#8.1

MICROECONOMICS COMPREHENSIVE EXAM SOLUTIONS

#22 (1) An allocation (x, y_A, y_B) is Pareto optimal if and only if it satisfies $(*)$ and either $(*A)$ or $(*B)$:

$(*)$ $x + y_A + y_B = 120$

$(*A)$ $2x = y_A$ and $x \geq y_B$: To increase u_A requires increasing both x and y_A, thus reducing y_B and hence u_B. To increase u_B requires increasing y_B, thus reducing either x or y_A and hence u_A.

$(*B)$ $2x \geq y_A$ and $x = y_B$: ~~To increase u~~ Interchange A and B in the $(*A)$ argument.

(a1) Pareto optimal: satisfies $(*)$, $(*A)$, $(*B)$.

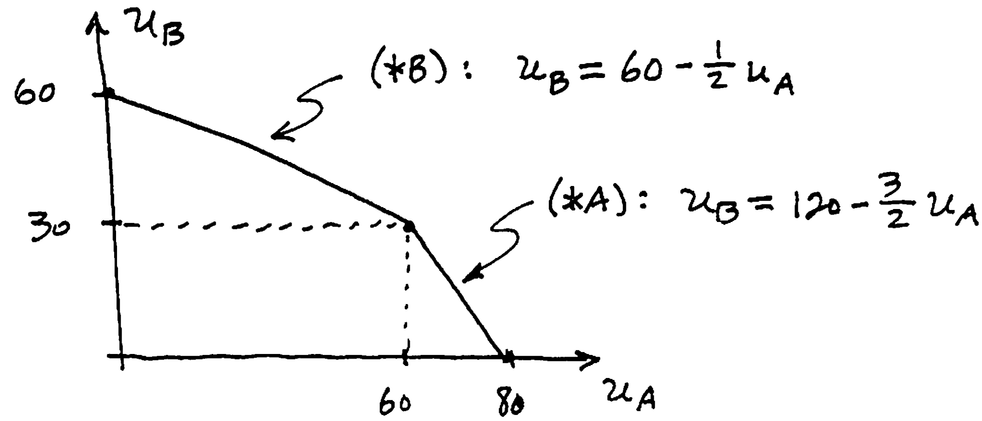
(a2) Satisfies $(*)$ but neither $(*A)$ nor $(*B)$. But at $(45, 30, 45)$ we satisfy $(*)$ and $(*B)$, and both u_A and u_B are larger.

(a3) Violates $(*)$: $x + y_A + y_B > 120$, so not feasible.

(a4) Violates $(*)$: $x + y_A + y_B < 120$, so wasteful. At $(39, 42, 39)$ we satisfy $(*)$ and $(*B)$ and both u_A and u_B are larger.

(b) IF (*) & (*A) SATISFIED: $y_A = u_A$, $x = \frac{1}{2}u_A$, $u_B = y_B$;
$\therefore u_B = y_B = 120 - x - y_A = 120 - \frac{1}{2}u_A - u_A = 120 - \frac{3}{2}u_A$.

IF (*) & (*B) SATISFIED: $y_B = u_B$, $x = u_B$, $u_A = y_A$;
$\therefore u_A = y_A = 120 - x - y_B = 120 - u_B - u_B = 120 - 2u_B$;
i.e., $u_B = 60 - \frac{1}{2}u_A$.



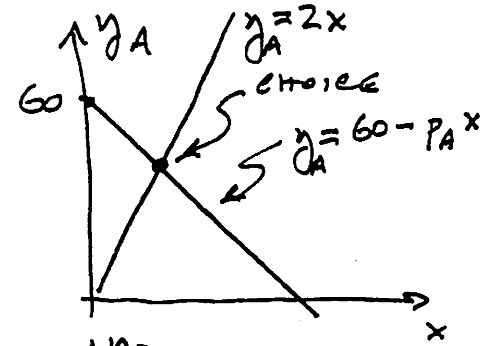
(c1) LET m_A AND m_B DENOTE THE CONTRIBUTIONS. NOTICE THAT i WILL INCREASE m_i IF $y_i > u_i$ (i.e., IF $y_A > 2x$ FOR A; IF $y_B > x$ FOR B), AND i WILL DECREASE m_i IF $y_i < u_i$ (i.e., IF $y_A < 2x$ FOR A; IF $y_B < x$ FOR B).

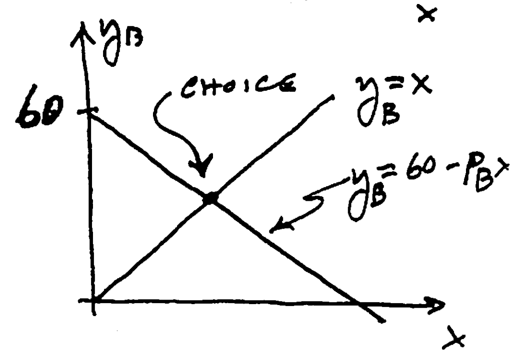
IS THERE AN EQUILIBRIUM AT WHICH $x = 30$? IF SO, THEN $y_A = 2x$ (i.e., $60 - m_A = 60$; i.e., $m_A = 0$) AND $y_B = x$ (i.e., $60 - m_B = 30$; i.e., $m_B = 30$). AT $(m_A, m_B) = (0, 30)$ WE HAVE $x = 30$, $y_A = 60 = 2x$, AND $y_B = 30 = x$, SO NEITHER DESIRES TO CHANGE HIS m_i — IT IS AN EQUIL'M, AND THE ONLY EQUIL'M AT WHICH $x = 30$.

WHAT IF $x > 30$? THEN EITHER $y_A < 60$ ($\therefore m_A \downarrow$) OR $y_B < 30$ ($\therefore m_B \downarrow$); $\therefore$ NOT AN EQUIL'M. AND IF $x < 30$, THEN EITHER $y_A > 60$ ($\therefore m_A \uparrow$) OR $y_B > 30$ ($\therefore m_B \uparrow$); $\therefore$ NOT AN EQUIL'M. THUS, THE ONLY EQUILIBRIUM IS $m_A = 0$, $m_B = 30$.

(c2) LINDAHL EQUILIBRIUM:

INDIVIDUAL PRICES p_A AND p_B THAT SATISFY $p_A + p_B = 1$ (i.e., EQUAL TO MC); EACH i CHOOSES x AND y_i TO MAXIMIZE u_i S.T. $y_i = 60 - p_i x$.





<u>FOR A:</u>

$y_A = 2x$ & $y_A = 60 - p_A x$;

i.e., $2x = 60 - p_A x$; i.e., $x = \frac{60}{2+p_A}$.

<u>FOR B:</u>

$y_B = x$ & $y_B = 60 - p_B x$;

i.e., $x = 60 - p_B x$; i.e., $x = \frac{60}{1+p_B}$.

$p_A + p_B = 1$, AND THEY CHOOSE THE SAME x AT EQUILIBRIUM:

$$\frac{60}{2+p_A} = \frac{60}{1+(1-p_A)};$$

i.e., $2 + p_A = 1 + (1 - p_A)$;

i.e., $2 + p_A = 2 - p_A$; i.e., $\boxed{p_A = 0, \ p_B = 1}$

$\therefore$ $x = 30$, $y_A = 60$, $y_B = 30$.

THIS IS THE ONLY LINDAHL EQUILIBRIUM.