TRANSLATING BETWEEN MAXIMIZATION AND MINIMIZATION

(P) $\max f(x)$ s.t. $g(x) \leq b$ FOC: $\exists \lambda \geq 0$ s.t. $\nabla f = \lambda \nabla g$.

DEFINE $\tilde{f}(x) = -f(x)$, $\tilde{g}(x) = -g(x)$, AND $\tilde{b} = -b$, AND THE PROBLEM

($\tilde{P}$) $\min \tilde{f}(x)$ s.t. $\tilde{g}(x) \geq \tilde{b}$ FOC: $\exists \lambda \geq 0$ s.t. $\nabla \tilde{f} = \lambda \nabla \tilde{g}$

i.e., $\nabla(-f) = \lambda \nabla(-g)$

i.e., $\nabla f = \lambda \nabla g$.

NOTE THAT IT'S CONVENIENT TO SAY THE STANDARD FORM FOR A MINIMIZATION PROBLEM ~~IS~~ HAS $\tilde{g}(x) \geq \tilde{b}$ RATHER THAN $\tilde{g}(x) \leq \tilde{b}$. THIS LEAVES THE FOC THE SAME, WITH $\lambda \geq 0$, AND λ IS STILL THE MARGINAL VALUE OF AN INCREASE IN $\tilde{b}$, THE RHS OF THE CONSTRAINT:

APPLYING THE ENVELOPE THEOREM TO THE VALUE FUNCTION:

$$\frac{d\tilde{v}}{d\tilde{b}} = \frac{d(-v)}{d(-b)} = \frac{dv}{db}; \quad \therefore \frac{d\tilde{v}}{d\tilde{b}} = \lambda.$$