Imperfect Competition

6.1 There are only two firms producing a particular product. The demand for the product is given by the relation $p = 24 - Q$, where p denotes the price (in dollars per unit) and Q denotes the total quantity sold by the two firms. The firms both have constant marginal cost: it costs Firm #1 eight dollars for each unit it produces and Firm #2 four dollars for each unit it produces. Neither firm has any fixed costs.

(a) Assume that each firm behaves as if its own decisions will not affect the quantity that the other firm tries to sell, and that each firm tries to maximize its own profit – in other words, each behaves as a Cournot duopolist. Determine the equilibrium price and quantity in the market, and determine each firm's production and profit.

(b) Determine the same items as in (a) under the assumption that the firms cooperate fully with one another.

(c) Determine the same items as in (a) under the assumption that each firm behaves as a price taker, taking the market price as given when making its decision.

6.2 There are only two limousine firms capable of driving passengers between the airport and downtown. The two firms' services are identical (in particular, each carries only a single passenger on each trip), but the firms' costs of production differ: it costs one of the firms only \$10 per trip and it costs the other \$20 per trip. The market demand for limousine trips from the airport to downtown is given by the equation $Q = 240 - 4p$, where Q denotes the number of trips purchased per week when the price is p dollars per trip.

(a) What is the Cournot equilibrium in this market?

(b) If the firms cooperate fully to maximize their joint profits, how many trips will each firm make?

(c) There are forty consumers. Each one has a weekly income of at least \$300 and each one's preference is described by the utility function $u(x, y) = y - (1/20)(60 - 10x)^2$, where x is the number of trips she makes per week and y is the number of dollars she has available to spend on other goods. Each firm cares only about its own profits. Find an allocation that makes all the consumers *and* each of the two firms strictly better off than in the Cournot equilibrium allocation.

(d) Determine the consumer surplus, producer surplus, and total surplus at each of the outcomes in (a), (b), and (c).

6.3 There are only two goods in the world, bread and wheat, and there are 102 people, 100 of them called "consumers" and two called "producers." Each consumer is endowed with 140 pounds of wheat and no bread and has preferences described by the utility function $u(x, y) = y - (1/2)x^2 + 20x$, where x and y are the number of loaves of bread and pounds of wheat that he consumes. The consumers all behave as price-takers, and the price of wheat is always \$1 per unit. Thus, each consumer's demand function for bread is $x = 20 - p$, where p is the price of bread (in dollars per loaf).

Each producer has no endowment of either good, but owns a machine that can turn wheat into bread, producing a loaf of bread for every eight pounds of wheat used as input. There is no other way for anyone in the economy to transform wheat into bread or bread into wheat. Each producer cares only for wheat; i.e., his preferences are described by $u(x, y) = y$.

(a) Suppose the two producers behave as Cournot duopolists. Determine the equilibrium price of bread, the amount produced by each producer, and the resulting consumption of bread and wheat by each of the economy's 102 participants.

(b) If the allocation in (a) is Pareto optimal, verify that it is. If it is not, find a Pareto optimal allocation in which all 102 members of the economy are strictly better off.

(c) What if each of the consumers were endowed with only 120 pounds of wheat?

6.4 There are only two firms producing a particular product. Demand for the product is given by the equation $Q = 24 - p$, where p denotes the price at which the product is sold (in dollars) and Q denotes the resulting quantity demanded. The two firms' products are perfect substitutes to all consumers, so both firms receive the same price for every unit sold. Firm 1's cost of production is \$15 per unit and Firm 2's is \$18 per unit; neither firm incurs any fixed costs.

(a) Determine the market price and each firm's production under each of the following assumptions:

(a$'$) The market is competitive.

(a$''$) The firms behave as Cournot duopolists. Include a diagram of the two firms' reaction curves.

(a$'''$) The firms collude. Include bounds on the monetary transfers between the firms.

(b) Now assume that Firm #1's unit cost has fallen to \$6. Draw the firms' reaction curves, and compare the market outcome under Cournot behavior with the outcome under collusive behavior.

(c) Compare the core in (a) and (b), assuming the only players are the two firms.

6.5 There are three firms selling a homogeneous good. Demand for the good is given by $p = 300 - Q$, where Q denotes the total quantity sold by all three firms. The firms all have constant per-unit costs of production: Firm 1's cost is \$20 per unit of output, Firm 2's is \$40 per unit, and Firm 3's \$80 per unit. Determine the Cournot equilibrium.

6.6 Firm 1 and Firm 2 are the only firms that can produce in a particular market. The firms' costs of production are described by the cost functions

$$C_1 = 4x_1 \qquad \text{and} \qquad C_2 = K + 2x_2.$$

for positive levels of output x_1 and x_2. Each firm's cost is zero if it produces at level zero. The market demand function for the (homogeneous) good produced by the firms is $Q = 12 - p$, where p is the price and Q is the quantity demanded.

(a) If the firms cooperate with one another, operating as a cartel, determine the levels of output x_1 and x_2 by each firm and the market price p, all as a function of Firm 2's fixed cost K.

(b) Now suppose that the two firms do not cooperate: each firm attempts to maximize its own profit, and each assumes that its own actions will have no effect upon the quantity that the other will offer for sale in the market. In a single diagram, draw each firm's reaction curve. What will be the Nash equilibrium outcome (each firm's output and the market price) if $K = 16$? How would your answer change (if at all) if $K > 16$?

(c) Now suppose that the two firms behave "competitively" — *i.e.*, each takes the market price as given and chooses an output level that will maximize its profit. What will be the equilibrium outcome?

6.7 There are only two firms in a market in which the demand curve for the product the firms produce is $Q_D = 24 - p$, where p is the market price of the product in dollars and Q_D is the quantity demanded. It costs Firm 1 six dollars for each unit it produces, and it costs Firm 2 eight dollars for each unit it produces. Neither firm has any fixed costs. Each firm has a capacity constraint: Firm 1 can produce no more than six units, and Firm 2 can produce no more than eighteen units.

(a) Determine the outcome (price, production levels, total profit) if the two firms cooperate to maximize joint profits. Draw the total and marginal cost curves for the joint-profit maximization problem

(b) Determine the outcome if each firm is a price-taker. Draw the market demand and supply curves.

(c) Determine the outcome if the firms behave as Cournot duopolists. Draw the firms' reaction functions.

6.8 There are two firms supplying a particular market. Consumers do not regard the two firms' products as perfect substitutes for one another: if the two firms charge the respective prices p_1 and p_2 for their products, then sales will be $q_1 = 12 - .3p_1 + .2p_2$ and $q_2 = 36 + .1p_1 - .4p_2$. The firms' cost functions are $C_1(q_1) = 10q_1$ and $C_2(q_2) = 20q_2$.

(a) Determine the Bertrand equilibrium

(b) Determine the Cournot equilibrium

(c) Determine the outcome if the two firms fully cooperate to maximize their joint profits.

Note: the Bertrand and cooperative outcomes are not nice integers. The Bertrand equilibrium is $(p_1, p_2) = (\$45.22, \$60.65)$, approximately; and the cooperative outcome is $(q_1, q_2 = (9.23, 12.05)$, approximately.

The fact that the Cournot and Bertrand outcomes differ might seem a little bit puzzling: if I take my rival's action as given, this leaves me facing a downward-sloping demand curve for my product, so it will not matter whether my "decision variable" is my price or my quantity, because one determines the other via the demand curve. What is the explanation of this seeming paradox?

6.9 Airhead and Bubbles are the only two firms producing natural spring water. The firms draw their waters from different springs (Airhead's water is "still" and Bubbles' is carbonated), so each firm has some "market power." Specifically, the demands for the two firms' waters are given by

$$q_A = 30 - 2p_A + p_B \qquad \text{and} \qquad q_B = 15 - 2p_B + p_A,$$

where p_i denotes the price per gallon charged by Firm i and q_i denotes the resulting number of gallons the customers of Firm i will purchase. Each firm's production is costless.

(a) Determine the equilibrium prices and quantities if each firm takes as given the price charged by the other firm (*i.e.*, find the Bertrand equilibrium). Draw the two firms' reaction curves in a single diagram.

(b) Now assume instead that each firm takes as given the *quantity* produced by the other firm, and determine the Cournot equilibrium. You'll first need to convert the demand functions for the two firms' products, given above, to the corresponding inverse demand functions, which give p_A and p_B as functions of the quantities q_A and q_B the two firms produce.

(c) The Bertrand equilibrium is often said to be the outcome if the firms "compete in prices" and the Cournot equilibrium the outcome if the firms "compete in quantities." But in either case, each firm is facing a residual demand curve for its product. So it doesn't matter whether Airhead, for example, "chooses price" or "chooses quantity": the residual demand curve dictates that choosing either one determines the other. In other words, it shouldn't matter whether the firms "choose prices" or "choose quantities" — the outcome ought to be the same. However, in (a) and (b) you should have found that the Cournot equilibrium differs from the Bertrand equilibrium: the two equilibria have different prices and different quantities. What's the explanation of this seeming paradox?

(d) Denote the Bertrand equilibrium prices and quantities that you calculated in (a) by $\widehat{p}_A$, $\widehat{p}_B$, $\widehat{q}_A$, and $\widehat{q}_B$. Determine Airhead's residual demand curve when it assumes that Bubbles will charge the price $\widehat{p}_B$. Then determine Airhead's residual demand curve if it instead assumes that Bubbles will produce its Bertrand equilibrium *quantity* $\widehat{q}_B$.

6.10 Consider a market in which all buyers are price-takers, each with the demand function $x = 40 - p$, where p denotes the price of the product and x the quantity the consumer purchases. There are two firms supplying this market, and the two firms are located near one another. Production generates air pollution, and the pollution in turn increases each firm's cost of production. Specifically, every unit produced by either firm adds one unit of pollution to the air, and the firms' cost functions are $C_1(q_1) = 2Aq_1$ and $C_2(q_2) = 3Aq_2$, where A is the total amount of pollution in the air, and where q_i is the per-capita production by Firm i (*i.e.*, the firm's production divided by the number of buyers in the market). The pollution level A is equal to the total per-capita production by the two firms, $q_1 + q_2$. Assume throughout that each firm understands how its own marginal cost is affected the amount of pollution and thus by the firms' production levels.

Determine the market price and each firm's production level under each of the following behavioral assumptions:

(a) Each firm takes the other's production level as parametric and maximizes profit.

(b) The two firms cooperate fully as a cartel, maximizing joint profits.

(c) Each firm behaves competitively — *i.e.*, is a price-taker — and maximizes profit.

6.11 Two firms produce similar but differentiated products. They choose their prices strategically, each firm taking the other's price as given, *i.e.*, unaffected by its own decisions. The demand for each firm's product and its cost are given by the equations

	Demand	Costs
Firm 1:	$q_1 = 120 - 30p_1 + 20p_2$	$C_1(q_1) = 4q_1$
Firm 2:	$q_2 = 240 + 10p_1 - 20p_2$	$C_2(q_2) = 8q_2$

(a) Determine the Bertrand equilibrium prices, output levels, and profits.

Inverting the demand functions given above, we obtain the following (equivalent) description of the demand for the firms' products:

Firm 1:	$p_1 = 18 - (1/20)q_1 - (1/20)q_2$
Firm 2:	$p_2 = 21 - (1/40)q_1 - (3/40)q_2$

(b) Determine the Cournot equilibrium output levels, prices, and profits. You should find that they're different than the ones you obtained in (a).

(c) Using the first demand system and treating the prices as the decision variables, determine the prices, output levels, and profits if the firms collude to maximize their total profit. Then do the same, using the second demand system and treating the quantities as the decision variables. You should obtain the same outcome both ways.

(d) Suppose Firm 2 is charging the price $p_2 = \$12$ and is producing the quantity $q_2 = 80$. Assume that Firm 1 takes Firm 2's price of \$12 to be unaffected by its own decision; determine Firm 1's residual demand curve, marginal revenue curve, and profit-maximizing output level and price. Now assume instead that Firm 1 takes Firm 2's output of 80 units to be unaffected by its own decision; determine Firm 1's residual demand curve, marginal revenue curve, and profit-maximizing output level and price. You should find that everything is different in the first case than in the second case.