(2) \[ u_A(x,y) = y + 60x - 2x^2 \quad u_B(x,y) = y + 30x - x^2 \]
\[ \text{MRS}_A = 60 - 4x_A \quad \text{MRS}_B = 30 - 2x_B \]
\[ (x_A^*, y_A^*) = (12, 160) \quad \text{MRS}_A = 12 \]
\[ (x_B^*, y_B^*) = (3, 100) \quad \text{MRS}_B = 24 \]
\[ u_A = 100 + 720 - (2)(144) = 532 \quad u_B = 100 + 30 - 9 = 181 \]

(a) After Bill receives one x-unit from Al, then their MRS's will be \( \text{MRS}_A = 16 \) and \( \text{MRS}_B = 22 \). Therefore any payment \( t \) from Bill to Al that satisfies \( 16 \leq t \leq 22 \) will make both of them better off than if they hadn't traded. But note that if \( t \) is very close to \( \text{MRS}_A = 12 \) or to \( \text{MRS}_B = 24 \), then one person will be worse off:
\[ \text{If } t = 12: \quad \dot{u}_A = 112 + 660 - (2)(121) = 530 < u_A \]
\[ \text{If } t = 24: \quad \dot{u}_B = 76 + 120 - 16 = 180 < u_B \]

This is an important point to understand.

(b) \( \text{MRS}_A = \text{MRS}_B \) (interior): \[ 60 - 4x_A = 30 - 2(15 - x_A) = 30 - 30 + 2x_A \]
\[ \text{i.e., } 6x_A = 60; \quad \begin{cases} x_A = 10 \end{cases}, \quad \begin{cases} x_B = 5 \end{cases} \]

Any distribution of the y-good is consistent with Pareto.

If \( y_A = 0 \), Pareto requires that
\[ \text{MRS}_A > \text{MRS}_B \quad \text{i.e., } 60 - 4x_A > 30 - 2(15 - x_A) \]
\[ \text{i.e., } 6x_A < 60; \quad x_A < 10. \]

If \( y_B = 0 \), Pareto requires that
\[ \text{MRS}_A < \text{MRS}_B \quad \text{i.e., } 60 - 4x_A < 2x_A \]
\[ \text{i.e., } 6x_A > 60; \quad x_A > 10. \]
(c) \text{Let } p_y = 1.

\text{Al's demand for } x: 60 - 4x_A = p_x \quad \text{i.e., } 4x_A = 60 - p_x \quad x_A = 15 - \frac{1}{4} p_x.

\text{Bill's demand for } x: 30 - 2x_B = p_x \quad \text{i.e., } 2x_B = 30 - p_x \quad x_B = 15 - \frac{1}{2} p_x.

\text{Aggregate demand for } x: \quad x = (15 - \frac{1}{4} p_x) + (15 - \frac{1}{2} p_x) = 30 - \frac{3}{4} p_x.

\text{At } x^* = 15, \quad 30 - \frac{3}{4} p_x = 15 \quad \text{i.e., } \frac{3}{4} p_x = 15 \quad \Rightarrow \quad p_x = 20 \quad \text{\textit{Equilibrium price of } x} \quad (\text{w/ } p_y = 1).

\Rightarrow \quad \begin{cases} x_A = 10, & y_A = 100 + (20)(2) = 140 \quad \text{\textit{Equilibrium allocation}} \\ x_B = 5, & y_B = 100 - (20)(2) = 60 \end{cases}