Exercise Set #6

1. When Venus is at the net, Martina can choose to hit the ball either cross-court or down-the-line. Similarly, Venus can guess that the ball will come cross-court or down-the-line and react accordingly. Here is a table giving the probability Martina will win the point as a function of the two players' choices:

		Venus	
		DL	**CC**
Martina	**DL**	.2	.8
	CC	.5	.1

Determine each player's maximin and minimax mixtures, and the Nash equilibrium mixtures. Determine each player's maximin and minimax values, and each player's probability of winning the point if they play according to the Nash equilibrium.

2. When a soccer player is kicking a penalty kick, he will choose to kick either to the goalie's left or to his right. The goalie will leap to one side or the other in an attempt to block the kick; he will leap before determining to which side the kick will come, but too late for the kicker to change direction, For many professional kicker vs. goalie matchups the following table gives a good approximation to the probabilities that the kicker will score a goal, as a function of the two players' choices (as described in the soccer paper by Palacios-Huerta that's on the course electronic reserve page):

		Goalie	
		L	**R**
Kicker	**L**	.60	.90
	R	.95	.75

Determine each player's maximin and minimax mixtures, and the Nash equilibrium mixtures. Determine each player's maximin and minimax values, and the probability that the kicker will score a goal if they play according to the Nash equilibrium.

3. Jennifer and Brad are each going to buy tickets to an event next Saturday night. Jennifer prefers a Leo De Caprio movie, Brad prefers an XFL football game. But in any case, each prefers to be with the other. Here is the payoff table describing the payoffs (utility) they receive as a function of the choices each player makes:

		Brad's Choice	
		XFL	**Leo**
Jennifer's Choice	**XFL**	**2 , 5**	**-1 , -1**
	Leo	**0 , 0**	**5 , 2**

Note that this is not a constant-sum game. We've analyzed this game before, and have found that it has two pure strategy Nash equilibria. But it also has a mixed strategy Nash equilibrium. Determine each player's maximin and minimax mixtures, and the Nash equilibrium mixtures. Determine each player's maximin and minimax values, and each player's expected payoff if they play according to the Nash equilibrium.

4. People, animals, and organizations often find themselves in a situation where each must choose to "fight" ("be tough") or "concede" ("give in"). Here are the payoffs for a two-player game of this kind, a game of "chicken":

		Column	
		Give Up	**Fight**
Row	**Give Up**	**0 , 0**	**0 , 4**
	Fight	**4 , 0**	**-1 , -1**

The game is not constant-sum. The game has two pure strategy equilibria and one mixed strategy equilibrium. Determine each player's maximin and minimax mixtures, and the Nash equilibrium mixtures. Determine each player's maximin and minimax values, and each player's expected payoff if they play according to the Nash equilibrium.

5. Replace the game in the preceding problem with this one, and answer the same questions:

		Give Up	**Fight**
Row	**Give Up**	**0 , 0**	**-1 , 2**
	Fight	**2 , -1**	**-2 , -2**

6. All the games we've seen so far with mixed strategy equilibria were 2x2. Here's a 2x3 game. When the Wildcats are on offense and the Devils are on defense, the Wildcats can choose to run or pass, and the Devils can choose a run defense, a pass defense, or a blitzing defense. The resulting average (i.e., expected) yardage gains by the Wildcats are as follows:

		Defense		
		RD	**PD**	**BD**
Offense	**R**	**1**	**6**	**5**
	P	**4**	**0**	**2**

This is a zero-sum game: the defense loses what the offense gains. You should verify that there are no dominated strategies and that there is no Nash equilibrium in pure strategies. (There are only six pure-strategy profiles to check.) We know (from Nash's Theorem) that the game must have at least one equilibrium, so it must have a mixed strategy equilibrium. A general result about mixed strategies in two-player games is that in any mixed strategy equilibrium each player places positive mixture weight on the same number of pure strategies. Therefore we know that in a mixed strategy equilibrium of this game the Defense will only mix over two of its pure strategies and will not use the other one. This fact will help you determine the equilibrium of the game.

Here's how you can use the above fact to determine the Nash equilibrium of this game (also the minimax and maximin solutions, since this is a constant-sum game). Analyze each of the three 2x2 games that you get when you eliminate one of the Defense's pure strategies. The equilibrium of the 2x3 game must be an equilibrium of one of these 2x2 games, because the Defense uses only two pure strategies in the 2x3 equilibrium. For each 2x2 game, find the Defense's minimax mixture and value. The Defense is clearly going to use the two pure strategies that yield the smallest minimax value (the smallest average yardage gain) – and that is clearly the minimax value for the 2x3 game as well. Now that you've determined the two strategies the Defense will use, you can easily determine the Nash equilibrium of that 2x2 game, which is the equilibrium of the 2x3 game as well. You should check that the Offense's mixture is indeed maximin for it in the 2x3 game, and that against this Offense mixture the pure strategy the Defense doesn't use yields it a worse expected payoff (higher average yardage) than the payoff from the two strategies it does use.

Solutions

1. Martina’s mixture on DL and CC is (.4, .6) and Venus’s is (.7, .3). These are their maximin mixtures, their minimax mixtures, and the Nash equilibrium mixtures. The probability Martina will win the point is .38, when they play Nash equilibrium, and this is Martina’s maximin value and her minimax value. Venus’s value is .62.

Note that DL looks more attractive to Martina than CC: her best chance of winning the point is .8, which occurs when she chooses DL and Venus guesses wrong; and her worst chance of winning the point is .1, when she chooses CC and Venus guesses correctly. But Martina’s minimax-maximin-Nash mixture nevertheless prescribes hitting cross-court more often than down-the-line: if she were to hit down-the-line more often, then Venus would be able to exploit that by guessing down-the-line even more often than in equilibrium and thereby reduce Martina to winning even less than 38% of the points when Venus is at the net. (If Martina hits down-the-line 80% of the time, what is Venus’s best response, and how often will Martina win the point?)

2. The kicker’s mixture on L and R is (.4, .6) and the goalie’s is (.3, .7). These are their maximin mixtures, their minimax mixtures, and the Nash equilibrium mixtures. The probability a goal will be scored is .81 when they play Nash equilibrium, and this is the kicker’s maximin value and his minimax value. The goalie’s value is .19.

3. The maximin mixtures on XFL and Leo are (5/8, 3/8) for Jennifer and (3/8, 5/8) for Brad; the maximin value for each is 10/8 (i.e., 5/4, or 1.25). The minimax and Nash equilibrium mixtures are (1/4, 3/4) for Jennifer and (3/4, 1/4) for Brad; the resulting values (expected payoffs) are again 5/4 for each of them. Despite the fact that the maximin values are the same as the minimax and Nash values, the maximin mixtures do *not* constitute a Nash equilibrium. You should be able to see why this is so: you should be able to determine each player’s best response to the other’s maximin mixture, and see that it is not the player’s own maximin mixture but instead a pure strategy. In other words, if one of the players is using his or her maximin mixture, then the other can do better by doing something else, and what that something else is should be intuitively clear. This example shows why maximin makes little sense as a prescription for play when the game is not strictly competitive; you should try to make sure you understand this point.

4. The maximin strategy for each player is to play G, and the maximin value is zero. The minimax and Nash equilibrium mixtures on G and F are (1/5, 4/5) for each player, and the resulting value is again zero. Just as in the preceding problem, the maximin strategies do not constitute an equilibrium. You should see why.

5. The maximin strategy for each player is to play G, and the maximin value is -1. The minimax and Nash equilibrium mixtures on G and F are (1/3, 2/3) for each player, and the resulting value is -2/3. Here the maximin values are less than minimax and Nash.

6. If the Defense doesn't use BD, its minimax mixture on RD and PD is (2/3, 1/3) and its minimax value is 2 2/3 yards gained, on average. If the Defense doesn't use PD, its minimax mixture on RD and BD is (1/2, 1/2) and its minimax value is 3 yards gained, on average. If the Defense doesn't use RD, then the Offense always uses R and the Defense always uses BD, which is its minimax strategy in this 2x2 game, with minimax value of 5 yards gained. So the best of these two-strategy mixtures for the Defense is to use RD and PD and ignore BD. Its minimax mixture is to use RD 2/3 of the time and PD 1/3 of the time, and its minimax value is 2 2/3 yards gained. The Offense's minimax mixture on R and P is (4/9, 5/9) which of course yields it 2 2/3 yards on average. Of course these are also the maximin strategies and values, and the Nash equilibrium. [This example is of course a vastly oversimplified version of football strategy.]