(2) $MRS^A = \frac{y}{x}$ AND $MRS^B = 2\frac{y}{x}$.

(a) $MRS^A = 2 = MRS^B$; $\therefore$ PARETO OPTIMAL IF NO WASTE.
$60 - (x_A + x_B) = 24$ IS INPUT; $\therefore$ OUTPUT IS 48;
$y_A + y_B = 48$, $\therefore$ PARETO OPTIMAL.

(b) $MRS^A = 1$, $MRS^B = 2$, $\therefore$ (SINCE INTERIOR) IT IS
NOT PARETO OPTIMAL. A PARETO IMPROVEMENT:
USE ONE THE WHEAT UNITS ALLOCATED TO MR. A
TO PRODUCE BREAD, AND GIVE THE BREAD TO MR. A;
i.e., $(x_A, y_A) = (19, 22)$, SO $u^A = 418$.

(c) $MRS^A = 2 = MRS^B$, BUT $60 - (x_A + x_B) = 30$ IS INPUT,
$\therefore$ OUTPUT IS 60, BUT $y_A + y_B = 50 < 60$;
$\therefore$ NOT PARETO OPTIMAL. A PARETO IMPROVEMENT:
GIVE THE 10 WASTED BREAD LOAVES TO SOMEONE,
e.g. $(x_A, y_A) = (20, 45)$ AND $(x_B, y_B) = (10, 15)$.

#4

(3) (a) $\max \lambda_1 u^1(x_1, y_1)$ S.T. $x_i, y_i \geqq 0$ $(i=1,2)$ AND TO
$x_1 + x_2 \leqq \bar{x} : \sigma_x$ $\quad y_1 + y_2 \leqq \bar{y} : \sigma_y$ $\quad u^2(x_2, y_2) \geqq c : \lambda_2$.

KUHN-TUCKER CONDITIONS:

x_i: $\lambda_i u^i_x \leqq \sigma_x$ ("=" IF $x_i > 0$)

y_i: $\lambda_i u^i_y \leqq \sigma_y$ ("=" IF $y_i > 0$)

σ_x: $x_1 + x_2 \leqq \bar{x}$ ("=" IF $\sigma_x > 0$)

σ_y: $y_1 + y_2 \leqq \bar{y}$ ("=" IF $\sigma_y > 0$)

λ_2: $u^2(x_2, y_2) \geqq c$ ("=" IF $\lambda_2 > 0$)

CAN BE REWRITTEN AS

$MRS^1 = MRS^2$ IF $x_1, x_2, y_1, y_2 > 0$

$MRS^1 \leqq MRS^2$ IF $x_1 = 0$ OR $y_2 = 0$; $x_2, y_1 > 0$

$MRS^1 \geqq MRS^2$ IF $x_2 = 0$ OR $y_1 = 0$; $x_1, y_2 > 0$

NO MRS CONDITION IF $x_1 = y_1 = 0$ OR $x_2 = y_2 = 0$

..., AND $x_1 + x_2 = \bar{x}$, $y_1 + y_2 = \bar{y}$.

(b) $MRS^1 = \frac{1}{1+x_1}$, $MRS^2 = \frac{2}{1+x_2}$.

$\therefore MRS^1 \lesseqqgtr MRS^2$ AS $\frac{1}{1+x_1} \lesseqqgtr \frac{2}{1+x_2}$; i.e., $1 + x_2 \lesseqqgtr 2 + 2x_1$;

i.e., $x_2 \lesseqqgtr 1 + 2x_1$.

BUT ALSO $x_1 + x_2 = \bar{x} = 10$; i.e., $x_2 = 10 - x_1$;

$\therefore MRS^1 \lesseqqgtr MRS^2$ AS $10 - x_1 \lesseqqgtr 1 + 2x_1$; i.e., AS $x_1 \gtreqqless 3$,

WHEN $x_1 + x_2 = 10$. & $x_2 \lesseqqgtr 7$.

IF $y_1, y_2 > 0$: $MRS^1 = MRS^2$; i.e., $x_1 = 3$, $x_2 = 7$;

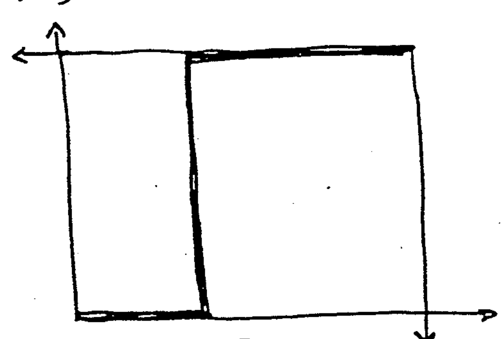
$MRS^1 = MRS^2 = \frac{1}{4}$.

IF $y_1 = 0$: $MRS^1 \geqq MRS^2$; i.e., $x_1 \leqq 3$, $x_2 \geqq 7$,

$x_1 + x_2 = 10$. $MRS^1 \geqq \frac{1}{4}$, $MRS^2 \leqq \frac{1}{4}$.

IF $y_2 = 0$: $MRS^1 \leqq MRS^2$; i.e., $x_1 \geqq 3$, $x_2 \leqq 7$,

$x_1 + x_2 = 10$. $MRS^1 \leqq \frac{1}{4}$, $MRS^2 \geqq \frac{1}{4}$.



$x_1 = 3$

NOTE: σ_x, σ_y THAT SATISFY THE K-T CONDITIONS ARE ANY σ_x, σ_y S.T. $MRS^1 \leqq \frac{\sigma_x}{\sigma_y} \leqq MRS^2$ OR $MRS^1 \geqq \frac{\sigma_x}{\sigma_y} \geqq MRS^2$.

THEN λ_2 NEED ONLY SATISFY $\lambda_2 u_j^2 \leqq \sigma_j$, $j = x, y$.