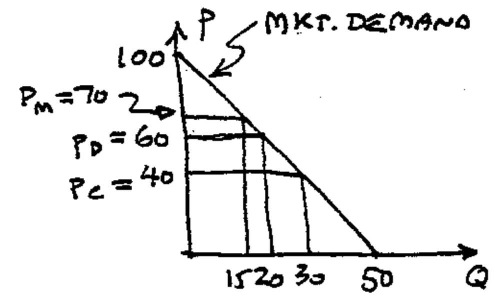
Duopoly Example

(Cournot Analysis)



THE MARKET SITUATION:

$$p = 100 - 2Q, \quad Q = q_1 + q_2$$

$$c_i(q_i) = 40q_i, \quad i = 1,2.$$

EACH FIRM TAKES OTHER'S QUANTITY CHOICE AS "GIVEN," UNAFFECTED BY OWN CHOICE.

PRODUCT IS HOMOGENEOUS.

FIRM 1'S PROBLEM:

$$p = 100 - 2(q_1 + q_2)$$
$$= [100 - 2q_2] - 2q_1$$

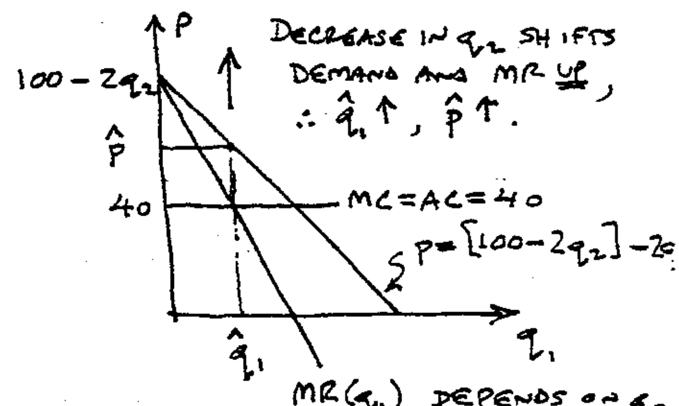
$$\therefore MR(q_1) = [100 - 2q_2] - 4q_1$$

$MR = MC$: $100 - 2q_2 - 4q_1 = 40$

i.e., $4q_1 + 2q_2 = 60$

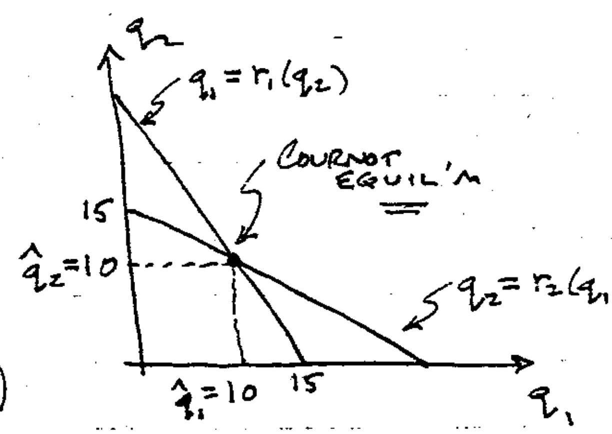
i.e., $\boxed{q_1 = 15 - \frac{1}{2}q_2}$

FIRM 1'S "REACTION FUNCTION" OR "REACTION CURVE."



FIRM 2: $2q_1 + 4q_2 = 60$

i.e., $\boxed{q_2 = 15 - \frac{1}{2}q_1}$



(THEY'RE THE SAME BECAUSE FIRMS' COSTS ARE THE SAME — AND BECAUSE $p = 100 - 2(q_1 + q_2)$ [GOOD IS HOMOGENEOUS].)

CAN SOLVE THE TWO EQUATIONS SIMULTANEOUSLY, OBTAINING

$$q_1 = 10, \quad q_2 = 10. \quad \therefore Q = 20, \quad p = 60.$$

[PUT BACK INTO INDIVIDUAL FIRM'S PROBLEM TO CHECK THAT THIS IS RIGHT.]

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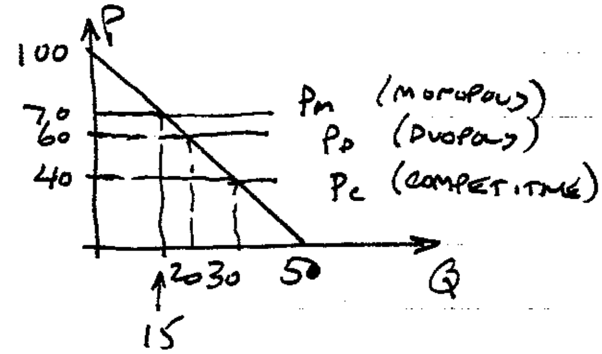
(Cournot Analysis)

$P = 100 - 2Q, \quad Q = q_1 + q_2$

$C_i(q_i) = 40q_i, \; i=1,2.$



$$\pi_1(q_1, q_2) = Pq_1 - C_1(q_1)$$
$$= [100 - 2(q_1+q_2)]q_1 - 40q_1$$
$$= 100q_1 - 2q_1^2 - 2q_2q_1 - 40q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 4q_1 - 2q_2 - 40 = 60 - 4q_1 - 2q_2$$
$$= 0 \Leftrightarrow 4q_1 = 60 - 2q_2; \text{ i.e., } q_1 = 15 - \tfrac{1}{2}q_2.$$

SIMILARLY, $\frac{\partial \pi_2}{\partial q_2} = 0 \Leftrightarrow q_2 = 15 - \frac{1}{2}q_1$.

$\therefore$ EQUILIBRIUM IS $q_1 = q_2 = 10, \quad Q = 20, \quad P = \$60;$

$\pi_1 = \pi_2 = Pq_i - C(q_i) = (\$60)(10) - (\$40)(10) = \$200.$

IF A MONOPOLY (CARTEL; COLLUSION):

$MR = 100 - 4Q; \quad MR(Q) = MC(Q): \; 100 - 4Q = 40;$

i.e., $60 = 4Q$; i.e., $\underline{Q = 15}$

$Q = 15, \; P = \$70, \; \pi = \$450.$

NOTE THAT $\$450 > \$200 + \$200$.

IF COMPETITIVE: $P = MC = \$40; \; Q = 30; \; \pi = 0.$

Monopoly in the Example

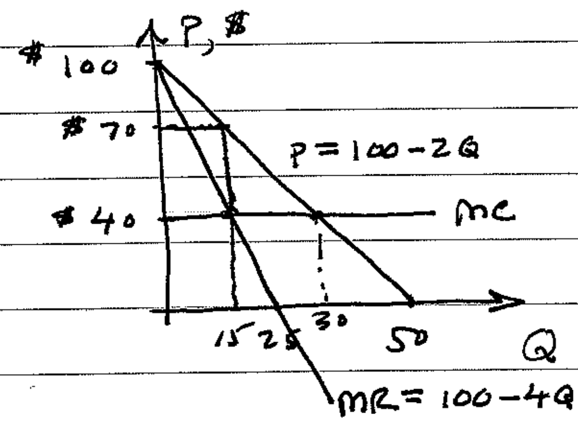
$$P = 100 - 2Q$$

$$R(Q) = P \cdot Q = (100 - 2Q)Q = 100Q - 2Q^2$$

$$R'(Q) = 100 - 4Q = MR$$

$$C(Q) = 40Q$$

$$MC = 40$$



$MR = MC$: $100 - 4Q = 40$

$\therefore \; 4Q = 60$

$Q = 15$, $P = \$70$

$R = \$1050$, $C = \$600$

$\pi = \$450 = PS$

$CS = \frac{1}{2}(\$30)(15) = \$225$

Total Surplus =

$= CS + PS$

$= \$225 + \$450$

$= \$675$.

Competitive Outcome

$P = MC = \$40$, $Q = 30$

$R = \$1200$, $C = \$1200$

$\pi = 0$, $PS = 0$

$CS = \frac{1}{2}(\$60)(30) = \$900$

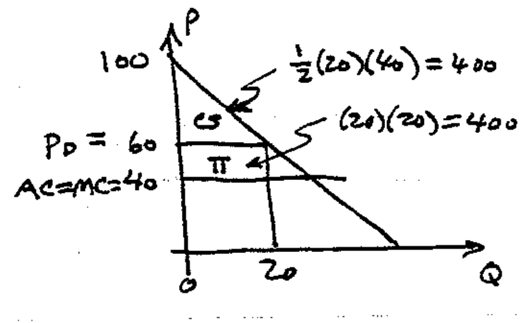
Total Surplus =

$= CS + PS$

$= \$900 + 0 = \$900$

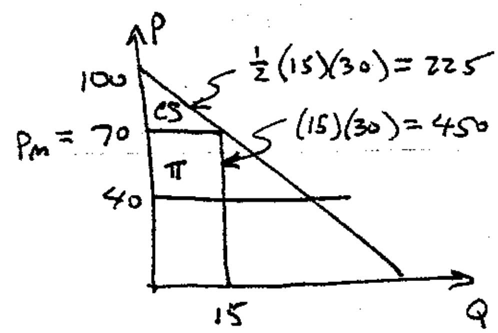
Duopoly:

Consumer surplus = 400
Producer surplus = 400
Total surplus = 800



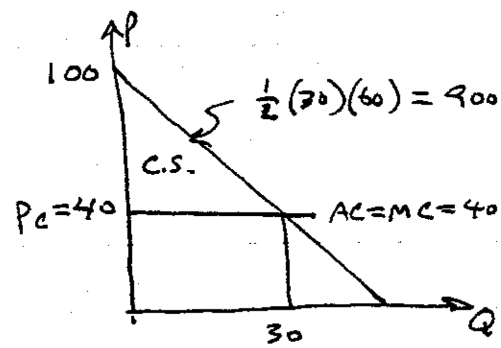
Monopoly:

Consumer surplus = 225
Producer surplus = 450
Total surplus = 675



Competitive:

Consumer surplus = 900
Producer surplus = 0
Total surplus = 900



ASSUME THE DEMAND CURVE WAS IN PER-CAPITA TERMS (ALSO THE MC — i.e., ALL QUANTITIES ARE PER-CAPITA). THEN A TYPICAL UTILITY FUNCTION IS

$$u(x,y) = y - x^2 + 100x$$

$$MRS = 100 - 2x \text{ — i.e., } p = 100 - 2x. \quad (*)$$

$u(p) = -px - x^2 + 100x$, WHERE x, p SATISFY $(*)$.

<u>DUOPOLY:</u> $x = 20$, $px = \$1200$;

$$u = -1200 - (20)^2 + (100)(20)$$
$$= -1200 - 400 + 2000 = \underline{400}.$$

$$u + \pi_1 + \pi_2 =$$
$$= 400 + 200 + 200 = \underline{\underline{800}}$$

<u>MONOPOLY:</u> $x = 15$, $px = \$1050$;

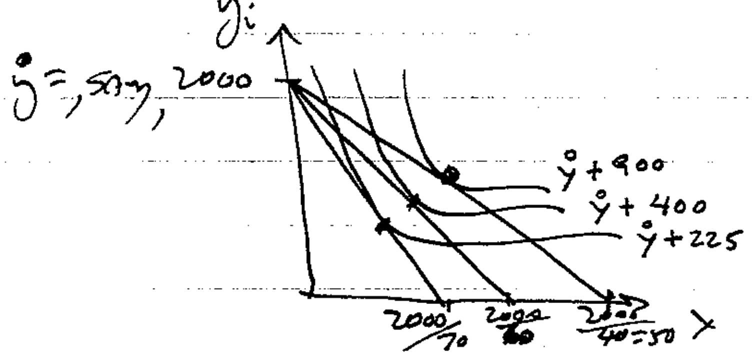
$$u = -1050 - (15)^2 + (100)(15)$$
$$= -1050 - 225 + 1500 = \underline{225}.$$

$$u + \pi = 225 + 450 = \underline{\underline{675}}$$

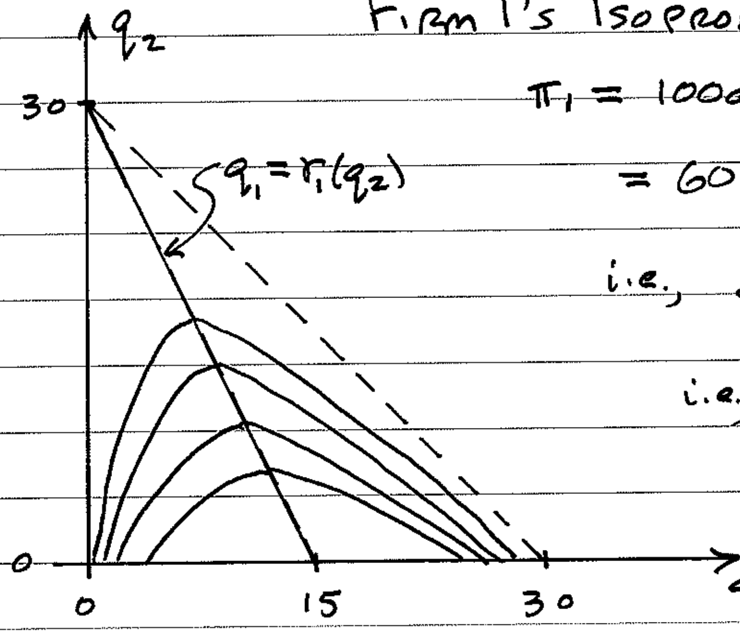
<u>COMPETITIVE:</u> $x = 30$, $px = \$1200$;

$$u = -1200 - (30)^2 + (100)(30)$$
$$= -1200 - 900 + 3000 = \underline{900}.$$

$$u + \pi = 900 + 0 = \underline{\underline{900}}$$



FIRM 1'S ISOPROFIT CURVES:



$$\pi_1 = 100q_1 - 2q_2q_1 - 2q_1^2 - 40q_1$$

$$= 60q_1 - 2q_2q_1 - 2q_1^2$$

i.e., $2q_2q_1 = 60q_1 - 2q_1^2 - \pi_1$

i.e., $\boxed{q_2 = 30 - q_1 - \frac{\pi_1}{2q_1}}$

FOR ANY GIVEN VALUE OF q_2, THE BEST CHOICE FOR q_1 (i.e, THE ONE ON THE BEST ISOPROFIT CURVE AVAILABLE) IS $q_1 = r_1(q_2)$.

PUTTING THE TWO FIRMS' ISOPROFIT MAPS TOGETHER: