Exercise Set #5

1. Use Excel (or another spreadsheet program) to reproduce the table of Row-player (Pursuer) winning probabilities that appears on page 5 of the lecture notes for Lecture #14. This is much easier than it appears at first. You can create the top row (the s-values) quickly, as follows:

(a) enter the first value (s=15); let's suppose that's in cell E8;
(b) move one cell to the right (cell F8) and enter the formula **=E8+1** ;
(c) copy the F8 cell entry and paste it into as many cells in that row as you wish.

You can use the same idea to quickly create the left column (the p-values). And you can get all the cumulative binomial probabilities into your spreadsheet by putting the correct formula into the upper left cell and then copying-and-pasting it from there into all the other cells. And presto, you have the same table as the one in the lecture notes. You're going to use this spreadsheet in some subsequent problems.

2. Look at the Matching Pennies game on pages 37 and 106 of Dutta's textbook. Note first of all that this is a game of pursuit and evasion, like the one we used for our 97-point matches: the Row player is the Pursuer and the Column player is the Evader. It's intuitively clear that either player in the Matching Pennies game can assure herself of an expected value of zero by choosing Heads and Tails randomly, using a 50-50 mixture – for example, by flipping a coin to choose. Verify that indeed the Row player's Best Worst Case (or Maximin value) is 0, and that it is achieved by mixing 50-50.

3. Now suppose Pam and Eva play a "first player to win 50 points" winner-take-all match, in which each point is decided by playing Matching Pennies. Pam is the Row player (the Pursuer) and Eva is the Column player (the Evader). You'll probably find it easier to analyze this game if you change all the –1 entries in the payoff table to zeroes (to represent the player *not* winning the point). Note that the match will last for at most 99 points: we could have characterized it as a "best 50 out of 99" match. Create a spreadsheet like the one in Problem 1 for this match. (You can just alter a few things in the spreadsheet you already created.) Include s-values from at least 43 to 57, and p-values from at least .43 to .57. You should of course find that if Pam can win points with a .5 probability, then she will have exactly a 50% chance to win the match.

(a) What would be Pam's chance of winning the match if she had to win only 45 points in 99 tries?

(b) What would be Pam's chance of winning 50 points if she uses a mixture that chooses Heads with only a .48 probability instead of .50 and if Eva imposes Pam's Worst Case on her by always choosing Heads?

Penalty kicks in soccer are a game of pursuit and evasion: the kicker is the Evader and the goalie is the Pursuer. Kickers employ only two pure strategies: kick to the goalie's Left, or kick to his Right. Goalies also employ only two pure strategies: lunge Left, or lunge Right. The goalie lunges "simultaneously" with the kick: he lunges too late for the kicker to alter his kick, but too early to actually observe where the kick is coming; in other words, he "guesses" the direction the kicker will choose. For each of the four pure-strategy profiles, LL, LR, RL, and RR, there is a probability the kicker will succeed (score a point) and the residual probability that the goalie will succeed (prevent the point).

Suppose that the probabilities when Kidd kicks against the goalie Gomez are the same as in the 2x2 game we played repeatedly in our 97-point match:

		Kicker	
		L	**R**
Goalie	**L**	**1/3, 2/3**	**0, 1**
	R	**0, 1**	**2/3, 1/3**

Then we already know that the mixed strategies that give each player his Best Worst Case (Maximin) are for each to go Left with probability 2/3, and that if they do this the kicker will win the point 7/9 of the time.

4. Suppose Kidd's accuracy to the goalie's Left deteriorates: even when the goalie lunges to his Right, Kidd scores only 5/6 of the time. If neither Kidd nor Gomez adjusts, but each still chooses Left with 2/3 mixture probability, what is the probability Kidd will score a goal?

5. Determine for each player the mixture that will assure him of his best Worst Case probability of winning. Kidd's mixture is surprising: you should find that he *increases* the mixture weight he places on kicking to Gomez's Left, despite the fact that that's the direction in which his accuracy has deteriorated. With what probability does Kidd score a goal if the players use these mixtures?

6. A promoter has set up a $100,000 winner-take-all 97-point match between Kidd and Gomez: if Kidd scores 76 goals he wins the $100,000; if he doesn't score 76 goals (i.e., if Gomez prevents the goal 22 times), then Gomez wins the $100,000. With Kidd's new reduced accuracy to the goalie's Left, what's the highest probability with which Gomez can assure himself of winning the match?

Bring your spreadsheets to class on Wednesday. You may need them during the quiz that day.