

ARROW'S FRAMEWORK

WE CAN RECAST OUR SPECIFICATION OF THE ECONOMY (i.e., OF THE GIVEN ECONOMIC ALLOCATION PROBLEM) AS ONE INVOLVING ONLY PRIVATE GOODS: EACH INDIVIDUAL'S CONSUMPTION OF THE PUBLIC GOOD IS A DISTINCT COMMODITY, WITH A DISTINCT MARKET AND PRICE, AND THERE IS "JOINTNESS" IN PRODUCTION OF THESE GOODS — i.e., $x_1 = x_2 = \dots = x_n$ IS A TECHNOLOGICAL CONSTRAINT. THEN OUR WALRASIAN THEORY APPLIES. IN PARTICULAR, A LINDAHL EQUILIBRIUM IS NOW A WALRASIAN EQUILIBRIUM, AND IS CLEARLY PARETO EFFICIENT AND INDIVIDUALLY RATIONAL.

SPECIFICALLY: THERE ARE $n+1$ GOODS x_1, \dots, x_n, y , WITH PRICES p_1, \dots, p_n, p_y . AN EQUILIBRIUM IS A $(p_1^*, \dots, p_n^*, p_y^*)$ AND A $(x_1^*, \dots, x_n^*, y^*; Q_1^*, \dots, Q_n^*, z^*)$ THAT SATISFY

\uparrow y_1^*, \dots, y_n^*

(a) (a1) $\forall i: x_i^* \leq Q_i^*$, w/ EQUALITY IF $p_i^* > 0$.
 (a2) $z^* + \sum y_i^* \leq \sum y_i^0$, w/ EQUALITY IF $p_y^* > 0$.

(b) $(Q_1^*, \dots, Q_n^*, z^*)$ MAXIMIZES $\Pi(Q_1, \dots, Q_n, z) := \sum p_i^* Q_i - p_y^* z$
 SUBJECT TO $Q_i = f_i(z), \forall i$. [NOTE: SAME $f(\cdot), \forall i$]

(c) $\forall i: (x_i^*, y_i^*)$ MAXIMIZES $u^i(x_i, y_i)$ s.t. $x_i, y_i \geq 0$
 AND TO $p_i^* x_i + p_y^* y \leq p_y^* y_i^0 + Q_i \pi^*$.

NOTE THAT REVENUE IS $\sum p_i Q_i = (\sum p_i) Q = (\sum p_i) f(z)$;
 THE PRICE RECEIVED FOR A "UNIT OF OUTPUT" IS $\sum p_i$.

ARROW THUS MODELS THE PUBLIC GOOD AS A COLLECTION OF n REGULAR, "PRIVATE" GOODS WITH TWO SPECIAL FEATURES:

- (1) THE GOODS ARE PRODUCED THROUGH A PRODUCTION PROCESS IN WHICH THEY ARE ALL JOINT PRODUCTS, SO IT MUST BE THE CASE THAT $x_1 = x_2 = \dots = x_n$;
- (2) EACH PERSON CARES ONLY ABOUT "HIS" GOOD — i.e., PERSON i 'S UTILITY DOES NOT DEPEND ON x_j FOR ANY $j \neq i$.

THIS IS A STRAIGHTFORWARD WALRASIAN MARKET MODEL, AND THE WALRASIAN THEOREMS THEREFORE APPLY: THERE IS AN EQUILIBRIUM, AND ANY EQUILIBRIUM IS PARETO OPTIMAL.

BUT ARROW'S MODEL ALSO MAKES IT CLEAR THAT THE WALRASIAN MODEL'S PRICE-TAKING ASSUMPTION IS UNREALISTIC HERE: FOR EACH OF THE DISTINCT GOODS X_i , THERE IS ONLY ONE PERSON ON THE DEMAND SIDE OF THE MARKET. THE ONLY PERSON WHO CARES ABOUT THE GOOD X_i IS PERSON i . IT'S CLEARLY UNREALISTIC TO ASSUME THAT ANY OF THE PARTICIPANTS WILL TAKE THEIR OWN PRICE (OR LINDAHL COST SHARE) AS GIVEN. INDEED, THIS WAS EXACTLY ARROW'S MOTIVATION FOR MODELING THINGS THIS WAY — TO CLARIFY THIS POINT.