ECON 501B
FALL 2008 FINAL EXAM
SOLUTIONS

(1) $MRS_A = 100 - 2x$, $MRS_B = 150 - 3x$, $MRS_C = 250 - 5x$
$MC = 100$.

(a) $\max\limits_{x, y_A, y_B, y_C} \lambda_A u_A(x, y_A)$ s.t. $y_A + y_B + y_C + C(x) \leqq \mathring{y}$ $\quad : \sigma$

$u_B(x, y_B) \geqq \bar{u}_2$ $\quad : \lambda_B$

$u_C(x, y_C) \geqq \bar{u}_3$ $\quad : \lambda_C$

FOMC: (INTERIOR)

x: $\lambda_A u_{Ax} = \sigma C'(x) - \lambda_B u_{Bx} - \lambda_C u_{Cx}$ i.e., $\sum \lambda_i u_{ix} = \sigma C'(x)$

y_A: $\lambda_A u_{Ay} = \sigma$ i.e., $\lambda_A = \dfrac{\sigma}{u_{Ay}}$

y_B: $0 = \sigma - \lambda_B u_{By}$ i.e., $\lambda_B = \dfrac{\sigma}{u_{By}}$

y_C: $0 = \sigma - \lambda_C u_{Cy}$ i.e., $\lambda_C = \dfrac{\sigma}{u_{Cy}}$

COMBINING: $\sigma \dfrac{u_{Ax}}{u_{Ay}} + \sigma \dfrac{u_{Bx}}{u_{By}} + \sigma \dfrac{u_{Cx}}{u_{Cy}} = \sigma C'(x)$

i.e., $MRS_A + MRS_B + MRS_C = C'(x)$.

(b) $(100 - 2x) + (150 - 3x) + (250 - 5x) = 100$

i.e., $500 - 10x = 100$; i.e., $10x = 400$; i.e., $\boxed{x = 40}$.

ONLY RESTRICTION ON y_A, y_B, y_C: $y_A + y_B + y_C = \mathring{y} - 4000$.

(c) AT ANY $x > 0$: $MRS_C = \frac{5}{3} MRS_B = \frac{5}{2} MRS_A > MRS_A, MRS_B$.

$\therefore MRS_C = MC \Rightarrow MRS_A < MC$ & $MRS_B < MC$.

$m_C > 0 \Rightarrow MRS_C = MC$, $\therefore 250 - 5x = 100$, i.e., $\boxed{x = 30}$

$\therefore MRS_B < MC$ & $m_B = 0$, $MRS_C < MC$ & $m_C = 0$,

$\therefore m_C = \$3000$. $\therefore \boxed{(m_A, m_B, m_C) = (\$0, \$0, \$3000), \quad x = 30,}$

IF $m_A > 0$, THEN $MRS_A = MC$, BUT $MRS_C > MC$, NOT EQUIL'M. SAME IF $m_B > 0$.

(d) INCREASE x TO $x = 40$; i.e., $\Delta x = 10$.

LET'S TRY FUNDING THE INCREASE $\Delta x = 10$ BY HAVING EACH PERSON PAY HER MRS AT $x = 40$ FOR EACH OF THE 10 ADDITIONAL UNITS, BECAUSE WE KNOW THAT

(1) EACH PERSON'S MARGINAL BENEFIT FOR EACH OF THE INCREMENTAL UNITS EXCEEDS HER MRS AT $x = 40$,

(2) ΣMRS_i AT $x = 40$ IS EQUAL TO MC, WHICH IS CONSTANT, SO THIS METHOD WILL EXACTLY FUND EACH OF THE INCREMENTAL UNITS.

LET t_A, t_B, t_C DENOTE THE TOTAL EACH PERSON PAYS IN EITHER OUTCOME.

	NEW OUTCOME	NE
WE HAVE	$t_A = (20)(10) = 200$	$t_A = 0$
	$t_B = (30)(10) = 300$	$t_B = 0$
	$t_C = 3000 + (50)(10) = \cancel{500}\ 3500$	$t_C = 3000$
	4000	

NASH EQUILIBRIUM:

$$u_A = \mathring{y}_A - 0 + 3000 - 900 = \mathring{y}_A + 2100$$
$$u_B = \mathring{y}_B - 0 + 4500 - 1350 = \mathring{y}_B + 3150$$
$$u_C = \mathring{y}_C - 3000 + 7500 - 2250 = \mathring{y}_C + 2250$$

NEW OUTCOME:

$$u_A = \mathring{y}_A - 200 + 4000 - 1600 = \mathring{y}_A + 2200 \qquad \Delta u_A = +100$$
$$u_B = \mathring{y}_B - 300 + 6000 - 2400 = \mathring{y}_B + 3300 \qquad \Delta u_B = +150$$
$$u_C = \mathring{y}_C - 3500 + 10{,}000 - 4000 = \mathring{y}_C + 2500 \qquad \Delta u_C = +250$$
$$\Sigma \Delta u_i = 500$$

② (a) $MRS_R^A = MRS_R^B$: $\frac{1}{2} - \frac{1}{60}x_R^A = \frac{1}{3} - \frac{1}{60}x_R^B$

i.e., $30 - x_R^A = 20 - x_R^B$; i.e., $x_R^A = x_R^B + 10$.

SINCE $\mathring{x}_R = 30$, WE HAVE $\boxed{x_R^A = 20,\ x_R^B = 10}$.

$MRS_D^A = MRS_D^B$: $1 - \frac{1}{20}x_D^A = 1 - \frac{1}{40}x_D^B$

i.e., $40 - 2x_D^A = 40 - x_D^B$; i.e., $x_D^B = 2x_D^A$.

SINCE $\mathring{x}_D = 30$, WE HAVE $\boxed{x_D^A = 10,\ x_D^B = 20}$.

THE ONLY RESTRICTION ON x_0^A, x_0^B: $x_0^A + x_0^B = \mathring{x}_0 = 30$.

(b) ARROW-DEBREU EQUILIBRIA ARE PARETO EFFICIENT. FROM ABOVE: $MRS_R^A = MRS_R^B = \frac{1}{6}$ AND $MRS_D^A = MRS_D^B = \frac{1}{2}$.

∴ IF $p_0 = 1$, THEN $p_R = \frac{1}{6}$ AND $p_D = \frac{1}{2}$.

$x_0^A = \mathring{x}_0^A - \sum_s p_s(x_s^A - \mathring{x}_s^A) = 15 - (\frac{1}{6})(10) - (\frac{1}{2})(-10) = 15 - \frac{5}{3} + 5 = 18\frac{1}{3}$.

$x_0^B = \mathring{x}_0^B - \sum_s p_s(x_s^B - \mathring{x}_s^B) = 15 - (\frac{1}{6})(-10) - (\frac{1}{2})(10) = 15 + \frac{5}{3} - 5 = 11\frac{2}{3}$.

(c) $d_1 = \begin{bmatrix}1\\0\end{bmatrix}$ AND $d_2 = \begin{bmatrix}1\\1\end{bmatrix}$; ∴ THE SECURITIES' PRICES (USING THE ARROW SECURITY-PRICING FORMULA) ARE $q_1 = (1)p_R + (0)p_D = \frac{1}{6}$ AND $q_2 = (1)p_R + (1)p_D = \frac{2}{3}$.

THE INTEREST RATE r SATISFIES $\frac{1}{1+r} = q_2$, SO WE HAVE $\frac{1}{1+r} = \frac{2}{3}$; i.e., $1+r = \frac{3}{2}$; i.e., $r = \frac{1}{2} = 50\%$.

SINCE $d_{1D} = 0$, THE x_D^i-ALLOCATION REQUIRES THAT THE HOLDINGS y_2^i OF SECURITY #2 SATISFY

$y_2^A = x_D^A - \mathring{x}_D^A = -10$ AND $y_2^B = x_D^B - \mathring{x}_D^B = 10$.

THEN THE x_R^i-ALLOCATION REQUIRES THAT

$y_1^A = x_R^A - \mathring{x}_R^A - y_2^A = 10 - (-10) = 20$

$y_1^B = x_R^B - \mathring{x}_R^B - y_2^B = -10 - 10 = -20$.

(d) THE FIRST TWO SECURITIES' PRICES ARE UNCHANGED FROM (c). THE NEW SECURITY'S PRICE (IN EQUILIBRIUM) WILL BE $q_3 = 3p_R + 2p_D = (3)(\frac{1}{6}) + (2)(\frac{1}{2}) = \frac{3}{2}$.

EACH PERSON (A AND B) WILL HAVE TO HOLD SOME AMOUNT OF ONE OR BOTH OF THE FIRST TWO SECURITIES, SINCE THE THIRD SECURITY ALONE WILL NOT ENABLE ANYONE TO ATTAIN HIS EQUILIBRIUM ALLOCATION.

THIS IS NOT INCONSISTENT WITH THE FACT THAT THE THIRD SECURITY'S PAYOFFS DOMINATE THE OTHER TWO, BECAUSE THIS SECURITY HAS A CORRESPONDINGLY HIGHER PRICE THAN THE OTHER TWO. IN EQUILIBRIUM EACH PERSON IS INDIFFERENT AMONG ALL PORTFOLIOS MADE UP OF THE THREE SECURITIES IN COMBINATIONS THAT ATTAIN HIS ARROW-DEBREU CONSUMPTION STREAM.

③ MARKET DEMAND: $p = 40 - \frac{1}{100}Q$, i.e., $Q = 4000 - 100p$.

(a) $\frac{\partial \pi_1}{\partial q_1} = MR_1 - MC_1 = \left(40 - \frac{1}{100}q_2\right) - \frac{2}{100}q_1 - 30$

$= 10 - \frac{2}{100}q_1 - \frac{1}{100}q_2 = 0$ AT BEST q_1.

$\therefore\ 2q_1 + q_2 = 1000$; i.e., $\boxed{q_1 = 500 - \frac{1}{2}q_2}$.

$q_1 = r_1(q_2)$

$\frac{\partial \pi_2}{\partial q_2} = MR_2 - MC_2 = \left(40 - \frac{1}{100}q_1\right) - \frac{2}{100}q_2 - 32$

$= 8 - \frac{1}{100}q_1 - \frac{2}{100}q_2 = 0$ AT BEST q_2

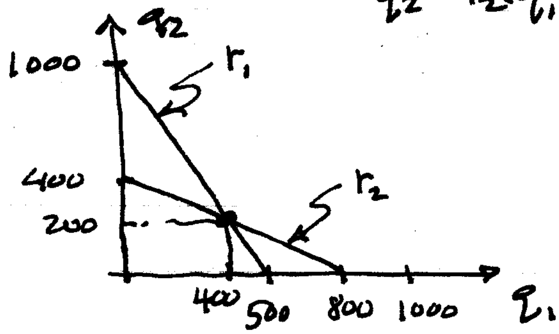
$\therefore\ q_1 + 2q_2 = 800$; i.e., $\boxed{q_2 = 400 - \frac{1}{2}q_1}$.

$q_2 = r_2(q_1)$

COURNOT EQUILIBRIUM:

SOLVING THE TWO EQUIL'M EQUATIONS (REACTION FUNCTIONS) SIMULTANEOUSLY YIELDS

$\boxed{q_1 = 400,\ q_2 = 200}$.



$\therefore\ \boxed{Q = 600 \text{ AND } p = \$34}$

$\pi_1 = (34 - 30)q_1 = (4)(400) = \$1600$

$\pi_2 = (34 - 32)q_2 = (2)(200) = \$400$.

(b) MAXIMIZATION OF TOTAL PROFIT $\pi_1 + \pi_2$ REQUIRES THAT $q_2 = 0$, SINCE EVERY UNIT OF OUTPUT COSTS LESS IF PRODUCED BY FIRM 1. FIRM 1'S REACTION CURVE, ABOVE, TELLS US THAT IF $q_2 = 0$ THEN π_1 IS MAXIMIZED AT $q_1 = 500$.

$\therefore\ Q = q_1 = 500,\ q_2 = 0,\ p = \$35,\ \pi_1 = (35 - 30)500 = \$2500$.

(c) $u(x,y) = y + 40x - \frac{1}{2}x^2$ $\therefore$ MRS $= 40 - x$, $\therefore$ $x = 40 - p$ is individual demand.

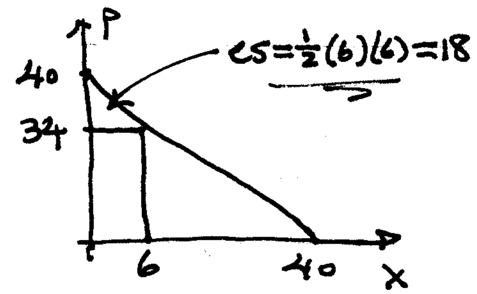
The Cournot outcome in (a):

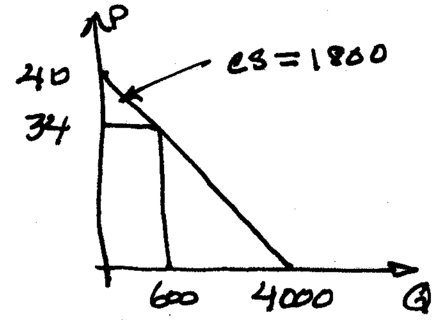
$p = \$34$, $x = 6$, $px = \$204$

$u = \mathring{y} - 204 + 240 - 18 = \mathring{y} + 18$.

Each buyer's CS is \$18;

total CS is \$1800.





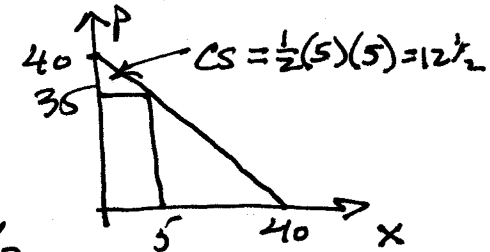
The collusive outcome in (b):

$p = \$35$, $x = 5$, $px = \$175$

$u = \mathring{y} - 175 + 200 - 12\frac{1}{2} = \mathring{y} + 12\frac{1}{2}$.

Each buyer's CS is \$12½;

total CS is \$1250.



(d) Any Pareto outcome will have $p = MC = \$30$,

$\therefore$ $x = 10$ for each consumer, and $px = \$300$.

$u = \mathring{y} - 300 + 400 - 50 = \mathring{y} + 50$.

Total CS is \$5000 an increase of \$3200 over (a) and \$3750 over (b).

But the firms' profits are now zero, a decrease of \$2000 relative to (a), and \$2500 relative to (b).

To compensate the firms, each consumer must give up a transfer of ~~[illegible]~~ $t < \$32$ (so the consumer's gain isn't entirely eliminated and $t > \$20$ (if comparing to (a)) or $t > \$25$ (compared to (b)).