Economics 519 Exam
University of Arizona
Fall 2013

Closed-book part: Don't consult books or notes until you've handed in your solutions to Problems #1 - #4. After you do consult books or notes, your solutions to Problems #1 - #4 will not be accepted.

1. Prove that a subset of a complete metric space is itself complete if and only if it is closed.

2. Prove that a compact subset S of a metric space (X, d) is a complete metric space. Don't assume that the space (X, d) is either compact or complete.

3. Is ℓ^∞_{++} an open subset of ℓ^∞? Verify your answer.

4. Provide counterexamples to show that each of the following two conjectures is false:

(a) If $S \subseteq \mathbb{R}^2$ is compact and has a nonempty interior, and $f : S \to S$ is continuous, then f has a fixed point.

(b) If $S \subseteq \mathbb{R}^2$ is bounded and convex and has a nonempty interior, and $f : S \to S$ is continuous, then f has a fixed point.

Open-book part: Be sure to turn in your solutions to Problems #1 - #4 before starting this.

5. In our growth theory example, we had a continuous production function $f : \mathbb{R}_+ \to \mathbb{R}_+$. In order to apply the Maximum Theorem, we said (but didn't prove) that the correspondence $\varphi : \mathbb{R}_+ \twoheadrightarrow \mathbb{R}_+$ defined by $\varphi(x) = [0, f(x)]$ is continuous (both UHC and LHC), where $[0, f(x)]$ is the closed interval from 0 to $f(x)$. Prove either that φ is UHC or that it's LHC; you needn't prove both.

6. Here is a *false* version of the Maximum Theorem (*i.e.*, it's a false conjecture):

Conjecture: Let X be a subset of $\mathbb{R}^l$; let E be a subset of $\mathbb{R}^m$; let $u : X \times E \to \mathbb{R}$ be a continuous function; and let $\varphi : E \twoheadrightarrow X$ be a UHC and compact-valued correspondence. Then the correspondence $\mu : E \twoheadrightarrow X$ defined by $\mu(e) = \{\ x \in \varphi(e) \mid x \text{ maximizes } u(\cdot, e) \text{ on } \varphi(e)\ \}$ is nonempty-valued, compact-valued, closed, and UHC, and the value function $v : E \to \mathbb{R}$ defined by $v(e) = \max_{x \in \varphi(e)} u(x, e)$ is continuous.

The actual Maximum Theorem requires that φ be continuous — *i.e.*, LHC as well as UHC. Provide a counterexample to show that even if both E and X are compact and μ is non-empty-valued, the conjecture is false: μ may fail to be UHC and v may fail to be continuous.