Public Goods and Other Externalities

8.1 Ms. Alpha and Mr. Beta live together. Each cares only about the cleanliness of the house they share and about the simoleans he or she consumes. Denote the level of cleanliness by x, and denote Ms. Alpha's and Mr. Beta's consumption of simoleans by y_A and y_B. Ms. Alpha's utility function is $u(x, y) = \min\{2x, y_A\}$ and Mr. Beta's is $u(x, y) = \min\{x, y_B\}$. It's only possible to convert simoleans into cleanliness at a rate of one simolean for each unit of cleanliness. If no simoleans are devoted to cleaning the house, the resulting level of cleanliness is $x = 0$. Ms. Alpha and Mr. Beta are endowed with a total of 120 simoleans; therefore the feasible allocations are the ones that satisfy $x + y_A + y_B = 120$.

(a) Four alternative allocations are described below. For each of the allocations do the following: Determine if the allocation is Pareto optimal; if a Pareto improvement exists, find a Pareto optimal allocation that makes *both* people strictly better off.

(a1) (x, y_A, y_B) = (30,60,30)
(a2) (x, y_A, y_B) = (60,20,40)
(a3) (x, y_A, y_B) = (40,50,40)
(a4) (x, y_A, y_B) = (36,40,36)

(b) Draw the utility-possibility frontier for these Ms. Alpha and Mr. Beta.

Assume for (c) and (d) that ownership of the 120-simolean endowment is divided equally between Ms. Alpha and Mr. Beta.

(c) Can the allocation (a1) above be supported as a "voluntary-contributions" equilibrium – *i.e.*, is the allocation a noncooperative equilibrium if the household's outcome is determined by each person voluntarily contributing simoleans to be used for cleaning? Determine the set of all voluntary-contributions equilibria.

(d) Can the allocation (a1) above be supported as a Lindahl equilibrium? Determine the set of all Lindahl equilibria.

8.2   There are two consumers, Ms. A and Ms. B, and two goods, quantities of which will be denoted by x and y. The x-good is a pure public good which can be produced at a constant marginal cost of c units of the y-good for each unit of the x-good produced. Each consumer's preference for the two goods has the same form, differing only in a parameter α: the consumer's marginal rate of substitution between the goods is 3 when $x < \alpha$ and is 1 when $x > \alpha$. The following utility function can be used to describe these preferences:

$$u(x, y) = \begin{cases} y + (x - \alpha) & = & x + y - \alpha, & \text{if } x \geqq \alpha \\ y + 3(x - \alpha) & = & 3x + y - 3\alpha, & \text{if } x \leqq \alpha. \end{cases}$$

The parameter values for Ms. A and Ms. B satisfy $0 < \alpha_A < \alpha_B < 10$. There are 100 units of the y-good to be used for consumption and/or production of the public good. An interior allocation is defined to be one in which each consumer receives a positive amount of the y-good.

(a) Let $c = 5$. Determine all the Pareto optimal allocations, if there are any. For every interior allocation (x, y_A, y_B) that is not Pareto optimal, find an allocation that is both Pareto optimal and a Pareto improvement upon (x, y_A, y_B).

(b) Again let $c = 5$. Determine all non-interior Pareto optimal allocations, if there are any. Explain why your answer is the right one.

(c) Let $c = 2$. Determine all the interior Pareto optimal allocations.

(d) Measuring c on the horizontal axis and x on the vertical axis, draw a diagram indicating, for each level c of marginal cost, the associated public good levels that are consistent with Pareto optimality at interior allocations.

(e) Let $c = 2$. Suppose that production of the public good results only from Ms. A and Ms. B voluntarily contributing some of their private-good holdings as input to the public good production process. Let z_A and z_B denote the contributions of private good by Ms. A and by Ms. B. Draw the two women's reaction curves and determine the Nash equilibrium. Is the Nash equilibrium allocation Pareto optimal?

8.3   A certain restaurant in town is known for refusing to give separate checks to customers. After a group has ordered and eaten together at this restaurant, the group is presented with a single check for the entire amount that the group has eaten. It has been suggested that the restaurant does this because, with a single check, those who dine in groups will be more likely to simply divide the charge equally, each person paying the same amount irrespective of who ordered the most; and that diners, knowing they will ultimately divide the charge equally, will order more than they would have ordered had each expected to pay only for his own order. Analyze this situation using the following model.

There are n diners in a group. Each has a utility function of the form $u_i(x_i, y_i) = y_i + a_i \log x_i$, where x_i represents the amount of food (in pounds) ordered and eaten by i, and y_i represents the amount of money that i has after leaving the restaurant. The restaurant charges p dollars for each pound of food, and the restaurant's profit is an increasing function of the amount of food that it sells at the price p. Each diner knows when he orders his food that the group will divide the check equally when it is time to pay.

Compare the outcome under this check-splitting arrangement with the outcome when each diner pays for his own order. Compare, in particular, the restaurant's profit in each case and the diners' welfare in each case. Is there an alternative arrangement that will make the diners better off than in either of these arrangements?

8.4   A group of n students has determined that they're in deep trouble in their economics course, so they have decided to hire a tutor to give them a group review session. The tutor will charge them p dollars per hour. Each member of the group has a utility function of the form $u_i(x, t_i) = -t_i + a_i \log x$, where x denotes the length of the review session, in hours, and where t_i denotes the amount he has to pay to the tutor. The tutor will be paid a total of $px = \sum_1^n t_i$. Assume that $a_1 > a_2 > \cdots > a_n$.

(a) Determine all the Pareto optimal allocations

(b) The group has agreed to decide on the length of the session by the following method: Each member of the group will announce his vote, a non-negative real number m_i; the length of the session will be the average (i.e., the mean) of all n votes; and each member will pay the same amount to the tutor — i.e., they'll share the cost of the tutor equally. What will be the outcome of this decision procedure, assuming that each member of the group knows the others' preferences and how the others will vote?

8.5   Ms. Alpha and Mr. Beta have just terminated their marriage. They have agreed that Mr. Beta will raise their only child, little Joey Alpha-Beta. The two parents hold no animosity toward one another, and each is intensely concerned about little Joey's welfare. Their preferences are described by the utility functions

$$u^A(x, y_A) = x^\alpha y_A \quad \text{and} \quad u^B(x, y_B) = x^\beta y_B,$$

where y_A and y_B denote thousands of dollars "consumed" directly by the respective parents in a year, and x denotes thousands of dollars per year consumed by Joey. (So, for example, if $\beta = 1/3$ and Mr. Beta consumes \$30,000 himself and Joey consumes \$27,000, then $x = 27$, $y_B = 30$, and $u^B(x, y_B) = 90$.) Joey's consumption is simply the sum of the support contributions from his mother and father, $s_A + s_B$, also in thousands of dollars. These contributions will be voluntary: neither parent has sought a legal judgment against the other. Assume throughout that $\alpha = 1/4$ and $\beta = 1/3$.

(a) Suppose Joey's mother is unable to contribute anything toward Joey's support, so that Mr. Beta must provide, out of his \$40,000 annual income, for both his own consumption, y_B, and Joey's consumption, x. Express Mr. Beta's budget constraint both analytically, and diagrammatically. Determine Mr. Beta's marginal rate of substitution between x and y_B at the choice he will make, and draw a diagram representing his choice problem. What levels of x and y_B will Mr. Beta choose?

(b) Actually, Ms. Alpha is going to contribute to Joey's support, but she is going to observe how much Mr. Beta contributes, s_B, and then choose her contribution, s_A. Suppose Mr. Beta does the same — *i.e.*, each parent takes the other's contribution as given. If Ms. Alpha's annual income is \$48,000 and Mr. Beta's is \$40,000, what will be their equilibrium contributions to Joey's support?

(c) Find an allocation of the parents' incomes that will make them both happier than the allocation in (b).

(d) Determine the two equations (*viz.*, the marginal condition and the "on-the-constraint" condition) that characterize the Pareto optimal allocations.

(e) Indicate some of the difficulties that a neutral third party (*e.g.*, a judge) might encounter in attempting to implement some method for arriving at a Pareto optimal allocation of the parents' incomes.

8.6   Three farmers (labeled $i = 1, 2, 3$) have recognized that any fertilizer sprayed in their neigh-
borhood is a public good to them. Fertilizer costs \$3 per gallon. The farmers' profits, as functions
of the amount x of fertilizer sprayed, are given by the functions $\pi_i(x) = \alpha \ln x$, where $\alpha_1 = 1$,
$\alpha_2 = 2$, and $\alpha_3 = 3$. (The π_i functions give the farmers' respective profits, in dollars, *not counting
what they pay for fertilizer*.) Each farmer is interested only in maximizing the profit he will be left
with after deducting his payment for fertilizer. An allocation here is a list $x, t_1, t_2, t_3)$ specifying
how much will be sprayed (that's x) and how much each farmer will pay (that's t_i for farmer i).
Efficiency clearly requires that $t_1 + t_2 + t_3 = 3x$.

(a) Determine which interior allocations are Pareto optimal.

(b) The farmers have agreed to use the following method to determine this month's allocation
of fertilizer and payments: each farmer will place a *request* r_i with the spraying company; the
company is then authorized to spray $x = r_1 + r_2 + r_3$ gallons and to charge the farmers the
amounts
$$t_1 = (1 + r_2 - r_3)x \qquad t_2 = (1 + r_3 - r_1)x \qquad t_3 = (1 + r_1 - r_2)x.$$
Notice that $t_1 + t_2 + t_3 \equiv 3x$. Determine the Nash equilibrium of this scheme (i.e., assume that
each farmer chooses his request taking the other two requests as given).

8.7   Three housemates, Amy, Bev, and Cathy are about to buy a satellite dish. They must decide
how large a dish to buy. Their preferences are as follows, where x denotes the diameter of the dish
(in meters) and t_i denotes the amount that person i pays (in dollars):
$$u_i(x, t_i) = \alpha_i \log x - t_i, \qquad i = A, B, C, \qquad \text{and} \quad \alpha_A < \alpha_B < \alpha_C.$$
Satellite dishes cost β dollars per meter of diameter (i.e., a dish of diameter x meters costs βx
dollars).

(a) Which decisions (x, t_A, t_B, t_C) are Pareto efficient?

(b) The housemates have decided to use the following procedure to decide upon x and t_A, t_B,
and t_C: each person will cast a vote for the size dish she would like; they will buy a dish the
size of the median of the three votes; and they will divide the cost of the dish equally. Votes
must be non-negative real numbers. Determine the Nash equilibrium (or, if there are multiple
Nash equilibria, determine them all), and indicate how the equilibrium outcome(s) compare to the
efficient outcomes.

(c) Does anyone have a dominant strategy? Explain.

8.8   A community with n households is contemplating improving its roads. Let x denote the level of improvement; any non-negative x can be chosen, but the cost of improvement level x will be cx dollars, where c is a positive number. Households do not all have the same preferences; denote household i's preference by the utility function $u_i(x, y_i)$, where y_i denotes the amount of money (in dollars) the household has available to spend after the road improvements have been paid for (thus, no y_i can be negative). Derive the Samuelson marginal condition for Pareto efficiency . . .

(a) for allocations in which all y_i are positive;

(b) for allocations in which one or more y_i is zero.

8.9   The tiny country of DeSoto has n households, each of which owns a car. The residents of DeSoto have only two interests in life — driving their cars and consuming the economy's only tangible commodity, simoleans. Each household has a utility function of the form

$$u_i(x_i, y_i) = y_i + v_i(x_i) - a_i H,$$

where y_i denotes consumption of simoleans, x_i denotes miles driven, and H denotes the level of hydrocarbons in the air. Cars use simoleans for fuel: every mile that a car is driven uses up c units of simoleans, but the burning of simoleans also puts b units of hydrocarbons into the air for every mile that the car is driven. In other words, $H = (x_1 + \cdots + x_n)b$. Use A to denote the sum of all households' a_i parameters and X to denote the total of all miles driven by all households, and assume that each function v_i is strictly concave and increasing. Consider only those allocations in which each x_i and each y_i is positive. Each household has a positive endowment of simoleans.

(a) Give the n marginal conditions that characterize the Pareto optimal allocations, and interpret them in words.

(b) Give the n marginal conditions that characterize the Walrasian equilibrium, and interpret them in words.

(c) Determine whether, in the equilibrium, all families necessarily drive "too much," all families necessarily drive "too little," or whether the miles driven might be either too large or too small depending upon the data of the problem.

8.10   The de Beers Brewery uses water from the Pristine River in its brewing operations. Recently, the United Chemical Company (also called Chemco) has opened a factory upstream from de Beers. Chemco's manufacturing operations pollute the river water: let x denote the number of gallons of pollutant that Chemco dumps into the river each day. De Beers's profits are reduced by x^2 dollars per day, because that's how much it costs de Beers to clean the pollutants from the water it uses. Chemco's profit-maximizing level of operation involves daily dumping of 30 gallons of pollutant into the river. Altering its operations to dump less pollutant reduces Chemco's profit: specifically, Chemco's daily profit is reduced by the amount $(1/2)(30 - x)^2$ if it dumps x gallons of pollutant per day. There are no laws restricting the amount that Chemco may pollute the water, and no laws requiring that Chemco compensate de Beers for the costs imposed by Chemco's pollution.

(a) Coase's argument holds that the two firms will reach a bargain in which the Pareto efficient level of pollution will be dumped. Determine the efficient level of pollution. If efficiency requires that $x < 30$, then determine the range of the bargains the two firms could be expected to reach — i.e., the maximum and minimum dollar amounts that de Beers could be expected to pay to Chemco in return for Chemco's agreeing to dump only x (less than 30) gallons per day.

(b) Now suppose a law is passed that requires anyone who pollutes the Pristine River to fully compensate any downstream firm for the damages caused by the polluter's actions. How does this change the Pareto efficient level of pollution? How does this change the pollution level that Chemco and de Beers will agree to? How does it change the payments that one of the firms will make to the other?

(c) Now suppose that de Beers is not the only firm harmed by Chemco's pollution: there are more than one hundred firms whose profits are reduced by the pollution. How would this affect your answers in (a) and (b)? (You will not be able to give a precise quantitative answer here, because you do not know exactly how much each firm is damaged by the pollution. But describe qualitatively how the answers to (a) and (b) will change.)

8.11   The Simpsons and the Flanders are next-door neighbors. The Simpsons enjoy listening to music, played very loud. The Flanders prefer quiet. Using x to denote the volume of the Simpsons' music in "decibooms" (tens of decibels), and using y_S and y_F to denote their monthly consumption of other goods (in dollars' worth), the preferences of the Simpson and Flanders households are described by the following utility functions:

$$u_S(x, y_S) = y_S + 9x - \frac{1}{2}x^2 \qquad \text{and} \qquad u_F(x, y_F) = y_F - x^2.$$

Each family's monthly income is $3,000. Note that their marginal rates of substitution are given by $MRS_S = 9 - x$ and $MRS_F = -2x$.

(a) Determine the Pareto efficient volume of the Simpsons' music.

Suppose there is no law restricting the volume at which people can play music; thus, the Simpsons have the right to play their music at whatever volume they like. According to Coase's argument, the two families will reach an agreement about the volume of the music, and one family will pay some monetary compensation to the other.

(b) According to Coase's argument, what volume of music will the families agree upon? Which family will receive compensation?

(c) Determine the Lindahl equilibrium — the music volume and the amount of money that one family transfers to the other, again assuming that there is no law restricting the volume of music.

(d) Determine a core outcome when there is no law restricting music volume.

(e) Now suppose a law has been passed that imposes a $500 fine on anyone whose neighbor justifiably complains about loud music. According to Coase's argument, what volume of music will the Simpsons and Flanders now agree to? What is the Lindahl outcome? Determine a core outcome.

(f) Now suppose there are 20 students living in a dormitory, half of whom have preferences described by u_S above, and half of whom have preferences described by u_F. Everyone has a stereo, and x is the volume of the stereo that is played the loudest. There is no restriction on the volume at which stereos may be played. What is the Pareto efficient level of x? What is the Lindahl outcome? Is Coase's solution more likely than with only two individuals, or less likely? Why?

8.12   Ann, Bob, and Carol are renting a house together and they must decide what level of cable TV service they will subscribe to. They can choose any non-negative level of service x, for which they will be charged $6x$ dollars. Unfortunately, the three roommates do not have the same preferences for cable TV service: each one's preferences are described by a utility function of the form $u_i(x, y_i) = y_i + r_i \log x$, where y_i denotes dollars available to spend on other goods, and the values of r_i for Ann, Bob, and Carol are $r_A = 4$, $r_B = 8$, and $r_C = 36$. Each of the roommates is endowed with 40 dollars.

(a) Determine all service levels that are consistent with Pareto optimality when every $y_i > 0$ ($i = A, B, C$).

(b) Determine the Lindahl allocations and prices.

(c) Suppose the housemates use the following procedure to determine the service level x they will purchase and how they will pay for their purchase: each housemate will announce (or "vote for") the service level m_i he or she claims to most prefer; then they will purchase a service level x equal to the median of the three votes, and they will share the cost, $6x$, equally (i.e., each will pay $2x$).

(c′) Suppose $m_A = 2$ and $m_B = 4$. Draw Carol's "budget set" – i.e., the set of all bundles (x, y_C) Carol can obtain for herself via her vote m_C, assuming that Ann and Bob don't change their votes. Which bundle will she choose, and what vote(s) could she cast in order to achieve that bundle?

(c″) Suppose $m_A = 2$ and $m_C = 12$. Draw Bob's budget set. Which bundle will he choose, and what vote(s) could he cast in order to achieve that bundle?

(c‴) Determine the Nash equilibrium allocation(s).

8.13   A large university assigns three graduate students to each office. Each office has a thermostat by which the temperature in the office can be set at any level from 60° to 90° Fahrenheit. The university recognizes that the three office-mates generally will not prefer the same temperature, and to avoid arguments and lawsuits the university is going to mandate a rule by which office-mates are to decide on the temperature for their office. Two rules have been proposed, each of which requires each office-mate to state which temperature he desires. These "votes" are required to be not less than 60 nor greater than 90. Denote student i's vote by m_i. The rules differ in the way the three votes are used to determine the temperature:

The Median Rule: The temperature will be set at the median of the three votes.

The Mean Rule: The temperature will be set at the mean of the three votes.

Assume that each graduate student's preference for alternative temperatures can be described by a strictly concave real function u_i on the interval [60,90]: student i prefers temperature x to temperature y if $u_i(x) > u_i(y)$. Let β_i denote the temperature student i likes best (i.e., the maximizer of u_i). Note that $\beta_i \in [60, 90]$. Without loss of generality, assume that $\beta_1 \leq \beta_2 \leq \beta_3$.

(a) Verify that each student has a dominant strategy if the Median Rule is used. In any given office, will these dominant strategies be the unique Nash equilibrium?

(b) Suppose the Mean Rule is used and consider an office in which β_2, the median value of β_i, is at least 80. (Note that this is the median most-preferred temperature, not necessarily the median vote.) Also assume that $\beta_1 < \beta_2 < \beta_3$. Determine a Nash equilibrium in this office. Is it the unique Nash equilibrium?

(c) Determine which, if either, of the outcomes in (a) and (b) are Pareto optimal.

(d) Now assume that each student's preference is described by a utility function of the form $U_i(x, y_i) = y_i + u_i(x)$, where u_i is as described above, and where y_i denotes the amount of money i has. Assume, moreover, that it's possible to transfer money from one student to another. How will this change your answers to the questions posed in (a), (b), and (c)?

8.14   100 men have access to a common grazing area. Each man can choose to own either no cows, one cow, or two cows to provide milk for his family. The more cows the grazing land is required to support, the lower is each cow's yield of milk. Specifically, each man obtains

$$Q_i = (250 - X)x_i \text{ quarts of milk per year,}$$

where x_i denotes the number of cows the man owns, and X denotes the total number of cows owned by all 100 men (i.e., $X = x_1 + \cdots + x_{100}$). Each man wants to obtain as much milk as he can, but no man has the resources to own more than two cows.

(a) How many cows do you predict each man will own? Explain your prediction. Indicate, in particular, whether your prediction is some sort of equilibrium, and if so whether it is a unique equilibrium, and whether this equilibrium is one that would be likely to be reached quickly, or only after a long period of time during which the men learn how one another behaves. If your prediction is not some sort of equilibrium, explain why you have predicted as you have.

(b) Assume that the men can make transfers of milk among themselves (in particular, that men with more cows can give milk to those with fewer cows to compensate them for owning fewer cows). Is your prediction in (a) Pareto efficient for the 100 men? If so, verify it. If not, then find a Pareto optimal allocation of milk to the men that makes everyone strictly better off, and a pattern of cow ownership and transfer payments (in quarts of milk) that will support that allocation.

(c) Now suppose that there are only two men whose cows share a common grazing area, and that again each man can choose to own either no cows, one cow, or two cows. Each cow's daily yield of milk, in quarts, depends on how many cows in total are grazing, as follows:

Total cows grazing:	1	2	3	4
Each cow's daily yield:	8	5	3	2

What are the Pareto efficient individually rational allocations of milk (recall that an allocation is "individually rational" if each man is at least as well off as he would be by "unilateral" action)? What are all the patterns of cow ownership and transfer payments that support these allocations? Determine all the core allocations of milk to the two men.

(d) For the situation described in part (c), answer all the questions posed in (a).

8.15   Ozone City, located on the Left Coast, has n residents, all of whom do a lot of driving. A simple model of the situation has only two goods, gasoline (gallons denoted by x) and dollars (quantities denoted by y). The market for gasoline is competitive, and it costs the typical firm β dollars to deliver a gallon of gasoline at the pump. All the gasoline combustion produced by all the driving causes serious pollution of Ozone City's air: the pollution level, denoted by s, is given by the equation $s = \alpha x$, where x is the total gallons of gasoline sold (all of which is used in driving). Each resident i's preferences are described by a differentiable utility function $u^i(x_i, y_i, s)$, where x_i denotes the gallons of gasoline he buys and y_i the number of dollars he consumes. Of course, the partial derivatives of u^i satisfy $u^i_x > 0$, $u^i_y > 0$, and $u^i_s < 0$.

(a) Derive the marginal conditions that characterize the Pareto efficient outcomes.

   For the remainder of this problem, assume that each resident, when making his decision, ignores the effect of his own purchase, x_i, upon the total $x = \sum x_j$.

(b) Determine the marginal conditions that the market outcome will satisfy if there are no interventions such as taxes or subsidies. Can you determine, without knowing the specific utility functions, whether the market outcome involves "too much" or "too little" driving?

(c) Determine a tax-and-rebate arrangement that would induce a Pareto efficient outcome via individual market decisions. Describe any difficulties one would likely encounter in implementing the tax-and-rebate arrangement.

(d) This model will not allow one to analyze the individual's decision whether to purchase a more fuel-efficient car. How would you change the model to allow this kind of analysis?

8.16 There are n people people in the economy, and only two goods that they care about consuming, food and leisure. Each person owns a machine that produces Kz units of food if someone gives up z of his leisure hours to operate the machine; the coefficient K is the amount of knowledge in the economy. There is a third use to which a person can put his time (in addition to working and consuming his time in the form of leisure): he can spend his time "adding to knowledge." Everyone's production coefficient, K, is equal to the sum of the knowledge gained by all the members of the economy. (To keep things simple, assume that the economy only operates once; equivalently, in each market period all knowledge from previous periods is forgotten.

Use the following notation: x_i is i's consumption of food; y_i is i's consumption of leisure (hours); z_i is the number of hours i works producing food, either with his own machine or with others' machines; k_i is the number of hours i devotes to gaining knowledge; $K = \alpha \sum_1^n k_i$; and p is the market price of food. The market price of labor is \$1.

Determine each of the following:

(a) The constraints that characterize the feasible allocations.

(b) The first-order conditions that characterize the Pareto optimal allocations.

(c) The constraint(s) imposed upon an individual by competitive markets, assuming that there is no market for knowledge – any knowledge that an individual gains is automatically in the public domain.

(d) The first-order conditions that characterize the individual's choice in the marketplace.

(e) Derive whatever economic implications you can from the first-order conditions in (b) and (d) – give as complete an analysis of the situation as you can.

(f) How would the market outcome be changed if there is only one machine and it's in the public domain – i.e., a single machine into which anyone can put z hours of work and obtain Kz units of food in return? What if the single machine is owned by just one of the individuals in the economy?

8.17   Activities that generate negative externalities, such as pollution, will generally be carried out at a level greater than Pareto efficiency would require. On the other hand, it is often argued, reducing the externality-generating activity will result in a loss of jobs. You should now be able to produce some insight into this issue by constructing your own simple model. Combine the ideas in your simple model of a pollution-generating activity with the ideas in the exercises you have done that concern the welfare differences between monopoly and competition. As in the pollution model, let s denote the level of pollution, let x_i denote the amount i consumes of the pollution-generating product, and let y_i denote the amount i consumes of the good that is also used as input in the production of the x-good – and, in particular, let this latter good be i's "leisure (non-working) time," so that the amount $z_i = \mathring{y}_i - y_i$ is the amount of time he sells as an employee to the producers of the x-good. Make up a numerical example (I suggest using a constant-returns-to-scale production technology) in which you can determine the outcome and utility levels determined by Pigovian taxes and transfers (ignoring the incentive issues associated with determining the taxes and transfers). Determine, in particular whether, by "gainers" compensating "losers," a reduction in the pollution-generating activity can be a Pareto improvement. (A more complete model would include two produced products (one polluting, one not) and two kinds of labor used in the production process (some consumers endowed with one kind of labor and other consumers with the other kind). If we moved from the unregulated market to the Pigovian outcome, what would happen to the output levels of the two products, to the incomes of the two types of labor, and to their utility levels?

8.18   Acme Nurseries and Badweiser Brewery are located adjacent to one another. Each imposes an external cost on the other: the fertilizer that Acme uses increases Badweiser's costs, and the air pollution from Badweiser's production increases Acme's costs. Specifically, if x_A and x_B are Acme's and Badweiser's production levels, then their profits (in dollars per hour) are given by the functions π_A (for Acme) and π_B (for Badwesier):

$$\pi_A(x_A, x_B) = (30 - x_B)x_A - x_A^2 \quad \text{and} \quad \pi_B(x_A, x_B) = (30 - x_A)x_B - x_B^2.$$

(a) On a single diagram draw the two firms' reaction functions. Calculate the Nash equilibrium, assuming that each firm takes the other's production level as given.

(b) If the firms were somehow able to choose their production levels cooperatively (for example, if they were owned by the same person), what would those levels be?

(c) Suppose the firms are not owned by the same person. Describe the Coase "Theory of Social Cost" argument as it applies to this situation, and determine the outcome(s) predicted by the Coase argument – the production levels and any payments from one firm to the other.

(d) Consider two possible situations: In one case the two firms make their production decisions once and for all, at a single date; in the other case, the two firms make their production decisions repeatedly, day after day, year after year. Would the Coase argument be more likely in one situation than the other, and if so, why? Would your answer be the same if this were a so-called "pecuniary" externality – for example, if the firms were Cournot duopolists, selling an identical, costless-to-produce product in a market where the price is $p = 30 - (x_A + x_B)$, so that the externality occurs through the effects on revenue instead of on costs?

(e) Suppose there is a law stating that any firm polluting the water must fully reimburse any firm whose costs are increased by that pollution, but that there is no such law covering air pollution. Determine the outcome(s) predicted by the Coase argument – the production levels and any payments from one firm to another. What if the law states that the polluting firm need only reimburse half of the costs it imposes on others?

8.19.   Alice is a musician; Bart is not. Let x denote the number of hours per day that Alice devotes to writing, playing, and recording his music. It costs Alice \$4 for every hour he spends producing music. Let y_A and y_B denote Alice's and Bart's dollar expenditures on goods other than music. Each is endowed with more than \$100 per day. Alice's and Bart's preferences for Alice's music are described by the utility functions

$$u_A(x, y_A) = \begin{cases} y_A + 8x - \frac{1}{2}x^2 & ,x \leqq 8 \\ y_A + 32 & ,x \geqq 8 \end{cases} \quad \text{and} \quad u_B(x, y_B) = \begin{cases} y_B + 12x - \frac{1}{2}x^2 & ,x \leqq 12 \\ y_B + 72 & ,x \geqq 12 \end{cases}$$

Note that Alice's and Bart's marginal rates of substitution are

$$MRS_A = \max\{0, 8 - x\} \quad \text{and} \quad MRS_B = \max\{0, 12 - x\}.$$

(a) How much will Alice produce if he doesn't even know Bart exists?

(b) Suppose that Alice produces the amount of music in (a) and Bart has found a way to pirate the music by downloading it from Alice's computer. Alice has no way to prevent Bart from this "free riding" piracy. How much consumer surplus do Alice and Bart obtain? (Don't forget that it costs Alice \$4 for every hour he devotes to producing music.)

(c) Derive the marginal condition that characterizes the Pareto allocations (x, y_A, y_B).

(d) What is the Pareto amount of music for Alice to produce? What is the total surplus at this level of music?

(e) Now suppose Bart and Alice agree on a transfer payment t from Bart to Alice, in return for Alice producing the Pareto amount of music. Determine the range of payments that yield core allocations, and determine each one's consumer surplus as a function of t.

(f) Now suppose Alice has found a way to protect his music from Bart's piracy. From now on, Bart will have to pay p dollars for every unit of music he downloads — i.e., Bart will have to pay Alice px dollars to download x units of music. If Alice chooses the price p that maximizes his profit (revenue from Bart, minus the cost of production), what price will he charge and how much music will he produce? What will be Alice's and Bart's consumer surpluses? How much profit will Alice earn? (**Important:** This is *not* the price that maximizes Alice's utility.)

(g) Knowledge of the Envelope Theorem should tell you how Alice can increase his utility by charging a different price than in (f). Don't try to determine the utility-maximizing price and production, but indicate whether Alice should increase or decrease the price and production from the profit-maximizing levels in (f) in order to maximize his utility, and explain why this is easy to determine.