Exercise Set #7

1. A state is considering mandatory drug testing for all state employees. It is known that five percent of the state's population uses drugs, and it is reasonable to assume that state employees are typical of the population as a whole. It is also known that when a drug user is tested, there is a 10% chance the test will come back "negative," i.e., it will say, incorrectly, that she is not a user; and that when someone who is "clean" (i.e., is not a user) is tested, there is a 2% chance her report will be a (false) positive, saying incorrectly that she is a user.

(a) What is the expected rate of false positives -- i.e., what proportion of the people who test positive can be expected to actually be clean?

(b) What is the expected rate of false negatives, the proportion of people who test negative who are actually users?

(c) Out of 1000 people tested, how many can we expect will test positive, and how many of those are likely to actually be clean?

2. A recent newspaper article reported that three-quarters of all teenage suicides are by children whose parents are divorced. Assume that one-third of all children's parents have divorced. Every year one teenager in 15,000 commits suicide.

(a) What is the annual rate of teenage suicide among children of divorced parents?

(b) Out of 60,000 teenagers, how many are (on average) from divorced families, how many commit suicide, and how many are from intact families but nevertheless commit suicide?

3. George has just moved from New York, where he did not need to own a car, to Tucson, where a car is a necessity. George is therefore shopping for a used car. He has found one that he thinks he will like, but he has consulted Consumer Reports and discovered that there is some chance that this model will need serious repairs, costing $4000. George's friends have told him about two mechanics he could take the car to, before buying it, to have it checked out. Machanic Al will charge $200 for checking the car out, and if the car needs the repair work, Al will very likely find that out. Mechanic Bob will check the car out for free, but if repairs will be needed Bob is considerably less likely than Al to detect that fact. George will have to pay $18,000 to purchase this car, and he figures that if he passes this one up, he will soon find another for $20,000.

Draw the decision tree for George's problem, labeling all branches and nodes as completely as possible, given the information you have. (Note that you haven't yet been given any probabilities, so you can't yet actually solve George's problem for him.)

4. According to the New York Times, Marilyn vos Savant's description of the Monty Hall Problem was not clear regarding the rules of the game. In fact, there are at least four different rules that Monty could have been using to choose the door he would open. (We are told only that he has in fact opened one of the doors not chosen by the constestant.) The four possible rules are: (A) Monty can open any door he likes. (B) He opens one of the three doors at random. (C) He opens at random one of the two doors not chosen by the contestant. (D) He opens a door which is neither the constestant's choice nor the one with the prize.

Give as complete an analysis as you can of the probabilites and the contestant's best choice under rules (B), (C), and (D). Rule (A) will require a game theoretic analysis, which we will defer for now. Under which of the other rule(s), if any, is Ms. vos Savant correct, under which are her critics correct, and under which are neither correct?

5. Demonstrate, using the New York Times's "extra credit" problem, that when using information it is critical to know *how the information was generated*, by analyzing the problem under the assumption that the way you have discovered that at least one of the woman's children is a boy is that she has said to you "Let me introduce you to Monty, one of my two children."