THE ARROW-DEBREU CONTINGENT CLAIMS MODEL:
AN INTERPRETATION OF THE WALRASIAN GE MODEL

IF S=1 IT'S OUR INTERTEMPORAL MODEL

	THE BASIC GE MODEL	THE ARROW-DEBREU MODEL
BUNDLES, CONSUMPTION PLANS	$x \in \mathbb{R}^{\ell}_{+}$	$(x_0, (x_s)_{s\in S}) \in \mathbb{R}^{C}_{+} \times \mathbb{R}^{SC}_{+} = \mathbb{R}^{(1+S)C}_{+}$
PREFERENCE	$\succsim$ on $\mathbb{R}^{\ell}_{+}$ or $u: \mathbb{R}^{\ell}_{+} \to \mathbb{R}$	$\succsim$ on $\mathbb{R}^{(1+S)C}_{+}$ or $u: \mathbb{R}^{(1+S)C}_{+} \to \mathbb{R}$
PRICES	$p \in \mathbb{R}^{\ell}_{+}$	$(p_0, (p_s)_{s\in S}) \in \mathbb{R}^{(1+S)C}_{+}$
BUDGET CONSTRAINT	$p \cdot x \leqq p \cdot \mathring{x}$	$p_0 \cdot x_0 + \sum_{s\in S} \underbrace{p_s \cdot x_s}_{= \sum_{c\in C} p_{sc} x_{sc}} \leqq p_0 \cdot \mathring{x}_0 + \sum_{s\in S} p_s \cdot \mathring{x}_s$

FOR THE MOST PART WE'RE GOING TO KEEP THINGS SIMPLE BY SETTING $C = 1$:

IN OUR EXTENDED EXAMPLE: $S = \{H, L\}$, SO $1+S = 3$

PLAN: $(x_0, (x_s)_S) \in \mathbb{R}^{1+S}_{+}$

UTILITY FUNCTION: $u: \mathbb{R}^{1+S}_{+} \to \mathbb{R}$

BUDGET CONSTRAINT: $p_0 x_0 + \sum_{s\in S} p_s x_s \leqq p_0 \mathring{x}_s + \sum_{s\in S} p_s \mathring{x}_s$

NOT DOT PRODUCTS

(★) → THE DEFINITIONS AND THEOREMS FROM OUR BASIC GE MODEL STILL APPLY HERE.

INTERPRETATION:

X_{sc} IS THE QUANTITY OF COMMODITY c ONE IS CONTRACTING TO HAVE DELIVERED TO HIM (AT A GIVEN TIME IN THE FUTURE) IF AND ONLY IF STATE s OCCURS. DELIVERY IS "CONTINGENT" ON STATE s OCCURRING, AND $(X_{sc})_{s \in S}$ IS THEREFORE A RANDOM VECTOR, FOR ANY $c \in C$.

P_{sc} IS THE PRICE ONE MUST PAY TODAY FOR A UNIT CLAIM ON sc (i.e., TO OBTAIN $X_{sc} = 1$), AND THE PRICE RECEIVED TODAY FOR A PROMISE TO DELIVER ONE UNIT "TOMORROW."

ALL TRANSACTIONS (AND DOLLAR PAYMENTS) TAKE PLACE TODAY. ALL DELIVERIES X_{sc} TAKE PLACE TOMORROW IF AND ONLY IF THE DESIGNATED STATE s OCCURS. THE QUANTITIES (DELIVERIES) X_{sc} ARE CONTINGENT; THE PRICES P_{sc} (AND PAYMENTS) ARE NOT CONTINGENT.

THE <u>CONTINGENT CLAIMS MODEL</u> IS SIMPLY A REINTERPRETATION OF OUR STANDARD WALRASIAN MODEL:

EACH CONSUMER'S UMP IS THE SAME AS BEFORE:

$$\max u(x) \text{ s.t. } p_0 \cdot x_0 + \sum_{s\in S}\sum_{c\in C} p_{sc} x_{sc} \leq p_0 \cdot \mathring{x}_0 + \sum_{s\in S}\sum_{c\in C} p_{sc} \mathring{x}_{sc}.$$

NOTE THAT THERE IS STILL JUST A <u>SINGLE</u> CONSTRAINT.

AN EQUILIBRIUM IS THE SAME AS BEFORE:

A PAIR $(p, x) \in (\mathbb{R}^C_+ \times \mathbb{R}^{S\times C}_+) \times (\mathbb{R}^{nC}_+ \times \mathbb{R}^{n(S\times C)}_+)$

THAT SATISFIES THE (U-MAX) AND (M-CLR) CONDITIONS. THE CONDITIONS ARE UNCHANGED.

NOTE:

$$p = (p_0, (p_s)_{s\in S}) \in \mathbb{R}^C_+ \times \mathbb{R}^{S\times C}_+,$$

WHERE $p_0 = (p_{0c})_{c\in C} \in \mathbb{R}^C_+$

$p_s = (p_{sc})_{c\in C} \in \mathbb{R}^C_+, \ \forall s \in S$

$$x = ((x^i_0, (x^i_s)_{s\in S}))_{i\in N} \in \mathbb{R}^{nC}_+ \times \mathbb{R}^{n(S\times C)}_+$$

WHERE $x^i_0 = (x^i_{0c})_{c\in C} \in \mathbb{R}^C_+$

$x^i_s = (x^i_{sc})_{c\in C} \in \mathbb{R}^C_+, \ \forall s \in S.$

ONLY THE NOTATION IS CHANGED, FROM $k = 1, \ldots, \ell$ TO $c \in C$ AT $t=0$ AND $sc \in S\times C$ AT $t=1$.

THE PRICE OF AN EVENT-CONTINGENT CLAIM:

... IS JUST THE SUM OF THE A-D PRICES OF THE CORRESPONDING <u>STATE</u>-CONTINGENT CLAIMS:

$$P_E = \sum_{s \in E} P_s \quad \text{FOR AN EVENT } E \subseteq S$$

$$\left(\text{i.e., } P_{Ec} = \sum_{s \in E} p_{sc} \text{ FOR A GOOD } c \in C\right).$$

FOR EXAMPLE: (W/ ONLY ONE GOOD)

$S = \{H, M, L\}$; LET $E = \{H, M\}$ AND $E' = S = \{H, M, L\}$.

THEN $P_E = P_H + P_M$ AND $P_{E'} = P_H + P_M + P_L$.

THE PRICE OF A CONTRACT FOR DELIVERY IF $s \in E = \{H, M\}$

THE PRICE OF A CONTRACT FOR <u>CERTAIN</u> DELIVERY, i.e., IF $s \in E' = S$.

i.e., THESE ARE THE PRICES FOR DELIVERY OF <u>EACH UNIT</u> IN THE RESPECTIVE EVENTS.

BUT ARE THERE REALLY MARKETS LIKE THIS — i.e., A MARKET (FOR EACH $s \in S$ AND EACH $c \in C$) IN WHICH ONE CAN BUY OR SELL A CONTRACT FOR DELIVERY OF x_{sc} UNITS OF GOOD c IF AND ONLY IF STATE s OCCURS? AT A PRICE p_{sc} TO BE PAID TODAY? GENERALLY THERE AREN'T.

THE ARROW-DEBREU MODEL ASSUMES THERE ARE — NOT AS A MODEL OF REALITY, BUT AS A BENCHMARK AGAINST WHICH TO EVALUATE ALTERNATIVE MARKET STRUCTURES, AND AS A CONCEPTUAL DEVICE TO HELP US UNDERSTAND AND MODEL MARKETS (FOR COMMODITIES AND FOR SECURITIES) WHEN THERE IS UNCERTAINTY.