Introductory Notes on Public Goods for Intermediate Microeconomics

Let's begin with an extremely simple example of a public good. Suppose there are only two people who live on the shore of Lake Magnavista. Amy likes to water ski and Bev likes to sunbathe. Both activities are seriously affected by the level of the water in the lake. When there is a lot of water in the lake, it's good for water skiing but the water line is so high that there is no beach for sunbathing. When there is much less water, the sunbathing is good but the lake is too shallow for water skiing. Therefore Amy prefers that the lake have lots of water, and Bev prefers that it have much less water. Fortunately, it's possible to raise or lower the water level costlessly, by opening a dam at one end of the lake or at the other end. Unfortunately, it's not clear at what level the water ought to be set.

In order to have a measure of the amount of water in the lake, let's use the water's depth at a specified location on the lake: let x denote the water's depth (in feet) at that location. Amy's and Bev's preferences are described by the following utility functions:

$$u^A(x, y_A) = y_A - (15 - x)^2 \quad \text{and} \quad u^B(x, y_B) = y_B - \frac{1}{2}(6 - x)^2,$$

where x denotes the water level and y_A and y_B are Amy's and Bev's daily consumption of other goods, measured in dollars. Suppose Amy and Bev each have incomes of \$100 per day. Note that Amy's and Bev's marginal rates of substitution are

$$MRS_A = 30 - 2x \quad \text{and} \quad MRS_B = 6 - x.$$

Amy's most-preferred water level is $\widehat{x}_A = 15$ and Bev's most-preferred level is $\widehat{x}_B = 6$. Figure 1 depicts their indifference maps. Notice that if the water level is above $\widehat{x}_B$, Bev would be willing to pay (*i.e.*, to give up some of the y-good) to have x *reduced*, and that Amy would similarly be willing to pay to reduce x if it's above her ideal level, $\widehat{x}_A$.

What makes this situation different than everything we've seen before is that the x variable can't be at different levels for different people. It's not like pizza or beer, where Amy can consume one quantity and Bev a different quantity. In this case, the water level can be varied, but it will be the same for both of them. That's why we haven't used A and B subscripts on the x variable: it's just one variable, not two. The water level in this example is a **public good**.

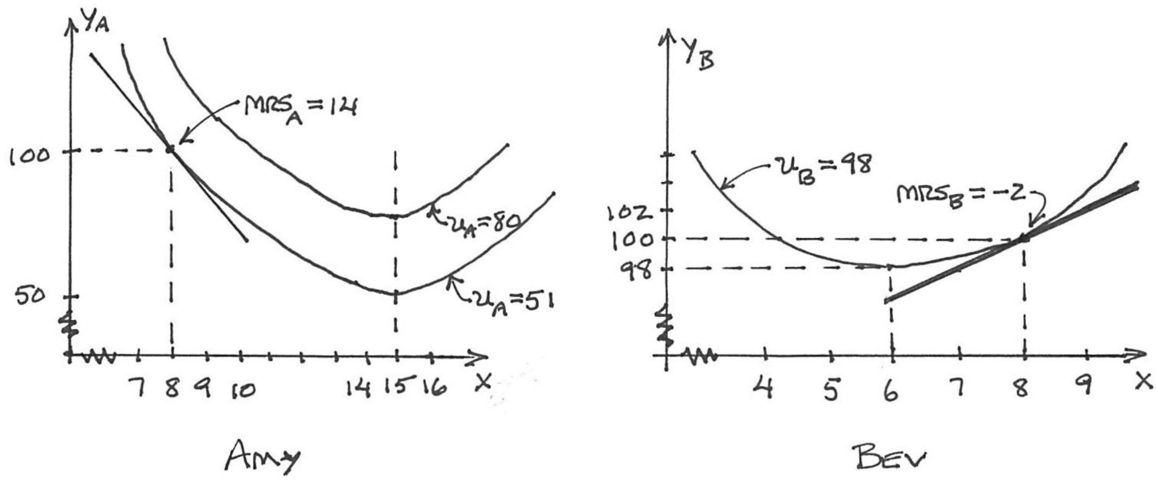

Figure 1

Let's try to determine which outcomes are Pareto efficient. Let's start by asking whether a water level of $x = 8$ feet is efficient. Figure 2 will be helpful here: it's a diagram similar to the Edgeworth Box. The difference is that here in this diagram the "corners" or "origins" for both consumers are placed on the left edge of the box, so that when x increases (*i.e.*, as we move toward the right), *both* persons' consumption of the x-good is increasing. This is in contrast to the Edgeworth Box, where as we move toward the right, Person A's consumption is increasing and B's is decreasing, because with "private goods" like pizza or beer, the more one person gets, the less is available for the other person.

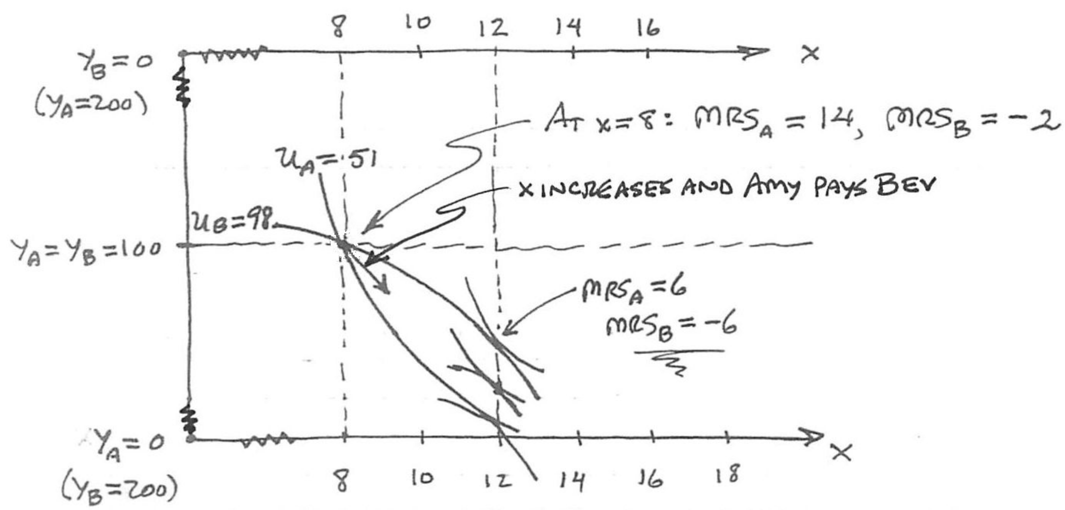

Figure 2

At $x = 8$, Amy's and Bev's MRS's will be $MRS_A = 14$ and $MRS_B = -2$. Amy would be willing to pay \$14 to increase the water level one foot and Bev would be willing to pay \$2 to decrease it by one foot — or Bev would be willing to accept a payment of \$2 as compensation for *increasing* the water level by one foot. Therefore, if we were to increase the water level by a foot, and if Amy were to *compensate* Bev by paying her, say, \$6, then they would both be better off. In fact, you can calculate that Amy's utility will increase from 51 to 58 and that Bev's utility will increase from 98 to $101\frac{1}{2}$.

So what about this new water level of $x = 9$ feet — is *it* efficient? We have $MRS_A = 12$ and $MRS_B = -3$, so we could increase the water level by another foot, with Amy paying another \$6 to compensate Bev. The \$6 payment is less than the \$12 Amy would have been willing to pay, and more than the \$3 Bev would have been willing to accept as compensation, so they're again both better off from the one-foot increase with \$6 compensation. You can calculate that their utilities will have increased again, to $u_A = 63$ and $u_B = 104$.

Now it's becoming clear that as long as Amy would be willing to pay more for an increase than Bev would be willing to accept as compensation, then such a bargain — increasing the water level, with Amy compensating Bev — will make them both better off. In other words, the water level is not Pareto efficient so long as $MRS_A > -MRS_B$ — *i.e.*, so long as $MRS_A + MRS_B > 0$. When $MRS_A + MRS_B > 0$ the **marginal social value** of an increase in x is positive, so x should be increased. Similarly, we could show that if $MRS_A + MRS_B < 0$, then x should be decreased (because the marginal social value of an increase is negative, so the marginal social value of a *decrease* in x is positive).

The Pareto efficient outcomes are therefore the ones that satisfy the Test Condition $MRS_A + MRS_B = 0$. In our example, it's easy to solve for the efficient water level in the lake:

$$MRS_A + MRS_B = (30 - 2x) + (6 - x) = 36 - 3x;$$

therefore the Test Condition yields

$$36 - 3x = 0 \quad \textit{i.e.}, \quad x = 12.$$

The efficient water level is 12 feet, where $MRS_A = 6$ and $MRS_B = -6$.

Now we know the water level that's Pareto efficient. But what level will actually be chosen? As usual, that depends on the institutional arrangements that are used for choosing the water level. For example, the affected parties might vote on the level they want. The outcome of that institution can be analyzed using game theory, which we won't do here. Let's suppose instead that Bev owns the lake, or at least that she has the right to choose the water level. What will the water level be? It seems that Bev will choose the level that she likes best, namely $x = 6$ feet.

But we've already seen that at such a low water level, Bev would be better off to allow a higher level if Amy will compensate her appropriately. Indeed, just exactly as in our discussion earlier in the semester of bargaining between two parties, we would expect them to arrive at a mutually agreeable bargain in which there are no more gains to be had from further trade or bargaining — *i.e.*, at a Pareto efficient outcome. In our example this means that the water level will be 12 feet, with Amy paying Bev some amount as compensation for the increase from 6 to 12 feet. In Figure 3 they will end up on the vertical line at $x = 12$, with Amy paying Bev an amount that leaves their y consumptions between their initial indifference curves, the ones that pass through the allocation in which $x = 6$ and $y_A = y_B = 100$.

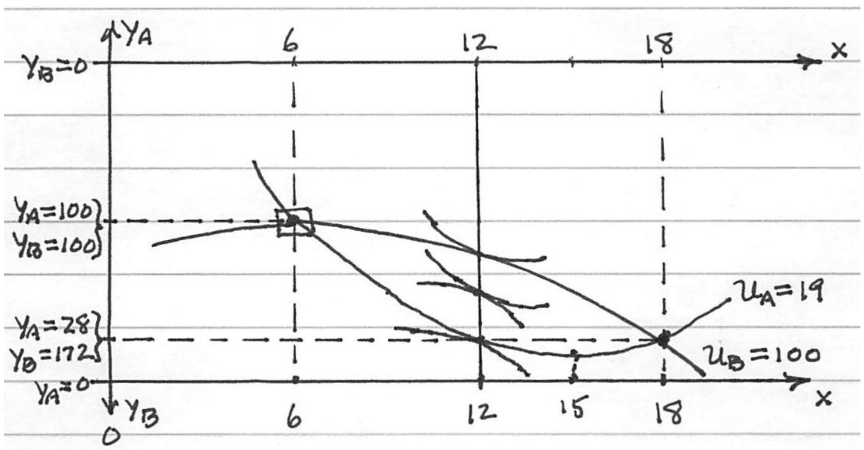

Figure 3

What if instead Amy has the right to choose the water level? At first it seems as if she would choose the level *she* likes best, $x = 15$ feet. But the same argument as in the preceding paragraph tells us that again we should expect the two women to bargain with one another to set the water level at $x = 12$ feet, but this time with Bev paying some compensation to Amy for setting the water level below what Amy would like the most.

It was the economist Ronald Coase who pointed out that the assignment of legal rights will affect only who pays compensation, and how much, but will not affect the level of the activity that affects both parties. He was awarded the Nobel Prize for this idea, which has had a profound effect on law and, to a lesser extent, legislation in recent decades.

What if the public good is not costless, as the water level was? Then the Pareto Efficiency Test Condition (PETC) is that the marginal social value of a one-unit increase in the amount of the public good must be equal to the marginal cost of producing that additional unit — *i.e.*, that the sum of all the MRS's must be equal to MC:

$$(PETC) \qquad MRS_1 + MRS_2 + \cdots + MRS_n = MC.$$

Notice that in our lake example the MC of changing the water level was zero, so this Test Condition is actually the same as the one we used in the example, just generalized to account for costly public goods as well as costless ones.

Another example: Suppose Amy and Bev are plagued by mosquitoes, but it's possible to control the number of mosquitoes by spraying regularly. However, the mosquito spray can't be confined to the property of just one of the women: any spray that's applied affects the entire lake shore equally. Suppose Amy's and Bev's preferences are described by the same utility functions as in our water-level example, where x now denotes the number of gallons that are sprayed each week, and y_A and y_B still denote the amounts Amy and Bev spend on other goods. If each gallon of spray were free, our problem would be exactly the same as before, because we would have $MC = 0$. But suppose instead that the spray costs \$24 per gallon. Then our Test Condition (PETC) yields the following:

$$MRS_A + MRS_B = MC: \quad (30 - 2x) + (6 - x) = 24; \qquad i.e., \; 36 - 3x = 24.$$

Therefore the Pareto efficient amount of mosquito spray is 4 gallons per week, which equates the marginal social value to the marginal social cost, \$24.

How much will be sprayed if Amy and Bev each choose on the basis of just their own personal benefits from the spray? Bev won't choose to spray at all, because even when $x = 0$ her MRS is much less than the price of the spray (\$6 vs. \$24). Amy will choose $x = 3$ gallons of spray, which equates her own MRS to the \$24 price of the spray. So the amount of spray that's chosen will be less than the Pareto efficient amount. Both women would be better off if an additional gallon were sprayed and Amy were to pay, say, \$22 of the cost of the additional gallon and Bev paid the remaining \$2.

A more striking example: Suppose there are 100 homeowners living on the lake shore, and that each one has $MRS = 6 - x$, where x is the number of tankfuls that are sprayed. Suppose that the cost of the mosquito spray is \$100 per tankful. Then the Pareto efficient amount of spray is $x = 5$ tankfuls:

$$\sum MRS_i = MC: \quad 100(6 - x) = 100; \qquad i.e., \; 600 - 100x = 100.$$

How much will actually be sprayed if each homeowner chooses on the basis of just his own benefit from the spray? At $x = 0$ each homeowner has $MRS = 6$ — *i.e.*, each would be willing to pay \$6 to increase x from $x = 0$ to $x = 1$ tankful sprayed. But the cost of such an increase — \$100 — far exceeds each person's MRS. Therefore no one will choose to spray. Even though the marginal social benefit is \$600 and the marginal cost is only \$100, no spray is forthcoming.