ECON 501B 2010 MIDTERM EXAM SOLUTIONS

(1) $u_A(x,y) = x^3 y \quad MRS_A = 3\frac{y}{x} \qquad (\mathring{x},\mathring{y}) = (50,40)$

$u_B(x,y) = x^2 y \quad MRS_B = 2\frac{y}{x}$

(a) IF $(x_A, y_A) = (30,20)$ AND $(x_B, y_B) = (20,20)$: $MRS_A = MRS_B = 2$.

(b) $MRS_A = MRS_B$

$$3\frac{y_A}{x_A} = 2\frac{y_B}{x_B} = 2\frac{\mathring{y} - y_A}{\mathring{x} - x_A}$$

$$3(\mathring{x} - x_A)y_A = 2(\mathring{y} - y_A)x_A$$

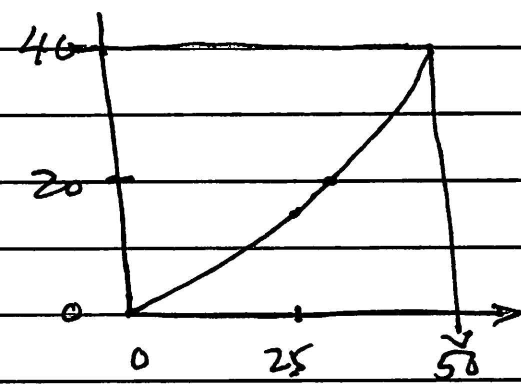
$$3\mathring{x}y_A - 3x_Ay_A = 2\mathring{y}x_A - 2x_Ay_A$$

$$3\mathring{x}y_A - x_Ay_A = 2\mathring{y}x_A$$

$$(3\mathring{x} - x_A)y_A = 2\mathring{y}x_A$$

$$y_A = \frac{2\mathring{y}x_A}{3\mathring{x} - x_A} = \frac{80x_A}{150 - x_A}$$

x_A	0	25	30	50
y_A	0	16	20	40



(c) ASSUME $(\mathring{x}_A, \mathring{y}_A) = (50,0)$ AND $(\mathring{x}_B, \mathring{y}_B) = (0,40)$:

THE ALLOCATION $((30,20),(20,20))$ WOULD REQUIRE THE TRADE $(\hat{x}_A, \hat{y}_A) = (-20,20)$, $(\hat{x}_B, \hat{y}_B) = (20,-20)$, AND THEREFORE A PRICE RATIO OF $p = \frac{p_x}{p_y} = 1$.

BUT AT $((30,20),(20,20))$ WE HAVE $MRS_A = MRS_B = 2 \neq p$;

$\therefore$ THIS CANNOT BE A MARKET EQUILIBRIUM.

HOWEVER, IF WE CHANGE $\mathring{x}_A$ AND $\mathring{x}_B$ SO THAT THE REQUIRED TRADE IS $(\hat{x}_A, \hat{y}_A) = (-10,20)$ AND $(\hat{x}_B, \hat{y}_B) = (10,-20)$, THEN WE WILL HAVE $p = 2 = MRS_A = MRS_B$, AND THE ALLOCATION $((30,20),(20,20))$ WILL BE AN EQUILIBRIUM ALLOCATION. SO WE NEED TO HAVE CAT CHANGE HER BEQUEST TO $\mathring{x}_A = 40$, $\mathring{x}_B = 10$.

② $(\overset{\circ}{x}_A, \overset{\circ}{y}_A) = (20, 40)$ AND $(\overset{\circ}{x}_B, \overset{\circ}{y}_B) = (30, 0)$:

(a) THE ALLOCATION $((30,20),(20,20))$ IS AGAIN AN EQUILIBRIUM, NOW WITH $(\overset{\Delta}{x}_A, \overset{\Delta}{y}_A) = (10, -20)$, $(\overset{\Delta}{x}_B, \overset{\Delta}{y}_B) = (-10, 20)$, AND $\rho = 2$. WE THEREFORE HAVE

INTEREST RATE: $r = 1 = 100\%$

AMY BORROWS $\overset{\Delta}{x}_A = 10$ AND PAYS BACK 20 TOMORROW.

BEV SAVES $\overset{\circ}{x}_B - x_B = 30 - 20 = 10$ AND IN RETURN RECEIVES $\overset{\Delta}{y}_B = 20$ TOMORROW.

(b) $V_0(x_A, y_A) = V_0(30, 20) = 30 + \frac{1}{1+r}20 = 30 + (\frac{1}{2})20 = 40$

$V_0(\overset{\circ}{x}_A, \overset{\circ}{y}_A) = 20 + (\frac{1}{2})40 = 20 + 20 = 40.$

$V_0(x_B, y_B) = V_0(20, 20) = 20 + \frac{1}{1+r}20 = 20 + (\frac{1}{2})20 = 30$

$V_0(\overset{\circ}{x}_B, \overset{\circ}{y}_B) = 30 + (\frac{1}{2})0 = 30.$

③ $q = f(z) = 5z - \frac{1}{5}z^2$; $f'(z) = 5 - \frac{2}{5}z$.

(a) $\max_{\lambda_A} u_A(x_A, y_A)$ s.t. $x_A, y_A, x_B, y_B, z \geqq 0$

$$x_A + x_B + z \leqq \overset{\circ}{x} \qquad \sigma_x$$
$$y_A + y_B \leqq \overset{\circ}{y} + f(z) \qquad \sigma_y$$
$$u_B(x_B, y_B) \geqq c \qquad \lambda_B$$

(b)
$$\left.\begin{aligned} x_A: &\ \lambda_A u_{Ax} = \sigma_x \\ y_A: &\ \lambda_A u_{Ay} = \sigma_y \end{aligned}\right\} \frac{u_{Ax}}{u_{Ay}} = \frac{\sigma_x}{\sigma_y}, \ MRS_A = \frac{\sigma_x}{\sigma_y}$$
$$\left.\begin{aligned} x_B: &\ \lambda_B u_{Bx} = \sigma_x \\ y_B: &\ \lambda_B u_{By} = \sigma_y \end{aligned}\right\} \frac{u_{Bx}}{u_{By}} = \frac{\sigma_x}{\sigma_y}, \ MRS_B = \frac{\sigma_x}{\sigma_y}$$
$$z: \quad 0 = \sigma_x - \sigma_y f'(z) \qquad f'(z) = \frac{\sigma_x}{\sigma_y}$$

$$\therefore MRS_A = MRS_B = f'(z) \; [= MRT].$$

(c) IF $(x_A, y_A) = (15, 15)$ AND $(x_B, y_B) = (30, 45)$, THEN WE HAVE $MRS_A = 3$, $MRS_B = 3$,

$z = 50 - 45 = 5$, $q = 60 - 40 = 20$.

WE MUST VERIFY THAT $q = f(z)$ AND THAT $f'(z) = 3$: $f(5) = (5)(5) - \frac{1}{5}(25) = 25 - 5 = 20 = q$ (OK)

$f'(5) = 5 - \frac{2}{5}z = 5 - 2 = 3$ (OK).

LAGRANGE MULTIPLIER VALUES:

$u_{Ax} = 3x_A^2 y_A = (3)(15)^2(15) = (3)(15)^3$

$u_{Ay} = x_A^3 = (15)^3$

$u_{Bx} = 2x_B y_B = (2)(30)(45) = (2)(2)(15)(3)(15) = (12)(15)^2$

$u_{By} = x_B^2 = (30)^2 = 4(15)^2$

LET $\lambda_A = 1$, $\sigma_x = (3)(15)^3$, $\sigma_y = (15)^3$, $\lambda_B = \frac{15}{4}$;

THEN ALL FIVE OF THE FOMC IN (b) ARE SATISFIED.

ALTERNATIVELY, CHOOSE ANY $\beta > 0$ AND MULTIPLY EACH OF THESE BY β; THE RESULTING $\lambda_A, \lambda_B, \sigma_x, \sigma_y$ WILL WORK AS WELL.

(d) ASSUME $(\mathring{x}_A, \mathring{y}_A) = (20, 0)$ AND $(\mathring{x}_B, \mathring{y}_B) = (30, 40)$:

TO ACHIEVE $(x_A, y_A) = (15, 15)$ AND $(x_B, y_B) = (30, 45)$,

WE MUST HAVE

$z = 5$, $q = 20$ AND $f'(z) = 3$;

THEREFORE $p = \dfrac{p_x}{p_y} = 3$, LET'S SAY $p_y = 1$ AND $p_x = 3$;

AND THEREFORE PROFIT IS $\pi = p_y q - p_x z$

$= (1)(20) - (3)(5) = 5.$

AMY'S BUDGET CONSTRAINT IS

$$p_x x_A + p_y y_A = p_x \mathring{x}_A + p_y \mathring{y}_A = (3)(20) + (1)(0) = 60$$

AND AT $(15, 15)$ WE HAVE $p_x x_A + p_y y_A = (3)(15) + (1)(15) = 60$

AND $MRS_A = 3 = \dfrac{p_x}{p_y}$, SO AMY IS MAXIMIZING HER UTILITY SUBJECT TO HER B.C.

BEV'S BUDGET CONSTRAINT IS

$$p_x x_B + p_y y_B = p_x \mathring{x}_B + p_y \mathring{y}_B + \pi = (3)(30) + (1)(40) + 5 = 135$$

AND AT $(30, 45)$ WE HAVE $p_x x_B + p_y y_B = (3)(30) + (1)(45) = 135$

AND $MRS_B = 3 = \dfrac{p_x}{p_y}$, SO BEV IS MAXIMIZING HER UTILITY SUBJECT TO HER B.C.

BEV CONSUMES $x_B = 30 = \mathring{x}_B$ AND $y_B = 45 = \mathring{y}_B + \pi$.

SHE PURCHASES $z = 5$ UNITS OF X FROM AMY AND USES IT TO PRODUCE $q = 20$ UNITS OF Y; SHE SELLS 15 UNITS OF Y TO AMY AND 5 UNITS TO HERSELF (AS IN FISHER'S SEPARATION THEOREM), OR EQUIVALENTLY, SHE KEEPS THE 5 "SURPLUS" OR PROFIT UNITS OF Y FOR HERSELF.