Credit Market Only

The consumer can borrow or lend — e.g., sell or buy a bond. The return next period, or dividend, on each unit is $1+r$ in each state:

$$d = \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} \begin{matrix} \leftarrow \text{if } s=H \\ \leftarrow \text{if } s=L \end{matrix}$$

Let y denote units purchased today, at price q.

Individual's Market Behavior:

$$\max_{y, x_0, x_H, x_L} u(x_0, x_H, x_L) \text{ s.t. } \begin{aligned} x_0 + qy &\leq \mathring{x}_0 & \lambda_0 \\ x_H &\leq \mathring{x}_H + (1+r)y & \lambda_H \\ x_L &\leq \mathring{x}_L + (1+r)y & \lambda_L \end{aligned}$$

FOMC: (interior)

y: $0 = q\lambda_0 - (1+r)(\lambda_H + \lambda_L)$ i.e., $\frac{q}{1+r} = \frac{\lambda_H + \lambda_L}{\lambda_0}$

x_0: $u_0 = \lambda_0$

x_H: $u_H = \lambda_H$

x_L: $u_L = \lambda_L$

Combining the FOMC:

$$\frac{u_H + u_L}{u_0} = \frac{q}{1+r}$$

i.e. $MRS_H + MRS_L = \frac{q}{1+r}$

If $q = 1$: $MRS_H + MRS_L = \frac{1}{1+r}$.

Each unit costs \$1 ; y is amount saved today.

Henceforth let $q = 1$.

IF (y, x_0, x_H, x_L) SATISFIES ALL THREE CONSTRAINTS AS EQUATIONS, THEN WE HAVE

$x_s = \mathring{x}_s + (1+r)y, \ \forall s$

$$y = \mathring{x}_0 - x_0 \quad \text{(SAVING)}$$

$$x_H = \mathring{x}_H + (1+r)(\mathring{x}_0 - x_0) \qquad = 15 + (1+r)(15 - x_0)$$

$$x_L = \mathring{x}_L + (1+r)(\mathring{x}_0 - x_0) \qquad = 15 + (1+r)(15 - x_0).$$

IN OUR EXAMPLE

THE H AND L EQUATIONS CAN BE WRITTEN

$$x_0 + \frac{1}{1+r} x_H = \mathring{x}_0 + \frac{1}{1+r} \mathring{x}_H$$

$$x_0 + \frac{1}{1+r} x_L = \mathring{x}_0 + \frac{1}{1+r} \mathring{x}_L \ .$$

SO WE CAN WRITE THE MAXIMIZATION PROBLEM AS

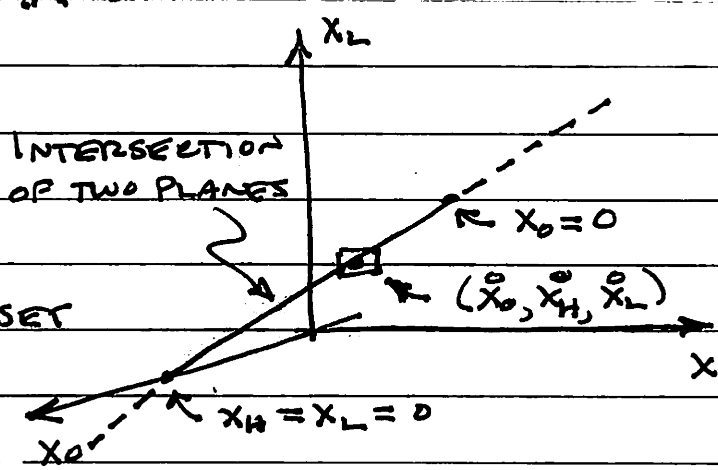
$$\max_{\underbrace{x_0, x_H, x_L}_{\in \mathbb{R}^3_+}} u(x_0, x_H, x_L) \ \text{s.t.} \ x_0 + \frac{1}{1+r} x_H = \mathring{x}_0 + \frac{1}{1+r} \mathring{x}_H$$

$$\text{AND} \ x_0 + \frac{1}{1+r} x_L = \mathring{x}_0 + \frac{1}{1+r} \mathring{x}_L$$

<u>TWO</u> CONSTRAINTS (TWO PLANES IN $\mathbb{R}^3$).

A

THE BUDGET CONSTRAINT IS NOT A PLANE IN $\mathbb{R}^3$ BUT THE INTERSECTION OF TWO PLANES IN $\mathbb{R}^3$ — i.e., A <u>LINE</u> IN $\mathbb{R}^3$:



MOREOVER, THIS LINE IS NOT GENERALLY A SUBSET OF THE A-D BUDGET CONSTRAINT (A PLANE).

MARKET EQUILIBRIUM: (w/ CREDIT MARKET ONLY!)

$$MRS^i_H + MRS^i_L = \frac{1}{1+r}, \quad i = A, B.$$

$$\therefore MRS^A_H + MRS^A_L = MRS^B_H + MRS^B_L \qquad (*)$$

NOTE: ALTHOUGH (*) IS IMPLIED BY THE PARETO CONDITION $MRS^A_s = MRS^B_s$ $(s = H, L)$, IT DOES NOT IMPLY THE PARETO CONDITION — IT CAN HOLD WITHOUT THE PARETO CONDITION BEING SATISFIED.

$\therefore$ THE CREDIT MARKET (ONLY) EQUILIBRIUM WILL NOT GENERALLY BY PARETO EFFICIENT.

IN OUR EXAMPLE:

$$\left(2 - \frac{1}{10}x^A_H\right) + \left(1 - \frac{1}{20}x^A_L\right) = \left(1 - \frac{1}{15}x^B_H\right) + \left(1 - \frac{1}{30}x^B_L\right) \qquad (1)$$

WE HAVE $x^A_H = \mathring{x}^A_H + (1+r)(\mathring{x}^A_0 - x^A_0) = 15 + (1+r)(15 - x^A_0)$;

ALSO $x^A_L = 15 + (1+r)(15 - x^A_0)$, BECAUSE $\mathring{x}^A_H = \mathring{x}^A_L = 15$ IN THIS EXAMPLE.

LET'S DENOTE x^A_H AND x^A_L, WHICH ARE THE SAME, BY x^A_1.

SIMILARLY, $\mathring{x}^B_H = \mathring{x}^B_L = 15$; LET'S DENOTE x^B_H AND x^B_L, WHICH WILL HAVE TO BE THE SAME, BY x^B_1.

NOW EQUATION (1) IS

$$3 - \frac{3}{20}x^A_1 = 2 - \frac{3}{30}x^B_1 .$$

BUT WE ALSO HAVE $x^A_1 + x^B_1 = 30$; i.e., $x^B_1 = 30 - x^A_1$.

SO NOW EQUATION (1) BECOMES

$$3 - \frac{3}{20} x_1^A = 2 - \frac{3}{30}(30 - x_1^A)$$

$$= 2 - 3 + \frac{1}{10} x_1^A = -1 + \frac{1}{10} x_1^A .$$

i.e., $\frac{5}{20} x_1^A = 4$; i.e., $x_1^A = 16$, $x_1^B = 14$.

THUS, WE HAVE $x_H^A = x_L^A = 16$ AND $x_H^B = x_L^B = 14$.

THESE ARE NOT THE CONSUMPTION LEVELS AT THE PARETO ALLOCATIONS WE OBTAINED EARLIER.

THUS,

→ THE CREDIT MARKET ALONE DOES NOT YIELD (IN OUR EXAMPLE) A PARETO EFFICIENT OUTCOME.

WE CAN ALSO SEE THE PARETO EFFICIENCY FAILURE BY CHECKING THE MRS VALUES:

$$MRS_H^A = 2 - \frac{16}{10} = \frac{2}{5} \qquad MRS_L^A = 1 - \frac{16}{20} = \frac{1}{5}$$

$$MRS_H^B = 1 - \frac{14}{15} = \frac{1}{15} \qquad MRS_L^B = 1 - \frac{14}{30} = \frac{8}{15}$$

WHICH ARE NOT THE MRS VALUES AT THE PARETO ALLOCATIONS.

WHAT IS THE EQUILIBRIUM INTEREST RATE?

$$\frac{1}{1+r} = MRS_H^i + MRS_L^i = \frac{3}{5}, \quad \text{so } 1+r = \frac{5}{3},$$

AND $r = \frac{2}{3} = 66\frac{2}{3}\%$.

HOW DOES THIS COMPARE WITH THE IMPLICIT INTEREST RATE AT THE PARETO ALLOCATIONS?

At the Pareto allocations we obtained

$$MRS^A_H = MRS^B_H = \frac{1}{5} \text{ and } MRS^A_L = MRS^B_L = \frac{2}{5},$$

So the implicit interest rate satisfies

$$\frac{1}{1+r} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} \text{ and } r = \frac{2}{3},$$

the same as the equilibrium interest rate in the credit market. That is just a coincidence! It's not typically true that the equilibrium interest rate with (only!) a credit market is consistent with Pareto efficiency.

Also note that we don't generally have $x^i_H = x^i_L$ as we do here. What we have, in general, is $x^i_s = \mathring{x}^i_s + (1+r)y^i$ for each $s \in S$ — i.e.,

$$x^i_s - \mathring{x}^i_s = (1+r)y^i;$$

the increment to $\mathring{x}^i_s$ provided by purchasing y^i units of the credit instrument is the same across states: We obtained $x^i_H = x^i_L$ because $\mathring{x}^i_H = \mathring{x}^i_L$.

For example, if $\mathring{x}^i_H = 25$ and $\mathring{x}^i_L = 10$, then

$$x^i_H = 25 + (1+r)y^i$$
$$x^i_L = 10 + (1+r)y^i.$$