# Economics 501B Midterm Exam
## University of Arizona
## Fall 2010

Amy and Bev are sisters. Their mother Cat has died and left them an inheritance consisting of 50 units of a good we'll call $X$ and 40 units of a good we'll call $Y$. Amy's and Bev's preferences are described by the utility functions

$$u_A(x,y) = x^3y \qquad \text{and} \qquad u_B(x,y) = x^2y.$$

1. Amy and Bev want to divide up the two goods so that each receives a bundle $(x_i, y_i)$.

(a) Verify that there is a Pareto allocation in which Amy receives the bundle $(x_A, y_A) = (30, 20)$.

(b) Use the EqualMRS Pareto condition to determine the set of all Pareto allocations — specifically, use the EqualMRS condition to derive an analytical expression that describes all the bundles $(x_A, y_A)$ that are part of a Pareto allocation. Draw a careful diagram of the Pareto set in the Edgeworth box.

(c) The Second Welfare Theorem says that a Pareto allocation can be supported as a Walrasian market equilibrium. Assume that Amy has inherited all 50 units of the $X$ good and Bev has inherited all 40 units of the $Y$ good. Will they attain the allocation $((x_A, y_A), (x_B, y_B)) = ((30, 20), (20, 20))$ as a market equilibrium (assuming they're both price-takers)? If so, verify it and determine the equilibrium price ratio. If not, explain why not, and determine a transfer of *only* the $X$ good — *i.e.*, a change in just the division of $X$ that Cat could have made in her bequest to her daughters — that will yield $((30, 20), (20, 20))$ as a market equilibrium allocation.

2. Now assume that $X$ is thousands of dollars inherited today and $Y$ is thousands of dollars placed into a trust fund that Amy and Bev will inherit "tomorrow". Amy has inherited $\mathring{x}_A = 20$ (thousand dollars) today and she will inherit $\mathring{y}_A = 40$ tomorrow. Bev has inherited $\mathring{x}_B = 30$ today but will not inherit anything tomorrow — the trust fund will all go to Amy.

(a) Determine an equilibrium interest rate at which they could trade dollars today for dollars tomorrow to achieve a Pareto allocation. How much will each person save, lend, or borrow?

(b) Determine the present value of the consumption stream each person achieves via this trade.

3. Amy and Bev have a production process that enables them to produce the $Y$ good using the $X$ good as input according to the production function $q = f(z) = 5z - \frac{1}{5}z^2$, where $z$ denotes the amount of $X$ used as input and $q$ denotes the resulting amount of $Y$ produced as output.

(a) Write down a parametric family of maximization problems that characterizes the Pareto allocations, in the sense that the Pareto allocations are the solutions of the maximization problems.

(b) Obtain the first-order marginal conditions (FOMC) that characterize the interior solutions of the maximization problems, and give the FOMC an economic interpretation.

(c) Verify that there is a Pareto allocation in which Amy receives the bundle $(x_A, y_A) = (15, 15)$ and Bev receives the bundle $(x_B, y_B) = (30, 45)$. Determine values of the Lagrange multipliers for which this allocation satisfies the FOMC you obtained in part (b).

(d) Assume that Amy has inherited 20 units of $X$ and no $Y$; Bev has inherited 30 units of $X$ and all 40 units of $Y$. Bev owns the production process and any profits it generates. Determine whether they could attain the allocation in (c) as a Walrasian market equilibrium. If not, explain why not. If so, determine the equilibrium price ratio and the amount of each good Amy and Bev trade with one another, and verify that each person's consumption bundle lies on her budget constraint.