The Walrasian Model and Walrasian Equilibrium

1.1 There are only two goods in the economy and there is no way to produce either good. There are n individuals, indexed by $i = 1, \ldots, n$. Individual i owns $\mathring{x}_1^i$ units of good #1 and $\mathring{x}_2^i$ units of good #2, and his preference is described by the utility function $u^i(x_1^i, x_2^i) = \alpha_1^i \log x_1^i + \alpha_2^i \log x_2^i$, where x_1^i and x_2^i denote the amounts he consumes of each of the two goods, and where α_1^i and α_2^i are both positive. Let ρ denote the price ratio p_1/p_2. Express the equilibrium price ratio in terms of the parameters $((\mathring{\mathbf{x}}^i, \alpha^i))_{i=1}^2$ that describe the economy.

1.2 Ann and Bob each own 10 bottles of beer and, altogether, they own 20 bags of peanuts. There are no other people and no other goods in the economy, and no production of either good is possible. Using x to denote bottles of beer and y to denote bags of peanuts, Ann's and Bob's preferences are described by the following utility functions:

$$u_A(x_A, y_A) = x_A y_A^4 \qquad \text{and} \qquad u_B(x_B, y_B) = 2x_B + y_B.$$

In each of the following cases, determine the market equilibrium price ratio and allocation and depict the equilibrium in an Edgeworth box diagram.

(a) Bob owns 20 bags of peanuts and Ann owns no peanuts.
(b) Bob owns 15 bags of peanuts and Ann owns 5 bags.
(c) Ann owns 20 bags of peanuts and Bob owns no peanuts.

1.3 Quantities of the economy's only two goods are denoted by x and y; no production is possible. Ann's and Ben's preferences are described by the utility functions

$$u^A(x, y) = x + y \qquad \text{and} \qquad u^B(x, y) = xy.$$

Ann owns the bundle (0,5) and Ben owns the bundle (30,5). Determine the Walrasian equilibrium price(s) and allocation(s).

1.4 There are two goods (quantities x and y) and two people (Al and Bill) in the economy. Al owns eight units of the x-good and none of the y-good. Bill owns none of the x-good, and three units of the y-good. Their preferences are described by the utility functions

$$u^A(x_A, y_A) = x_A y_A \quad \text{and} \quad u^B(x_B, y_B) = y_B + \log x_B.$$

Determine both consumers' demand functions and the market demand function, and the competitive (Walrasian) equilibrium price(s) and allocation(s).

1.5 There are two consumers, Al and Bill, and two goods, the quantities of which are denoted by x and y. Al and Bill each own 100 units of the Y-good; Al owns 12 units of the X-good and Bill owns 3 units. Their preferences are described by the utility functions

$$u_A(x_A, y_A) = y_A + 60x_A - 2x_A^2 \quad \text{and} \quad u_B(x_B, y_B) = y_B + 30x_B - x_B^2.$$

Note that their marginal rates of substitution are $MRS_A = 60 - 4x_A$ and $MRS_B = 30 - 2x_B$.

(a) Al proposes that he will trade *one unit* of the X-good to Bill in exchange for some units of the Y-good. Al and Bill turn to you, their economic consultant, to tell them how many units of the Y-good Bill should give to Al in order that this trade make them both strictly better off than they would be if they don't trade. What is your answer? Using marginal rates of substitution, explain how you know your answer will make them both better off.

(b) Draw the Edgeworth box diagram, including each person's indifference curve through the initial endowment point. Use different scales on the x- and y-axes or your diagram will be very tall and skinny.

(c) Determine all Walrasian equilibrium prices and allocations.

1.6 The Arrow and Debreu families live next door to one another. Each family has an orange grove that yields 30 oranges per week, and the Arrows also have an apple orchard that yields 30 apples per week. The two households' preferences for oranges (x per week) and apples (y per week) are given by the utility functions

$$u_A(x_A, y_A) = x_A y_A^3 \qquad \text{and} \qquad u_D(x_D, y_D) = 2x_D + y_D.$$

The Arrows and Debreus realize they may be able to make both households better off by trading apples for oranges.

Determine all Walrasian equilibrium price lists and allocations.

1.7 Amy and Bob consume only two goods, quantities of which we'll denote by x and y. Amy and Bob have the same preferences, described by the utility function

$$u(x,y) = \begin{cases} x + y - 1, & \text{if } x \geqq 1 \\ 3x + y - 3, & \text{if } x \leqq 1. \end{cases}$$

There are 4 units of the x-good, all owned by Amy, and 6 units of the y-good, all owned by Bob.

Draw the Edgeworth box diagram, including each person's indifference curve through the initial endowment point. Determine all Walrasian equilibrium prices and allocations.

1.8 There are r girls and r boys, where r is a positive integer. The only two goods are bread and honey, quantities of which will be denoted by x and y: x denotes loaves of bread and y denotes pints of honey. Neither the girls nor the boys are well endowed: each girl has 8 pints of honey but no bread, and each boy has 8 loaves of bread but no honey. Each girl's preference is described by the utility function $u_G(x,y) = \min(ax, y)$ and each boy's by the utility function $u_B(x,y) = x + y$.

Determine the Walrasian excess demand function for honey and the Walrasian equilibrium prices and allocations.

1.9 There are only two consumers, Amy and Bev, and only two goods, the quantities of which are denoted by x and y. Amy owns the bundle $(4,5)$ and Bev owns the bundle $(16,15)$. Amy's and Bev's preferences are described by the utility functions

$$u_A(x_A, y_A) = \log x_A + 4\log y_A \quad \text{and} \quad u_B(x_B, y_B) = y_B + 5\log x_B.$$

Note that the derivatives of their utility functions are

$$u_{Ax} = \tfrac{1}{x_A}, \qquad u_{Ay} = \tfrac{4}{y_A}, \qquad u_{Bx} = \tfrac{5}{x_B}, \qquad u_{By} = 1.$$

Determine a Walrasian equilibrium, and verify by direct appeal to the definition that the equilibrium you have identified is indeed an equilibrium.

1.10 A consumer's preference is described by the utility function $u(x, y) = y + \alpha \log x$ and her endowment is denoted by $(\mathring{x}, \mathring{y})$. Determine her offer curve, both analytically and geometrically.

1.11 There are two goods (quantities denoted by x and y) and two consumers (Ann and Bob). Ann and Bob each own three units of each good. Ann's preferences are described by the relation $MRS = y/x$ (you should be able to give a utility function that describes these preferences), but Bob's preferences are a little more complicated to describe:

If $y < \frac{1}{2}x$, then his indifference curve through (x, y) is horizontal.
If $y > 2x$, then his indifference curve through (x, y) is vertical.
If $\frac{1}{2}x < y < 2x$, then his MRS is x/y . (Note that this could be described by the utility function $u(x, y) = x^2 + y^2$ in this region.)

(a) Determine Bob's offer curve, both geometrically (first) and then analytically. (Note that Bob's demand is not single-valued at a price ratio of $\rho = 1$.)

(b) Show that there is no price that will clear the markets – i.e., there is no Walrasian equilibrium. Do this three ways:

by drawing the aggregate offer curve,
by drawing both individual offer curves in an Edgeworth box,
and by writing the aggregate demand function analytically, and showing that at each price the market fails to clear.

1.12 The demand and supply functions for a good are

$$D(p) = \alpha + \log \frac{a}{p^2} \qquad \text{and} \qquad S(p) = \beta - e^{-bp}.$$

where each of the parameters a, b, α, and β are positive. Determine how changes in the parameters will affect the equilibrium price and quantity. What is a natural interpretation of the parameter β?

1.13 The demand for a particular good is given by the function $D(p) = \alpha - 20p + 4p^2 - \frac{1}{6}p^3$ and the supply by $S(p) = 4p$. An equilibrium price of $p = 6$ is observed, but then α increases.

(a) Estimate the change in the equilibrium price if α increases by 2.

(b) Estimate the change in the equilibrium price if α increases by 1 percent. Determine the elasticity of demand at $p = 6$.

(c) Your answers to (a) and (b) should seem a little odd. What is it that is unusual here? Plot the excess demand function, and show why this unusual result is occurring here. How many equilibria are there in this market? Which of the equilibria have this unusual feature? Do you think this kind of demand function is possible?

1.14 The excess demand for a particular good is given by the function $E(p) = 3-(5+\alpha)p+5p^2-p^3$ for $p > 0$. For all positive values of α, determine how many equilibria there are and determine $\frac{\partial p^*}{\partial \alpha}$, where p^* denotes the equilibrium price.

1.15 Arnie has five pints of milk and no cookies. A cookie and a pint of milk are perfect substitutes to Arnie so long as he has no more than six cookies. He has no desire for more than six cookies: if he had more, he would sell or discard all but six. He always likes more milk. Bert has ten cookies and five pints of milk. He has no use for cookies — if he has cookies, he either sells them or discards them. He too always likes more milk.

(a) Provide a utility function that describes Arnie's preferences, and draw his indifference curve through the bundle $(6, 6)$.

(b) Determine all the Walrasian equilibrium price-lists and allocations. Verify that you've identified all the equilibria.

1.16 There are only two consumers, Ann and Bob, and only two goods, the quantities of which are denoted by x and y. Ann owns the bundle $(15, 25)$ and Bob owns the bundle $(15, 0)$. Ann's and Bob's preferences are described by the utility functions

$$u_A(x_A, y_A) = 6 \log x_A + \log y_A \quad \text{and} \quad u_B(x_B, y_B) = y_B + 30 \log x_B.$$

(a) Determine each consumer's demand function.

(b) Determine a Walrasian equilibrium price-list and allocation.

(c) Depict the equilibrium in an Edgeworth box diagram.

1.17 A market equilibrium price list $\widehat{\mathbf{p}}$ is required to satisfy the market-clearing equilibrium condition

$$\forall k = 1, \ldots, l: \quad z_k(\widehat{\mathbf{p}}) \leqq 0 \quad \text{and} \quad z_k(\widehat{\mathbf{p}}) = 0 \ \text{ if } \widehat{p}_k > 0. \tag{\textbf{Clr}}$$

where $\mathbf{z}(\cdot)$ is the market net demand function. Prove that if $\mathbf{z}(\cdot)$ satisfies Walras's Law, then $\widehat{\mathbf{p}}$ is an equilibrium price list if it merely satisfies

$$\forall k = 1, \ldots, l: \quad z_k(\widehat{\mathbf{p}}) \leqq 0. \tag{$*$}$$

In other words, if $\mathbf{z}(\cdot)$ satisfies Walras's Law and $\widehat{\mathbf{p}}$ satisfies $(*)$, then every good with a strictly positive price has exactly zero net demand.