Externalities: Pigovian Taxes and Subsidies

We'll model a situation in which the production of a consumption good X generates external effects on consumers. Examples are air pollution, water pollution, noise, etc. Let x denote quantities of the X good and let y denote dollars or quantities of a good Y that's a composite of all other goods. Let s denote the level of the externality.

Consumers:

There are n consumers, each represented by a utility function u^i and an endowment bundle $(\mathring{x}_i, \mathring{y}_i)$. We assume that $\mathring{x}_i = 0$ and $\mathring{y}_i > 0$. Each consumer's utility function has the form $u^i(x_i, y_i, s)$, where s is the level of the externality. We'll express marginal rates of substitution in terms of the Y good. Note that the externality is "good" for consumer i if $MRS^i_s > 0$ and is "bad" for i if $MRS^i_s < 0$.

Production:

There are m firms that can produce X, each one according to a production function $q_j = f_j(z_j)$, where z_j is the amount of the Y good firm j uses as input. Therefore we have $\sum_{j=1}^m z_j = \sum_{i=1}^n (\mathring{y}_i - y_i)$. We assume that $s = q = \sum_{j=1}^m q_j$; results are the same if $s = g(q)$.

We'll assume that all utility functions are increasing in x and y, that all production functions are increasing, and that all utility functions and production functions are continuously differentiable and concave.

Pareto Efficiency:

The Pareto maximization problem is

$$\max \sum_{i=1}^n \lambda_i u^i(x_i, y_i, s) \text{ subject to } x_i, y_i, z_j \geqq 0 \ \ \forall i, j \text{ and}$$

$$\sum_i x_i \leqq \sum_j f_j(z_j) \tag{σ_x}$$

$$\sum_i y_i + \sum_j z_j \leqq \sum_i \mathring{y}_i \tag{σ_y}$$

$$\sum_j f_j(z_j) \leqq s \tag{σ_s}$$

The first-order marginal conditions at an interior solution are

$$x_i : \qquad \lambda_i u^i_x = \sigma_x \quad \forall i \tag{1}$$

$$y_i : \qquad \lambda_i u^i_y = \sigma_y \quad \forall i \tag{2}$$

$$s : \qquad \sum_i \lambda_i u^i_s = -\sigma_s \tag{3}$$

$$z_j : \qquad 0 = \sigma_y + \sigma_s f'_j(z_j) - \sigma_x f'_j(z_j) \tag{4}$$

Equations (1) and (2) yield

$$\forall i: \ \frac{u_x^i}{u_y^i} = \frac{\sigma_x}{\sigma_y}, \quad i.e., \ MRS_x^i = \frac{\sigma_x}{\sigma_y}, \tag{5}$$

and equations (2) and (3) yield

$$\sigma_y \sum_{i=1}^n \frac{u_s^i}{u_y^i} = -\sigma_s, \quad i.e., \ \sum_{i=1}^n MRS_s^i = \frac{\sigma_s}{\sigma_y}. \tag{6}$$

Equations (4) can be rewritten as

$$\forall j: \ (\sigma_x - \sigma_s)f_j'(z_j) = \sigma_y, \quad i.e., \ \frac{\sigma_x}{\sigma_y} - \frac{\sigma_s}{\sigma_y} = \frac{1}{f_j'(z_j)}. \tag{7}$$

Combining equations (5), (6), and (7), we have

$$\forall i,j: \ MRS_x^i = MC^j - \sum_{h=1}^n MRS_s^h. \tag{$*$}$$

Recall that $MRS_s^i < 0$ if the externality is bad for consumer i and $MRS_s^i > 0$ if the externality is good for consumer i. Therefore the sum $\sum_1^n MRS_s^i$ represents the net marginal benefit of the externality, aggregated over all consumers. If $\sum_1^n MRS_s^i < 0$, then the marginal conditions ($*$) tell us that Pareto efficiency requires each consumer's marginal value for the x-good to be equal to the good's marginal social cost — the marginal cost of producing another unit of it, plus $|\sum_1^n MRS_s^i|$, the aggregate marginal damage the consumers suffer from producing another unit of the x-good. On the other hand, if the net effect of the externality is positive — i.e., $\sum_1^n MRS_s^i > 0$ — then Pareto efficiency requires that each consumer's marginal value for the good be less than its marginal production cost by the net amount of the marginal indirect benefit consumers receive via the externality, $\sum_1^n MRS_s^i$.

The Pigovian Tax

Now suppose the net marginal externality associated with producing the x-good is negative — i.e., $\sum_1^n MRS_s^i < 0$ — and suppose that in the market for the x-good all the consumers and producers are price-takers. At an equilibrium, then, we would have $MRS^i = p_x = MC^j$ for each consumer i and each firm j. Therefore too much of the x-good is being produced (recall that each u^i is increasing in x and y, so each MRS_x^i is decreasing in x): everyone's marginal value for the good is less than its social cost of production $MC^j - \sum_{i=1}^n MRS_s^i = MC^j + |\sum_{i=1}^n MRS_s^i|$, so Pareto efficiency requires that production be reduced.

As a solution to this inefficiency, the early-20th-century English economist A.C. Pigou developed what we now call a **Pigovian tax**. Let t denote the level of a per-unit tax imposed on purchases of the x-good, and let t be equal to $-\sum_{i=1}^n MRS_s^i$, the net marginal damages

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generated by the marginal unit of the x-good produced, at the Pareto efficient level of production and consumption. Now we should expect a consumer to purchase the good to the point where her MRS^i is equal to $p_x + t$ — i.e., $MRS^i = MC^j + |\sum_{i=1}^n MRS_s^i|$, thereby satisfying the efficiency condition ($*$). If the externality is beneficial — $\sum_1^n MRS_s^i > 0$ — we still set $t = -\sum_{i=1}^n MRS_s^i$, so in this case t is a per-unit subsidy paid to purchasers of the x-good.

Note that in either case — a negative or a positive externality — it's straightforward to balance the budget. In the case of a negative externality the aggregate of all the taxes collected, $\sum_{i=1}^n t_i x_i$, can be rebated to consumers as lump-sum per capita payments. In the case of a positive externality, consumers can be charged a lump-sum per capita tax (for example) to finance the per-unit subsidies, which total $\sum_{i=1}^n |t_i x_i|$.

Determining $\sum_1^n \mathbf{MRS_s^i}$:   The Pigovian analysis leaves open the question of how we can determine, or at least estimate, the value of $\sum_1^n MRS_s^i$. An approach called **contingent valuation** has been developed to accomplish this. The contingent valuation procedure first samples the relevant population: the individuals in the sample are asked to report how much (in dollar terms) they would value some particular increase or decrease in the amount of the externality s. This tells us the MRS_s^i for each individual i in the sample. Then the sample is used to calculate an estimate of $\sum_1^n MRS_s^i$, based on the demographics of the sample.

There is an obvious problem with this procedure, however. If the individuals are paid an amount that is based on their reported MRS_s^i, they will have an incentive to report very large negative values of MRS_s^i, in order to receive large payments. On the other hand, if they're *not* paid, each person's incentive is to report an MRS_s^i that will move the level of s in the direction he prefers, given his belief about the values of MRS_s reported by others in the sample.