Econ 431
Solution to Problem Set #4

(a) Consider the following Table:

Cows in field	1	2	3	4
Yield per cow	8	5	3	2
Total Yield	8	10	9	8

To maximize total milk production (social optimum) the men should graze 2 cows total. But consider the following payoff table:

		Mr. Two		
		$x_2=0$	$x_2=1$	$x_2=2$
	$x_1=0$	0,0	0,8	0,10
Mr. One	$x_1=1$	8,0	5,5	3,6
	$x_1=2$	10,0	6,3	4,4

Grazing two cows is a dominant strategy for each farmer. In equilibrium they each receive 4 quarts, even though it is possible to obtain 5. If Mr. One grazes two cows he can pay Mr. Two not to graze any, but he must pay him at least 4 quarts per day, that being what Mr. Two could get by grazing his own cows. Paying 5 or 6 quarts will work as well. But if the price for Mr. Two to graze no cows were 7 quarts, then Mr. One would prefer to make no deal, since he can be assured of getting 4 quarts with 2 cows rather than just getting 3 quarts. Alternatively, Mr. Two can graze the cows and pay Mr. One not to. Finally, each can agree to graze only one cow. This is probably the most sensible outcome. Each gets 5 quarts of milk and no transfers are necessary.

(b) There are two possible kinds of equilibria, cooperative and non-cooperative. If the men make no deal, then each will graze two cows and get 4 quarts per day, since grazing 2 cows is a strongly dominant strategy. But this equilibrium is unlikely, since it is easy to do better. Transfer payments of milk are possible. Communication is easy (since they are on the same field). It is easy to enforce a contract by withholding milk and easy to observe when the contract is broken (when the other guy grazes cows even though he promised not to.) So a cooperative equilibrium where one farmer grazes two cattle and pays the other to graze none or where each promises only to graze one cow is most likely, particularly in a situation where the men are repeating the game day in and day out.

(c) Mr. i gets $Qi=(250 - X)x_i = (250 - X_{-i} - x_i)x_i$, where X_{-i} is the total number of cows grazed by the <u>other</u> men. If Mr. i grazes one cow he gets $Q(1)=249 - Xi$. If he grazes two he gets $Q(2) = 2*(248 - X_{-i})$. Since X_{-i} can at most be 198, Q(2) is <u>always</u> greater than Q(1), so Mr.i will graze 2 cows. That is, whatever the choices by the other farmers, it is always a strongly dominant strategy to graze two cows rather than one. Hence, everybody will quickly wind up grazing two cows, since it is not necessary to observe or speculate on what one's neighbors will do in order to choose the best strategy. This is an equilibrium in the sense that everybody is best responding to the choice of the others while operating under the assumption that they cannot affect those choices.

(d) If every one grazes two cows, X=200, so Qi = 100, and total milk production is 10,000 quarts. Suppose the Dairy Council steps in to maximize total production. They want to maximize:

$$\begin{aligned} Q_{total} &= Q_1+\ldots+Q_{100}= (250-X)x_1 + \ldots +(250-X)x_{100} \\ &= (250-X)(x_1+\ldots+x_{100}) \\ &=(250-X)X \end{aligned}$$

taking the derivative:

$\partial Q/\partial X = 250 -2*X$ and setting this equal to zero indicates
X=125, so 75 men should graze one cow and 25 should graze two.

Total output Q = 15,625 quarts. Since each man gets at least 125 quarts this allocation is Pareto superior to the allocation in part (c). However, the 25 individuals grazing 2 cows are receiving 250 quarts. Someone grazing one can easily see that grazing another cow will increase his take from 125 to 248 quarts. First, notice that there is insufficient surplus among those with two cows to payoff <u>all</u> those with one so that there is not an incentive for them to graze another cow. Second, since there are so many cows and so many farmers, it may be difficult to tell if someone is cheating and grazing an extra cow. Hence it will be unlikely that individuals will coordinate to maximize total output.