Economics 501B Midterm Exam
University of Arizona
Fall 2008

1. Four gallons of gasoline and eight simoleons are to be allocated between Ann and Bill, whose preferences are described by the utility functions

$$u_A(x_A, y_A) = x_A y_A \qquad \text{and} \qquad u_B(x_B, y_B) = y_B - \tfrac{1}{8}(4 - x_B)^2 \ ,$$

where x denotes gallons of gasoline and y denotes simoleons.

Note that Ann's and Bill's marginal rates of substitution are given by

$$MRS_A = \tfrac{y_A}{x_A} \qquad \text{and} \qquad MRS_B = 1 - \tfrac{1}{4}x_B \ .$$

(a) Someone has suggested allocating the bundle $(4, 6)$ to Ann and $(0, 2)$ to Bill. Write down the maximization problem that characterizes Pareto allocations, the problem we've called (P-Max), using the utility functions and resources described here. Determine whether the problem's first-order conditions (FOC) are satisfied at the suggested allocation. (Include *all* first-order conditions, not just the marginal ones.) If the FOC are satisfied, determine the values of the Lagrange multipliers; if the FOC are not satisfied, show that appropriate Lagrange values cannot be found.

(b) Use the value of one of the Lagrange multipliers in (a) to determine how much u_A could be increased if we were to reduce u_B by one unit — for example, by reducing y_B from 2 to 1.

(c) At the allocation $(4, 1)$ to Ann and $(0, 7)$ to Bill we have $MRS_A = 1/4$ and $MRS_B = 1$. At the allocation $(3, \frac{3}{2})$ to Ann and $(1, \frac{13}{2})$ to Bill we have $MRS_A = 1/2$ and $MRS_B = 3/4$. Call these two allocation Z and Z', respectively. Use just the marginal rates of substitution (and the quasiconcavity of the preferences) to verify that Z' is a strict Pareto improvement on Z.

(d) Determine the set of *all* Pareto allocations and draw the set in an Edgeworth box diagram.

(e) Suppose Ann already owns the bundle $(4, 6)$ and Bill already owns the bundle $(0, 2)$. Determine *all* the Walrasian equilibrium price lists and allocations. Explain how you know you've identified all of them.

(f) Now suppose Ann owns the bundle $(4, 0)$ and Bill owns the bundle $(0, 8)$. Determine *all* the Walrasian equilibrium price lists and allocations. Explain how you know you've identified all of them.

2. Choose *one* of the following — either (a) or (b). Assume that A and B are subsets of Euclidean spaces.

(a) Prove that if A and B are convex sets, then the set $A \times B$ is also convex.

(b) Prove that if A and B are closed sets, then the set $A \times B$ is also closed.

3. Securities analysts at three investment firms provide weekly forecasts of the price of a particular security. None of the analysts ever provides a forecast larger than M and of course none of them ever provides a negative forecast. Each analyst consults the previous week's forecasts by the other two analysts to "shade" his own forecast. Specifically, using p_i to denote analyst i's forecast in a given week and p_i' to denote his forecast in the subsequent week, the forecasts are related as follows:

$$\begin{aligned} p_1' &= \lambda_{12}p_2 + \lambda_{13}p_3 \ , &\qquad \text{where } \lambda_{12} + \lambda_{13} = 1 \ , \\ p_2' &= \lambda_{21}p_1 + \lambda_{23}p_3 + \lambda_{20}M \ , &\qquad \text{where } \lambda_{21} + \lambda_{23} + \lambda_{20} = 1 \ , \\ p_3' &= \lambda_{31}p_1 + \lambda_{32}p_2, &\qquad \text{where } \lambda_{31} + \lambda_{32} < 1/2 \ . \end{aligned}$$

All λ_{ij}'s are strictly positive and do not change from week to week.

(a) Verify that there is a profile (p_1, p_2, p_3) of forecasts that is stationary, in the sense that it will lead to the same profile of forecasts in the following week. (Treat the forecasts p_i as continuous variables.) Make sure, in applying any theorem(s), that you indicate how each of the theorem's conditions is satisfied.

(b) Determine whether there is a stationary profile (p_1, p_2, p_3) in which all three analysts make the same forecast: $p_1 = p_2 = p_3$.

4. Let X be a set of alternatives for society and assume that each person $i \in N = \{1, \ldots, n\}$ has a preference over X represented by a utility function $u_i : X \to \mathbb{R}$.

(a) Prove that if an alternative $\widehat{x} \in X$ is Pareto efficient then it is a solution of the following constrained maximization problem:

$$(*) \qquad \max\ u_1(x) \ \text{subject to}\ x \in X \ \text{and to}\ u_i(x) \geqq u_i(\widehat{x}),\ i = 2, \ldots, n.$$

(b) Provide a counterexample to show that the converse of (a) is not true without imposing some condition(s) on the utility functions. You may wish to let $n = 2$ in your counterexample and to let X be a set of allocations to the members of N.