

**Economics 501B Final Exam**  
**University of Arizona**  
**Fall 2009**

1. Abby's income today is \$40 per hour. If universal health care legislation is passed within the next year, then her income tomorrow will be \$50/hour; but if the legislation fails to pass, her income tomorrow will be only \$30. Bill sells insurance, and his income today is \$20/hour. If health care legislation is passed, his income tomorrow will be \$10/hour, but if the legislation fails to pass, his income tomorrow will be \$15/hour. Abby's preferences are described by the utility function

$$u^A(x_0, x_H, x_F) = x_0 + 20 \log x_H + 5 \log x_F$$

and Bill's by the function

$$u^B(x_0, x_H, x_F) = x_0 + 10 \log x_H + 10 \log x_F,$$

where  $x_0$  denotes the individual's spending today,  $x_H$  denotes spending tomorrow if the legislation passes, and  $x_F$  denotes spending tomorrow if the legislation fails to pass (all measured per hour).

- (a) Determine the Pareto efficient allocation(s).
- (b) Determine the Arrow-Debreu allocation(s) and prices.

In (c), (d), and (e) you can solve directly, or you can appeal to the complete-markets security pricing formula.

- (c) In the Arrow-Debreu market structure, what is the (implicit) interest rate?
- (d) Suppose the only securities are shares in the firm Gamma Technologies and shares in the firm Delta Insurance. Each share of Gamma will yield \$1 if the legislation passes and \$2 if the legislation fails. Each share of Delta will yield \$1 if the legislation passes and nothing if it fails. Determine the equilibrium security prices and Abby's and Bill's holdings of securities.
- (e) In the market structure in (d), what portfolio would one have to hold in order to guarantee oneself a return of \$1 tomorrow, whether the legislation passes or not? What would be the cost of the portfolio? What would you say is the interest rate, and why?

2. Alice is a musician; Bart is not. Let  $x$  denote the number of hours per day that Alice devotes to writing, playing, and recording his music. It costs Alice \$4 for every hour he spends producing music. Let  $y_A$  and  $y_B$  denote Alice's and Bart's dollar expenditures on goods other than music. Each is endowed with more than \$100 per day. Alice's and Bart's preferences for Alice's music are described by the utility functions

$$u_A(x, y_A) = \begin{cases} y_A + 8x - \frac{1}{2}x^2 & , x \leq 8 \\ y_A + 32 & , x \geq 8 \end{cases} \quad \text{and} \quad u_B(x, y_B) = \begin{cases} y_B + 12x - \frac{1}{2}x^2 & , x \leq 12 \\ y_B + 72 & , x \geq 12 \end{cases}$$

Note that Alice's and Bart's marginal rates of substitution are

$$MRS_A = \max\{0, 8 - x\} \quad \text{and} \quad MRS_B = \max\{0, 12 - x\}.$$

- (a) How much will Alice produce if he doesn't even know Bart exists?
- (b) Suppose that Alice produces the amount of music in (a) and Bart has found a way to pirate the music by downloading it from Alice's computer. Alice has no way to prevent Bart from this "free riding" piracy. How much consumer surplus do Alice and Bart obtain? (Don't forget that it costs Alice \$4 for every hour he devotes to producing music.)
- (c) Derive the marginal condition that characterizes the Pareto allocations  $(x, y_A, y_B)$ .
- (d) What is the Pareto amount of music for Alice to produce? What is the total surplus at this level of music?
- (e) Now suppose Bart and Alice agree on a transfer payment  $t$  from Bart to Alice, in return for Alice producing the Pareto amount of music. Determine the range of payments that yield core allocations, and determine each one's consumer surplus as a function of  $t$ .
- (f) Now suppose Alice has found a way to protect his music from Bart's piracy. From now on, Bart will have to pay  $p$  dollars for every unit of music he downloads — *i.e.*, Bart will have to pay Alice  $px$  dollars to download  $x$  units of music. If Alice chooses the price  $p$  that maximizes his profit (revenue from Bart, minus the cost of production), what price will he charge and how much music will he produce? What will be Alice's and Bart's consumer surpluses? How much profit will Alice earn? (**Important:** This is *not* the price that maximizes Alice's utility.)
- (g) Knowledge of the Envelope Theorem should tell you how Alice can increase his utility by charging a different price than in (f). Don't try to determine the utility-maximizing price and production, but indicate whether Alice should increase or decrease the price and production from the profit-maximizing levels in (f) in order to maximize his utility, and explain why this is easy to determine.

3. Abby's and Beth's preferences are both described by the utility function  $u(x, y) = xy$ . Abby owns the bundle (4,1), Beth owns the bundle (1,4).

(a) Determine the Walrasian equilibrium allocation and prices. You needn't do this by deriving the equilibrium, but you should verify that what you have is an equilibrium.

(b) Determine the set of core allocations.

Now suppose that Abby and Beth are joined by a third person, Cathy, who has the same preference as the others, but who owns the bundle (2,2).

(c) Determine the Walrasian equilibrium for this three-person economy. Again, you only have to verify that the equilibrium you've identified is actually an equilibrium.

(d) Determine the set of core allocations in the three-person economy.