

#5.2

$$u^i(x, y) = xy, \forall i$$

$$(\bar{x}_B, \bar{y}_B) = (16, 4), (\bar{x}_L, \bar{y}_L) = (8, 8), (\bar{x}_K, \bar{y}_K) = (4, 16).$$

(a) PROPOSAL:  $(\hat{x}_B, \hat{y}_B) = (10, 7), (\hat{x}_L, \hat{y}_L) = (7, 10), (\hat{x}_K, \hat{y}_K) = (11, 11).$

CAN  $S = \{B, L\}$  IMPROVE UPON  $(\hat{x}, \hat{y})$ ?

WE HAVE  $\hat{u}_B = \hat{u}_L = 70, (\bar{x}, \bar{y})_S = (24, 12).$

LET  $(\tilde{x}_B, \tilde{y}_B) = (\tilde{x}_L, \tilde{y}_L) = (12, 6).$

THEN  $\tilde{u}^i = 72 > \hat{u}^i, i = B, L,$

SO  $S$  CAN IMPROVE UPON  $(\hat{x}, \hat{y})$  VIA  $(\tilde{x}^i, \tilde{y}^i)_S.$

NOTE THAT  $(\tilde{x}^i, \tilde{y}^i)_S$  IS PARETO FOR  $S$ :

$$(\tilde{x}_B, \tilde{y}_B) + (\tilde{x}_L, \tilde{y}_L) = (24, 12) = (\bar{x}_B, \bar{y}_B) + (\bar{x}_L, \bar{y}_L)$$

$$\text{AND } MRS_B = MRS_L = \frac{1}{2}.$$

(b) PROPOSAL:  $(\hat{x}_B, \hat{y}_B) = (11, 7), (\hat{x}_L, \hat{y}_L) = (7, 11), (\hat{x}_K, \hat{y}_K) = (10, 10).$

CAN  $S = \{B, L\}$  IMPROVE UPON  $(\hat{x}, \hat{y})$ ?

NO, BECAUSE THE PARETO-FOR- $S$  ALLOCATION

IN (a) GIVES EACH OF THEM  $u_i = 72$  AND

THIS PROPOSAL GIVES THEM EACH  $u_i = 77.$

(c) NEITHER OF THE PROPOSALS IN (a) AND (b) IS IN

THE CORE, BECAUSE NEITHER ONE IS PARETO

EFFICIENT (FOR  $N = \{B, L, K\}$ ): IN EACH PROPOSAL

ALL THREE MRS'S ARE DIFFERENT.