Economics 501B Final Exam
University of Arizona
Fall 2008

1. Anna, Beth, and Carl share a common swimming pool. Pool cleaning services all charge \$100 per visit to clean the pool. Let x denote the number of visits per year by a pool cleaning service, and let y_i denote the amount i has available to spend on other goods. The three residents' preferences are described by the utility functions

$$u^A(x, y^A) = y_A + 100x - x^2, \quad u^B(x, y_B) = y_B + 150x - \frac{3}{2}x^2, \quad u^C(x, y_B) = y_C + 250x - \frac{5}{2}x^2.$$

Note that their marginal rates of substitution are

$$MRS_A = 100 - 2x, \quad MRS_B = 150 - 3x, \quad MRS_C = 250 - 5x.$$

Let $\mathring{y}_i$ denote the initial endowment of resident i, and assume that $\mathring{y}_i$ is at least \$20,000 for each resident.

(a) Derive the Samuelson condition on marginal rates of substitution that characterizes the Pareto efficient interior allocations (x, y_A, y_B, y_C).

(b) Determine the set of all interior Pareto allocations.

(c) Assume that the number of pool-cleaning visits is determined by the following method: At the beginning of the year each resident contributes to a fund that will pay for the cleaning visits. Let m_i denote the amount that resident i contributes. Determine the Nash equilibrium when this procedure is used.

(d) Find a Pareto allocation that is a strict Pareto improvement on the outcome in (c).

2. Assume that half the people in the economy choose according to the utility function

$$u^A(x_0, x_R, x_D) = x_0 + \frac{1}{2}x_R - \frac{1}{120}x_R^2 + x_D - \frac{1}{40}x_D^2$$

and the other half according to the utility function

$$u^B(x_0, x_R, x_D) = x_0 + \frac{1}{3}x_R - \frac{1}{120}x_R^2 + x_D - \frac{1}{80}x_D^2,$$

where x_0 represents consumption today, x_R represents consumption tomorrow if the Republicans win the election that will take place in the interim, and x_D represents consumption tomorrow if the Democrats win the election. Everyone is endowed with 15 units of the consumption good today. If the Republicans win the election the Type A people will be endowed with 10 units of the good and the Type B people with 20 units. If the Democrats win it will be just the reverse: the Type A people will be endowed with 20 units and Type B people with 10 units. Storage of the consumption good from today until tomorrow is not possible.

In your answers, consider only allocations that give all type A people the same consumptions and all type B people the same consumptions, so that you will be able to completely describe an allocation with the six variables $x_0^A, x_R^A, x_D^A, x_0^B, x_R^B, x_D^B$. Note that the marginal rates of substitution are

$$MRS_R^A = \frac{1}{2} - \frac{1}{60}x_R^A, \quad MRS_D^A = 1 - \frac{1}{20}x_D^A, \quad MRS_R^B = \frac{1}{3} - \frac{1}{60}x_R^B, \quad MRS_D^B = 1 - \frac{1}{40}x_D^B.$$

(a) Determine the interior Pareto allocations.

(b) Determine the Arrow-Debreu prices and the Arrow-Debreu equilibrium allocation.

(c) Assume that Arrow-Debreu contingent claims markets don't exist, but that there are two securities. One security is an insurance policy that pays off one unit of the consumption good tomorrow if the Republicans win the election and nothing if the Democrats win. The other security is a bond that pays off one unit of the good tomorrow whichever party wins the election. Determine the equilibrium prices of the securities and the equilibrium amounts of each security that each person will buy or sell today.

(d) In addition to the securities in (c), there is a third security that pays off 3 units of the good if the Republicans win and 2 units if the Democrats win. Determine the equilibrium prices of all three securities. Note that the new security pays off strictly more in each state than either of the other two. Will anyone hold either of the other two securities in this case? Explain why or why not.

3. Two firms produce an identical product and they are the only firms in their market. Let p denote the price at which the product sells and let Q denote the total quantity the two firms sell: $Q = q_1 + q_2$, where q_i is the quantity sold by firm i. The market inverse demand function is $p = 40 - \frac{1}{100}Q$. Firm 1's cost function is $C_1(q_1) = 30q_1$ and Firm 2's cost function is $C_2(q_2) = 32q_2$.

(a) Determine each firm's Cournot reaction function, draw the two reaction curves in a single diagram, and determine the Cournot equilibrium price and quantities. Determine each firm's profit at the equilibrium.

(b) What would be the price and quantities if the two firms were to collude, jointly choosing their output quantities to maximize joint profits (i.e., total profit)? What would be the two firms' profits, before any side payments from one firm to the other?

(c) Suppose there are 100 consumers, each one with a preference represented by the utility function $u(x, y) = y + 40x - \frac{1}{2}x^2$, where x denotes consumption of the homogeneous product the two firms produce and y denotes the amount of money the consumer spends on other goods. Determine the total consumer surplus in each of the two outcomes described in (a) and (b).

(d) Assuming the owners of the two firms care only about the profits their firms earn, find a Pareto optimal Pareto improvement on either the outcome in (a) or the outcome in (b), whichever one you find easier to do.