore MRS = \frac{6}{x}.$$

NOTE THAT IT DEPENDS ONLY ON X.

(1) $p_x = \$2,\ p_y = \$1,\ I = \$12$:

(c) $2x + y = 12$

(m) $\frac{6}{x} = 2$; i.e., $\boxed{x = 3; \therefore y = 6}$

∴ SAME BUNDLE AS THE FIRST HH!

(2) ~~$p_x$~~ $I\uparrow$ TO $I = \$18$:

(c) $2x + y = 18$

(m) $\frac{6}{x} = 2$; i.e., $\boxed{x = 3; \therefore y = 12}$

$\Delta I = +6$; $\Delta x = 0$, $\Delta y = +6$

"NO INCOME EFFECT" ON X.

(3) $p_x\downarrow$ TO $p_x = \$1.50$:

(c) $\frac{3}{2}x + y = 12$

(m) $\frac{6}{x} = \frac{3}{2}$; i.e., $\boxed{x = 4; \therefore y = 6}$

$\Delta p_x = -\frac{1}{2}$; $\Delta x = +1$, $\Delta y = 0$.

## ANOTHER HH:

A HH WITH $u(x,y) = y + 8\sqrt{x}$, $= y + 8x^{1/2}$

~~IN THE SAME THREE SITUATIONS.~~

FIRST WE GET THE MRS EXPRESSION, OR FUNCTION:

$$u_y = 1, \quad u_x = 4x^{-1/2} = \frac{4}{\sqrt{x}};$$

$\therefore MRS = \frac{4}{\sqrt{x}}$, WHICH DEPENDS ONLY ON $x$.

(1) $p_x = \$2$, $p_y = \$1$, $I = \$12$:

(c) $2x + y = 12$

(m) $\frac{4}{\sqrt{x}} = 2$; i.e., $\sqrt{x} = 2$; i.e., $\boxed{x = 4; \therefore y = 4}$

(2) $I \uparrow$ TO $I = \$18$:

(c) $2x + y = 18$

(m) $\frac{4}{\sqrt{x}} = 2$; i.e., $\boxed{x = 4, \therefore y = 10}$

$\Delta I = +6$; $\underline{\Delta x = 0}$, $\Delta y = +6$

NO INCOME EFFECT

(3) $p_x \downarrow$ TO $p_x = \$1.50$:

(c) $\frac{3}{2}x + y = 12$

(m) $\frac{4}{\sqrt{x}} = \frac{3}{2}$; i.e., $\sqrt{x} = \frac{8}{3}$; $\therefore \boxed{x = \frac{64}{9} = 7\frac{1}{9} \quad \therefore y = 1\frac{1}{3}}$

$\Delta p_x = -\frac{1}{2}$; $\Delta x = +3\frac{1}{9}$

$\Delta y = -2\frac{2}{3}$ ← ! "AS IF" $I \uparrow$, BUT $y \downarrow$: SUBSTITUTION.

9a

Economics 361 Fall 1995

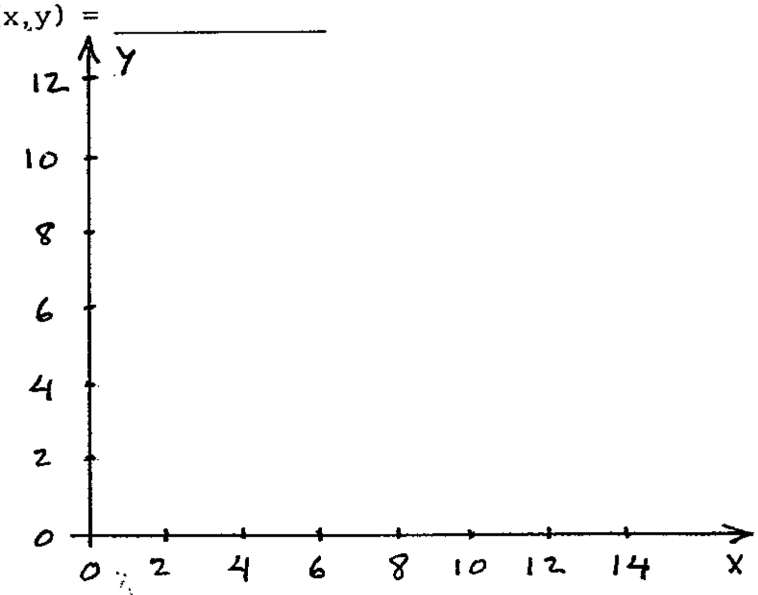
Quiz #2

1. For the utility function $u(x,y) = y + 10\sqrt{x}$ , determine the MRS at the bundle $(x,y) = (25,16)$. MRS = ___________

2 Elaine enjoys doughnuts and coffee according to the utility function $u(x,y) = 4x + 3y$, where x denotes the number of doughnuts she eats and y denotes the cups of coffee she drinks.

(a) What is Elaine's MRS for doughnuts?

(b) In a very expensive coffee shop, where doughnuts cost \$3 each and coffee costs \$3 a cup, Jerry gives the cashier \$12 and tells Elaine to choose whatever \$12 combination of doughnuts and coffee she wants. On the diagram below, draw Elaine's budget constraint and two of her indifference curves. What combination does she choose? (x,y) = ____________



# "Boundary" Choices

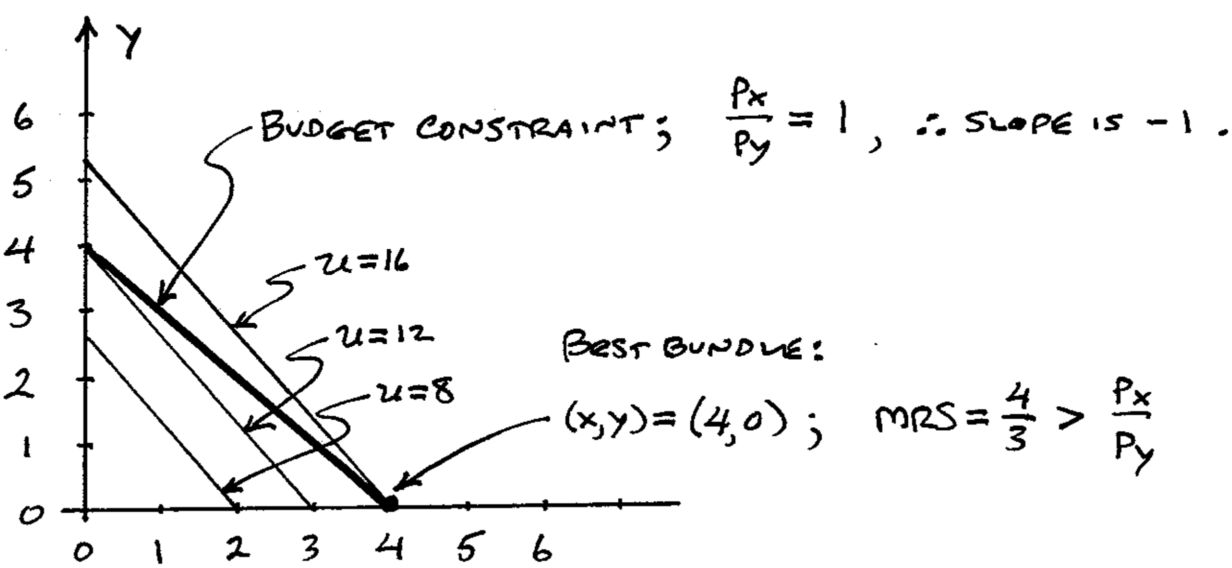
In our Elaine and Jerry example (Quiz #2; also see text exercise #3.5), Elaine chose four doughnuts and no coffee — i.e., she chose the bundle $(x,y) = (4,0)$.

Because Elaine's utility function is $u(x,y) = 4x + 3y$, and her budget constraint is $3x + 3y = 12$, we see that

$$MRS = \frac{4}{3} \quad \text{and} \quad \frac{P_x}{P_y} = 1.$$

Thus, $MRS > \frac{P_x}{P_y}$ at all possible bundles, and therefore no bundle can satisfy the (M) test condition, namely, the marginal condition.

The diagram of Elaine's decision problem looks like this:



SINCE $MRS > \frac{P_x}{P_y}$, ELAINE WOULD LIKE TO <u>INCREASE</u> $x$ (HER DOUGHNUT CONSUMPTION), BY <u>REDUCING</u> $y$ (HER COFFEE CONSUMPTION), USING THE PROCEEDS FROM HER COFFEE REDUCTION TO PURCHASE MORE DOUGHNUTS.

BUT SHE CAN'T DO THIS, BECAUSE THE BUNDLE SHE IS CONTEMPLATING, NAMELY $(x,y)=(4,0)$, <u>ALREADY INCLUDES NO COFFEE</u>. IT'S CLEAR, THEN, THAT . . .

WHEN $y=0$, A BUNDLE CAN BE A BEST CHOICE EVEN IF $MRS > \frac{P_x}{P_y}$, BECAUSE <u>ALL</u> OF $I$ IS ALREADY BEING SPENT ON THE "HIGHER-VALUE" GOOD $x$.

SIMILARLY,

WHEN $x=0$, A BUNDLE CAN BE A BEST CHOICE EVEN IF $MRS < \frac{P_x}{P_y}$, BECAUSE <u>ALL</u> OF $I$ IS ALREADY BEING SPENT ON THE "HIGHER-VALUE" GOOD $y$.

CONSEQUENTLY, OUR TEST CONDITIONS ARE

(C) $\quad P_x x + P_y y = I$

(M) $\quad MRS = \frac{P_x}{P_y}$, IF $x>0$ & $y>0$ ("INTERIOR" BUNDLES)

AND $\left[\begin{array}{l} MRS \geq \frac{P_x}{P_y}, \text{ IF } y=0 \\ MRS \leq \frac{P_x}{P_y}, \text{ IF } x=0 \end{array}\right]$ ("BOUNDARY" BUNDLES).

## Another Example:

$$u(x,y) = y + 6\log x \qquad \left(MRS = \frac{6}{x}\right).$$

$$P_x = \$2,\ P_y = \$3, \text{ AND } I = \$12.$$

THEREFORE THE TEST CONDITIONS ARE

(c) $2x + 3y = 12$

(m) $\frac{6}{x} = \frac{2}{3}$ ; i.e., $\boxed{x = 9.}$

BUT AT $x = 9$ WE FIND THAT $p_x x = 18$ — i.e. \$18 IS SPENT ON $x$, WHICH EXCEEDS $I$. JUST LIKE ELAINE, THIS CONSUMER WOULD LIKE TO INCREASE X-CONSUMPTION (TO $x=9$ IN THIS CASE) BUT SPENDING ALL OF $I$ ON THE X-GOOD YIELDS (IN THIS CASE) ONLY $x = 6$.

DOES THE BUNDLE $(x,y) = (6,0)$ SATISFY THE TEST CONDITIONS? YES: IT CLEARLY SATISFIES (C) — IT'S ON THE BUDGET CONSTRAINT; AND IT SATISFIES THE "BOUNDARY" VERSION OF THE MARGINAL CONDITION

$$MRS \geqq \frac{P_x}{P_y} \text{ IF } y = 0,$$

BECAUSE $MRS = 1$ AND

$$\frac{P_x}{P_y} = \frac{2}{3}.$$