MODELING UNCERTAINTY AND INFORMATION

$s \in S$: STATES OF THE WORLD (SAMPLE SPACE; ELEMENTARY EVENTS)

- ONE AND ONLY ONE WILL OCCUR.
- WE DON'T KNOW TODAY (WHEN MAKING DECISION) WHICH ONE IT WILL BE.

S CAN BE FINITE, INFINITE, A CONTINUUM.
(WE WILL CONSIDER ONLY FINITE SETS S)

A DECISION-MAKER'S "BELIEF" ABOUT S IS REPRESENTED BY A PROBABILITY MEASURE ON S. (WE WILL NOT NEED TO EXPLICITLY CONSIDER PROBABILITIES.)

$(x_s)_{s \in S} \in \mathbb{R}_+^{CS} = (\mathbb{R}_+^C)^S$: A PLAN/CONTINGENT PLAN/STRATEGY.

NOTE THAT $(x_s)_S$ IS JUST A FUNCTION FROM S TO $\mathbb{R}_+^C$ — WE COULD WRITE IT $x(s)$ INSTEAD OF x_s. THIS MAKES IT CLEAR THAT A PLAN $(x_s)_S$ IS A RANDOM VARIABLE (OR OFTEN CALLED A RANDOM VECTOR), $x: S \to \mathbb{R}_+^C$.

TEMPORAL RESOLUTION OF UNCERTAINTY

← BECOMING INFORMED; OBTAINING INFORMATION

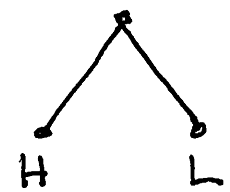
MODEL VIA

(1) A SET $\mathcal{T} = \{0, 1, \ldots, T\}$ OF DATES, AND

(2) A TREE STRUCTURE OR PARTITION OF S.

↑ GEOMETRIC ↑ ALGEBRAIC

EXAMPLE 1:



$t=0$

$t=1$

$S = \{H, L\}$

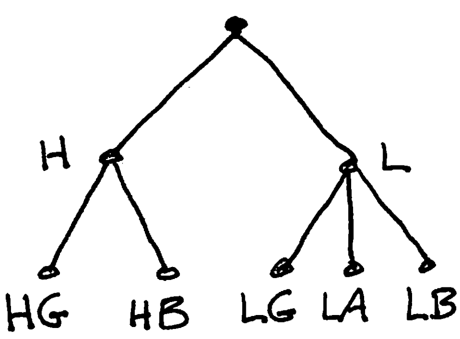
$S = \{H, L\}$, $\mathcal{T} = \{0, 1\}$

PARTITIONS:

$\mathcal{S}_0 = \{S\}$

$\mathcal{S}_1 = \{\{H\}, \{L\}\}$

EXAMPLE 2:



$t=0$

$t=1$

$t=2$

$S = \{HG, HB, LG, LA, LB\}$, $\mathcal{T} = \{0, 1, 2\}$

$\mathcal{S}_0 = \{S\}$

"H" "L"

$\mathcal{S}_1 = \{H, L\} = \{\{HG, HB\}, \{LG, LA, LB\}\}$

$\mathcal{S}_2 = \{\{HG\}, \{HB\}, \{LG\}, \{LA\}, \{LB\}\}$

$\mathcal{S}_0$ IS COARSEST

$\mathcal{S}_2$ IS FINEST

$t' > t \Rightarrow \mathcal{S}_{t'}$ IS A REFINEMENT OF $\mathcal{S}_t$, i.e.,

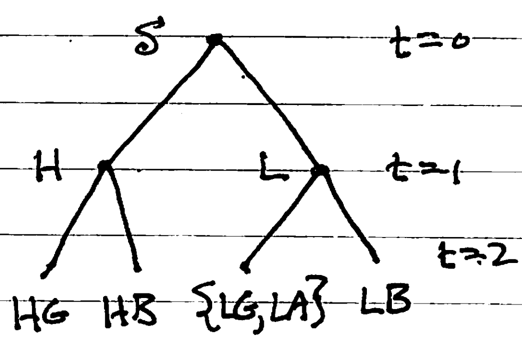
$$\forall E' \in \mathcal{S}_{t'} : \exists E \in \mathcal{S}_t : E' \subseteq E.$$

FOR EXAMPLE, WE CAN'T HAVE $\mathcal{S}_2$ INCLUDE THE EVENT $\{HG, LG\} = E'$ IF HG AND LG ARE IN __DIFFERENT__ EVENTS IN $\mathcal{S}_1$.

ASYMMETRIC INFORMATION:

THE TERM "ASYMMETRIC INFORMATION" MEANS PEOPLE HAVE DIFFERENT INFORMATION. FOR EXAMPLE, SUPPOSE ONE PERSON'S INFORMATION IS AS IN EXAMPLE 2 AND ANOTHER PERSON'S IS AS IN THE FOLLOWING EXAMPLE.

EXAMPLE 3:



$\mathcal{S}_0 = \{S\}$

$\mathcal{S}_1 = \{H, L\} = \{\{HG, HB\}, \{LG, LA, LB\}\}$

$\mathcal{S}_2 = \{\{HG\}\{HB\}, \{LG, LA\}, \{LB\}\}$

WE WOULD SAY THAT THIS PERSON HAS LESS INFORMATION AT $t=2$ THAN THE PERSON IN EXAMPLE 2, BECAUSE THE PARTITION $\mathcal{S}_2$ IN EXAMPLE 2 (LET'S DENOTE IT BY $\mathcal{S}_2'$) IS STRICTLY FINER THAN THE $\mathcal{S}_2$ IN THIS EXAMPLE.